Robust Localization in Multihop Wireless Sensor Networks

Kegen Yu and Y. Jay Guo
Wireless Technologies Laboratory
CSIRO ICT Centre, Australia

ABSTRACT

In this paper a hybrid localization scheme for multihop wireless sensor networks is presented. At first a relatively dense group of nodes is selected as a base. Next, the multidimensional scaling (MDS) method is applied to localize the group of nodes. Then, the robust quads (RQ) method is employed to localize other nodes, following which we make use of the robust triangle and radio range (RTRR) approach to perform the localization task. The RQ and the RTRR methods are used alternately until no more nodes can be localized by the two approaches. Simulation results demonstrate that the proposed hybrid localization algorithm performs well in terms of both accuracy and success rate of localization.

INDEX TERMS—hybrid localization, multidimensional scaling, robust quads, radio range, sensor networks.

I. INTRODUCTION

Recently, a wide range of algorithms for localization in wireless sensor networks have been reported. Depending on the applications, the algorithms can be either centralized [1, 2] or distributed [3–5]. In the centralized scheme, the algorithm is executed in a processing center or a location server. This centralized scheme requires infrastructure that challenges the ad-hoc nature of the network, and the involved long range communications could be time-consuming and energy inefficient. On the other hand, distributed localization avoids the problems of the centralized scheme by sharing the computational burden among the nodes so that the communications especially the long range ones are reduced greatly.

In some circumstances a certain number of nodes are equipped with a global positioning system (GPS) receiver so that their locations are known. Those nodes are often referred to as anchors and their locations are used to determine the positions of other ordinary sensor nodes which do not have a GPS receiver. Either distance or connectivity information can be used to locate the ordinary nodes [6–16]. In other circumstances, there is no anchor in the network so that no GPS measurements are available at any node. To localize sensor nodes in such anchor-free sensor networks, a number of algorithms have been proposed in the literature [3, 17–23].

In this paper, we investigate GPS/anchor free node localization in WSNs. We propose a hybrid localization scheme that combines the multidimensional scaling (MDS) and robust quads (RQ) methods [20, 22], and makes use of the robust triangle and radio range (RTRR) technique as well. The purpose is to exploit the advantages of the MDS and the RQ method, and to compensate for their drawbacks. More specifically, it aims to localize more nodes than the RQ method, and to achieve higher accuracy than the MDS method. A group of nodes are first localized by using the MDS approach to form a base. Then, the other nodes are localized starting from the base and by using the RQ and the RTRR approaches alternately until no more nodes can be localized by the two methods.

The remainder of the paper is organized as follows. Section II describes the hybrid node localization method. Section III shows some simulation results, and Section IV concludes the paper.

II. HYBRID NODE LOCALIZATION

It is assumed that the distance measurements between each pair of nodes that are within the radio range are available. That is,

\[ \hat{d}_{ij} = d_{ij} + \epsilon_{ij}, \quad 1 \leq i, j \leq N, \quad i \neq j, \]

where there are \( N \) nodes in the network, \( \epsilon_{ij} \) is the distance estimation error, and

\[ d_{ij} = \sqrt{(x_{i} - x_{j})^2 + (y_{i} - y_{j})^2} \]

where \((x_{i}, y_{i})\) and \((x_{j}, y_{j})\) are the coordinates of nodes \( i \) and \( j \), respectively.

In the first phase, we need to find out a group of nodes in the network that can communicate with each other and the number of the nodes in the group should be as large as possible. The reason we only choose nodes in the radio range of each other is that the MDS algorithm performs well when using one-hop distance estimates. On the other hand, the MDS algorithm may produces large errors when a few multihop/shortest path distance estimates are used. Then, we choose three nodes in the group to set up a local coordinate system. Without loss of generality, let the three nodes be nodes 1, 2, and 3, respectively. One of the three (say node 1) is set at the origin \( (x_{1}, y_{1}) = (0, 0) \) and the other (say node 2) is on the positive x-axis \( (x_{2}, y_{2}) = (d_{12}, 0) \) where \( d_{12} \) is the distance between nodes 1 and 2. The third node (say node 3) has a
positive y-axis coordinate, i.e.

\[ x_3 = \frac{x_2^2 + d_{13}^2 - d_{23}^2}{2x_2}, \]
\[ y_3 = \sqrt{d_{13}^2 - x_3^2}. \]

Note that since the distance estimates instead of the true values are available, we can only obtain the estimates of the coordinates \(x_2, x_3\) and \(y_3\).

Let us apply the MDS algorithm to localize the group of \(n\) selected nodes whose position matrix is defined as \([24]\)

\[ P = \begin{bmatrix} x_1 & x_2 & \ldots & x_n \\ y_1 & y_2 & \ldots & y_n \end{bmatrix}^T, \tag{3} \]

and the MDS algorithm can be described as follows.

1) Construct the squared-distance matrix \(D\) whose elements are

\[ [D]_{i,j} = \hat{d}_{ij}^2, \tag{4} \]

where \(\hat{d}_{ij}\) is the estimate of distance \(d_{ij}\).

2) Compute the matrix \(B = -\frac{1}{2}CDC\), where \(D\) is the squared-distance matrix whose elements are defined by (4) and

\[ C = I - \frac{1}{n}ee^T, \tag{5} \]

where \(I\) is the identity matrix of dimensions of \(n \times n\) and \(e\) is a column vector of length \(n\), whose elements are all ones.

3) Decompose \(B\) as \(B = U\Lambda U^T\), where \(\Lambda = \text{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_n\}\) is a diagonal matrix of eigenvalues of \(B\), and \(U\) is the matrix whose column vectors are the corresponding eigenvectors.

4) Normalize the eigenvectors to unit eigenvectors, sort the eigenvalues in a non-increasing order to form \(\Lambda'\), and rearrange the unit vector matrix accordingly, resulting in \(U'\).

5) The positions of the \(n\) nodes are estimated as the first two column components of the matrix \(V = U'\sqrt{\Lambda'}\), i.e.

\[ \hat{P} = V(1:1, n:2). \]

Since the position estimates \(\hat{P}\) are produced purely based on distance measurements, the resulting coordinate system can be rather different from the established one. Therefore, we need to transform the results to the established coordinate system defined by nodes 1, 2 and 3. The transformation can be performed such as by using the method proposed in [25].

The position estimation accuracy may be enhanced by using the optimization techniques [26–31]. This requires a node in the group has the computational power to run an iterative minimization algorithm.

After the \(n\) selected nodes are localized, we make use of the RQ approach to localize the other nodes in the network. An unknown node can be localized if it and three other nodes can form a robust quad [22]. A quad is robust if all the four triangles between the nodes are robust. The RQ method defines a parameter of robustness \(\gamma\) which may be set to \(3\sigma\), where \(\sigma\) is the standard deviation of the distance error. When the smallest angle and the shortest edge of a triangle satisfy

\[ d_s \sin^2 \theta_s > \gamma, \]

the triangle is defined to be robust. Fig. 1 shows an example of a robust quad, while Fig. 2 shows an example of a non-robust quad. When the positions of the three nodes of a robust quad are known, the fourth node can be robustly localized.

Assume that the known coordinates of the three nodes are \((\hat{x}_i, \hat{y}_i)\), \((\hat{x}_j, \hat{y}_j)\), and \((\hat{x}_k, \hat{y}_k)\), respectively. The simplest way to determine the position of the fourth node, \((x_\ell, y_\ell)\), is to use two known nodes (say \(i\) and \(j\)) to produce two solutions of the unknown position, and then to use the third known node (say \(k\)) to get rid of one solution. More specifically, defining

\[ x_{ij} = x_i - x_j, \quad y_{ij} = y_i - y_j, \]
\[ g = 0.5(d_{1\ell}^2 - d_{2\ell}^2 + x_{ij}^2 + y_{ij}^2 - (x_i^2 + y_i^2)), \]

the two solutions are given by

\[
\begin{cases}
 y_{(1)} = y_{(2)} = g/y_{ji}, & x_i = x_j, y_i \neq y_j; \\
 x_{(1)} = x_i + \delta_x, & \delta_x = \sqrt{d_{1\ell}^2 - (y_{(1)} - y_i)^2}, \\
 x_{(2)} = x_i - \delta_x, \\
 y_{(1)} = x_{(2)} = g/x_{ji}, & x_i \neq x_j, y_i = y_j; \\
 y_{(1)} = y_i + \delta_y, & \delta_y = \sqrt{d_{1\ell}^2 - (x_{(1)} - x_i)^2}, \\
 y_{(2)} = y_i - \delta_y.
\end{cases}
\]
or
\[
\begin{align*}
\frac{y_{\ell}^{(1)}}{y_{\ell}} &= \frac{b/a + c}{b/a - c}, \quad x_i \neq x_j, \quad y_i \neq y_j, \\
a &= 1 + (y_{ji}/x_{ji})^2, \quad b = y_i + (g/x_{ji} - x_i)y_{ji}/x_{ji}, \\
c &= \sqrt{(b/a)^2 - c_1/a}, \quad c_1 = y_i^2 - d_{\ell}^2 + (g/x_{ji} - x_i)^2, \\
y_{\ell}^{(1)} &= b/a - c, \\
x_{\ell}^{(1)} &= (g - y_{ji}y_{\ell}^{(1)})/x_{ji}, \\
x_{\ell}^{(2)} &= (g - y_{ji}y_{\ell}^{(2)})/x_{ji}.
\end{align*}
\]

(10)

Apparently, choosing two out of the three known nodes results in three different combinations so that three different solutions would be produced. Then the position estimate can be set to be the average of the solutions. In the event that a node is among more than one robust quad, the position estimate may be further improved by averaging all the available results. Note that to reduce the effect of the abnormal errors, before averaging, we check each position estimate by comparing the estimated distance (based on the position estimate) with the original distance measurement. If the distance difference is larger than a threshold, we drop the corresponding position estimate. This robust quad based localization is performed iteratively throughout the network until all connected robust quads are localized.

In the event that there are nodes whose locations can not be determined by the RQ method due to the absence of robust quads, we propose the robust triangle plus radio range (RTRR) approach to localize the remainder of the nodes. This RTRR technique can be described with the aid of Fig. 3. Node a is the neighbor of both nodes b and c which have been localized, but not necessary in connection to each other. Node d has been localized and it is the neighbor of node c, but not the neighbor of a. In the case that the triangle abc is robust, we can obtain two solutions for the position estimate of node a, that is, a' and a''. Node e is the neighbor of a and a', while node d is the neighbor of a'', but not the neighbor of a. We can eliminate the solution, a'', since a' is not the neighbor of node d, but the neighbor of e, whereas a'' is the neighbor of d, but not the neighbor of e. Therefore, we can make use of the neighbors which have been already localized of node a, b and c to help localize node a. More such information would result in more accurate judgement. Also, multiple robust triangles may produce multiple position estimates which can be further processed such as by averaging to obtain more accurate position information. The RTRR method is run over each un-localized node and then the RQ approach is applied if there are still node to be localized. The RQ and the RTRR algorithms are performed alternately until no more nodes can be localized by the two methods.

It is non-trivial to point out that once a node is localized, its position estimate can be refined by making use of the position information and distance measurements of all its localized neighbors, and by running a minimization algorithm, provided that the node has the computational power to do so.

### III. Simulation Results

In this section we evaluate the proposed hybrid localization method through simulation. We consider a network in which the nodes are randomly deployed in a squared area of dimensions of one kilometer by one kilometer. The radio/communication range of each node is 300 meters. The distance measurement error is modeled as a Gaussian distributed random variable of zero mean and a standard deviation equal to 5% of the true distance. Instead of directly using (6) to judge the robustness of a triangle, we slightly modify it as follows:

- The minimum angle of the triangle is greater than 30°.
- The shortest edge of the triangle is greater than 25% of the radio range.

It is found that the above criterion produces better results than using (6) with γ around 5σ.

Since the absolute positions of the nodes can not be determined in anchor-free sensor networks, we can only produce relative positions of the nodes. For this reason, we make use of the distance errors as the accuracy measure [25]. Specifically, in the case of N nodes, the accuracy is calculated according to

\[
\sqrt{\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} (d(i, j) - \hat{d}(i, j))^2}
\]

(11)

where \(d(i, j)\) is the true distance between nodes \(i\) and \(j\), and \(\hat{d}(i, j)\) is the distance between the estimated positions of nodes \(i\) and \(j\).

Fig. 4 shows the typical localization results based on the proposed algorithm under four different node configurations, each of which has 120 nodes. The dots denote the true node configuration under a predefined coordinate system and the circles denote the configuration of the estimated node locations. The two locations of the same node are connected by a solid line. The accuracy of the proposed algorithm is 6.68, 7.23, 8.82, and 8.30 meters, respectively, under the four different node configurations. The accuracy in the last configuration looks much worse than that in the third configuration. However, the computed accuracy of the last configuration is even better than that of the third one. The reason is that the distance (solid line) between the assigned location and
the estimated one of the same node does not represent the variations of the relative positions of the nodes in the last configuration. The accuracy of the MDS algorithm [20] is 18.4, 17.07, 19.88, and 31.26 meters, respectively. In the MDS algorithm, the distance between two nodes that are out of the radio range is computed based on the shortest path distance. Clearly, the proposed algorithm outperforms the MDS method considerably. The number of nodes that have been successfully localized by the proposed algorithm is 119, 120, 118, and 118, respectively. The un-localized nodes result from that the condition(s) of localizing a node is (are) not satisfied. For instance, a node can not communicate with at least two other nodes so that its location can not be determined based on distance measurements. Another example is that a node can not form a robust quad with other three nodes so that its location can not be determined if only using the RQ method. On the other hand, the number of nodes that have been localized by the RQ method, including the nodes that form the base, is 108, 108, 114, and 23, respectively. Compared to the RQ method, the proposed algorithm can successfully localize more nodes. This is because of the fact that even when the RQ algorithm stops due to the lack of robust quads, the RTRR approach can resume the localization process. This attributes to that the conditions of the RTRR approach can usually be satisfied more easily than those of the RQ method. Note that all nodes can be localized by using the MDS method provided that any pair of nodes is connected via one-hop or multihop path.

Fig. 5 shows the cumulative distribution of the absolute values of the distance errors when there are 150 and 90 nodes in the network, respectively. The accuracy of both the proposed algorithm and the MDS method is plotted for comparison. For each number (150 and 90) of nodes, 100 different node deployment realizations are examined. As expected, more nodes in the network produce better position estimation accuracy. This is due to the fact that a higher node density results in more neighboring nodes such that more information can be employed to localize the unknown nodes. It can be seen that the accuracy of the proposed algorithm is considerably higher than that of the MDS method.

Table I shows the average success rate of localization of the proposed method and the RQ method. The success rate of localization is defined as the ratio of the number of nodes that have been localized to the number of all nodes in the network. Clearly, the success rate of the proposed algorithm is higher than that of the RQ method, especially when the node density is relatively low.

IV. CONCLUSIONS

In this paper we investigated node localization in multihop WSNs. A hybrid localization scheme was proposed and numerically evaluated. The proposed approach combines the MDS, the RQ, and the RTRR approach to achieve both good

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3When a node is within the radio range of another node, its location is constrained to a circular region with a radius equal to the radio range.
accuracy and success rate of localization. The effectiveness of the proposed algorithm has been demonstrated by simulation.

V. ACKNOWLEDGEMENT

The authors would like to thank their colleague, Carol Wilson, for useful discussions.

REFERENCES


TABLE I
SUCCESS RATE OF LOCALIZATION OF THE PROPOSED ALGORITHM AND THE RQ METHOD WHEN THERE ARE EITHER 150 OR 90 NODES IN THE NETWORK.

<table>
<thead>
<tr>
<th></th>
<th>Proposed Method</th>
<th>RQ Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>150 nodes</td>
<td>99.47 (%)</td>
<td>95.24</td>
</tr>
<tr>
<td>90 nodes</td>
<td>96.48 (%)</td>
<td>53.13</td>
</tr>
</tbody>
</table>

Fig. 5. The cumulative distribution probability of the distance error of the proposed algorithm and the MDS method when there are 150 and 90 nodes in the network, respectively. 'proposed (150)' denotes results for the proposed algorithm with 150 nodes in the network.