Matchmaking using Fuzzy Analytical Hierarchy Process, Compatibility Measure and Stable Matching for Online Matrimony in India

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ABSTRACT

Use of online matrimony for matchmaking is rapidly growing in India. One of the major difficulties faced by the users of such websites is the long time taken to realize the matches. We propose an integrated approach to matchmaking in e-matrimony environment using fuzzy analytical hierarchy process considering multiple criteria involved in the process. The objective of the study is to enable users to search their partners effectively and efficiently and narrow down to the desired matches so that the chances of matchmaking through online portal are maximized. A compatibility index, that is match metric, is developed that enhances the probability of matchmaking with reduction in lead time. Furthermore, Gale-Shapley stable matching algorithm is used to help customers obtain the desired shortlist of profiles with a suggested stable match. An illustrated hypothetical example of matchmaking using an Indian matrimony portal is presented, and the use of the proposed methodology is demonstrated.

KEY WORDS: online matrimony; fuzzy AHP; compatibility index; stable matching; matchmaking

1. INTRODUCTION

Most of the people in this world, if not all, would have some thoughts/expectations concerning life partners. Given these expectations, finding a perfect partner is not an easy job. In Indian culture, marriage means a lifetime commitment, and it is expected that the person is wholly committed and dedicated to his or her life partner. In the past, marriages in India would traditionally be arranged typically by the parents who, with the help of relatives and other persons, would finalize the match for their ward. In today’s world, however, with changes in society, and family people being located at different places because of jobs etc., more and more people take the decision themselves about their life partners taking their families in confidence. In this process, they get good support from the newspaper advertisements and online matrimonial websites in lieu of some charges. US-based EmPower Research has published a report that states that the online matrimony industry in India may reach 21 million registrations with revenues of $63 million by 2010–2011. According to this research, the popularity of online matrimony services in India has grown mainly because of convenience and cost-savings. If e-marital sites can extend the trust gained by identifying a spouse/partner for a subscriber to many other, lifelong services, the e-matrimony industry may attain very strong, Web loyalty among the Indian population.

As far as matrimonial agencies and newspaper advertisements are concerned, Indians are comfortable with that, but still, the online matrimonial websites have a meagre 1.5% of penetration in total market share of matrimonial business. Shaadi.com and Bharatmatrimony.com are two major players today in the world of online matrimonial business. Gordon and Gupta (2003) showed that growth acceleration of the services in the 1990s was mostly because of the fast growth in communication services, financial services, business services (IT) and community services.

These matchmaking Web portals provide extensive search facility to every customer, but actual time to realize the match with one of the prospective profiles for interaction is longer because of few reasons like certain good profiles getting curtailed because of a crisp key-based search process, and usually a profile, liked by customer, might dislike the customer’s profile in response. Therefore, further contacting them is a waste of time, that is lead time of whole matchmaking process is prolonged.

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We propose an integrated fuzzy analytical hierarchy process (FAHP), compatibility measure and stable matching-based technique to address these issues. Different criteria are fuzzily defined in terms of satisfaction level. Typical AHP procedure is employed for calculating profile scores. Compatibility measure is developed to reduce the efforts of the customer in searching and sorting the profile from mutual point of view. Gale-Shapley algorithm suggests a stable match that specifies the higher probability of getting positive response from the contacted profile.

Section 2 explains the findings from the literature concerning the online matrimony business in India and the techniques used in helping the process of matchmaking. Section 3 is devoted towards current profile search process in Web portals and the proposed search process with AHP, compatibility index (CI) and stable matching algorithm. Section 4 describes an illustrated example of matchmaking through Internet portal in India. The last section concludes the findings and highlights the possible improvements in the matchmaking process.

2. ONLINE MATRIMONY BUSINESS IN INDIA

Arranged marriages have been the tradition in Indian society for centuries. Even today, an overwhelming majority of Indians have their marriages planned by their parents and other respected family members. Although most marriages are arranged, some couples in India are opting for love marriage in urban areas. An arranged marriage is effectively the result of a wide search by both the girl’s family and the boy’s family. Banerjee et al. (2009) studied the role played by caste, education and other social and economic attributes in arranged marriages among middle-class Indians. Even in modern India, there is a strong preference towards within-caste marriage. Parents generally discuss expectations with their son/daughter before starting searching for a match. These expectations are shared with relatives and family friends who often bring in valuable suggestions. Indian matrimonial sites attempt to provide databases that can be queried to find matches using similar attributes.

Acceptance of online matchmaking as a culturally legitimate approach to mate selection and consumer spending on these services continues to rise. The online dating industry is clearly growing in importance as an industry not only because it is becoming a popular and efficient way for busy singles to find love interests but also because of the rich and valuable information that it provides for potentially reducing the rising divorce rate and other types of unsuccessful relationships. Therefore, it is crucial that online matching services purporting to use empirically validated matching systems actually do validate their systems and release their findings to the public (Chang et al., 2006).

Given the varied and complex matchmaking process followed in India that is based on diverse aspects such as caste, religion, economic standard, job status, age, height, complexion, family background etc., online matrimony services have become quite successful. Pathak (2005) stated an importance of the role of computers in future matchmaking at the hands of marriage bureaus or when independently used by the interested parties. Online matrimony is an organized Web-based matrimonial service facilitating wishful young men and women to find their suitable life partners. In India, organized marriage services business is worth 10 billion. Online matrimony caters to people spread across the globe to find their suitable partner living in a remote place by matching his or her specific interests and requirements. Dugar et al. (2010) indicated that affirmative action policies in India, which seek to enhance the income of the lower-caste population, are not likely to produce any significant change in intercaste marriage.

To deal with the behaviour of the clients of a marriage bureau, Vaillant (2004) provided rough estimates of the lengths of the personal partner searches of individuals who seek remarriage using a marriage bureau. Few studies related to persons looking for partnership based on multiple attributes in other domains are carried out. Chang et al. (2006) proposed a fuzzy multiple attribute decision-making method based on the fuzzy linguistic quantifier to select supply chain partners at different phases of product life cycle. Li and Murata (2009) proposed a novel method to match buyers and suppliers in B2B e-marketplace based on priority and multi-objective optimization. They analysed matchmaking in a stable bilateral market, where each buyer or supplier is matched with trade partners. Batabyal and DeAngelo (2008) studied matchmaking from the perspective of a matchmaker. They analysed the circumstances under which a matchmaker optimally accepts or rejects individual matching assignments. AHP for decision making uses objective mathematics to process the inescapably subjective and personal preferences of an individual or a group in making a decision. With AHP and its generalization, one constructs hierarchies, then makes judgments or performs measurements on pairs of elements with respect to a controlling element to derive ratio scales that are then synthesized throughout
the structure to select the best alternative (Saaty, 1980; Salo and Hamalainen, 1997; Triantaphyllou, 2001). Few review papers described, the application area in which AHP has been successfully used either via stand-alone tool or in combination with other techniques (Vaidya and Kumar, 2006; Sipahi and Timor, 2010). Hajeeh and Lairi (2009) employed the AHP because of the multiplicity of objectives and surveyed women from different ethnic, religious and residential backgrounds to explore the most preferred criteria. Thomaidis and Mavrakis (2006) applied AHP method with geopolitical, economic and technical criteria to define the most preferred route of the transcontinental gas pipeline that branches in SE Europe to transport Caspian gas further into the targeted markets of Europe. Korkmaz (2008) proposed AHP and two-sided matching-based decision support system to assist detailers in the context of assignment of military personnel to positions. Matchmaking business has recently caught great attention in business schools too. Harvard Business Review published eHarmony (Prod. no: 709424-HCB-ENG), a case, on such business with a successful differentiation strategy. It offers a unique product, which combines an extensive relationship communication system. Vi et al. (2010) proposed a mathematical approach to optimizing marriage by allocating spouses in such a way that would reduce the likelihood of divorce or separation. Hitsch et al. (2010) used a novel data set obtained from an online dating service to draw inferences on mate preferences and to investigate the role played by these preferences in determining match outcomes and sorting patterns.

Various researchers have been looking into economic, empirical modeling aspects of this matchmaking. We have not come across any work that focuses on the matchmaking process in e-matrimony environment.

3. PROCESS OF ONLINE MATCHMAKING

In India, marriage is viewed as ‘Strategic decision’ in the life of humankind because of its non-repetitive nature (in normal conditions) and having long-term effect on the life of an individual and the following generations. It is not an easy decision to make, and to lay the proper foundation for any marriage, careful and studied steps that require wisdom and thorough planning are necessary. Therefore, matchmaking is indeed a multi-criterion decision-making process. Through any website portal, a customer provides his or her details as input and expects prospective profiles as a result of search he or she makes using keywords like height range, complexion, age difference, eating habits, profession, educational qualification etc. Each user provides his or her authentic information through online registration. Most of the online matrimony portals have person verification requirements as a part of their registration process that can be briefed in Figure 1.

Although the website provides extensive search facility to the member of the portal, the actual time to realize the match with one of the prospective profiles for interaction is longer because of the following reasons:

- It is a keyword-based search (0 or 1 type).
- All factors may not be considered by a customer.
- Few good profiles might get curtained because of a specific (crisp) key-based search process.
- Many times person liked by customer might be disliked by prospective partner; therefore, proposing to such profiles is waste of time.

To overcome the problems faced in the current search and proposal process, we propose a new integrated method as depicted in Figure 2. This integrated approach based on FAHP, CI and stable matching algorithm-based process is elaborated stepwise in the following:

**Step 1 Defining attributes in terms of fuzzy sets**

User would like to describe his or her own requirements (attributes) like complexion and salary in terms of more fair, around 50000 etc. Such linguistic expressions need to be converted into fuzzy sets as described in the example. The classical AHP also performs pairwise comparison of candidates attribute-wise. It is cumbersome here to ask and gather each user’s needs attribute-wise. Belot and Francesconi (2010) found that both women and men value physical attributes, such as age and weight, and those choices are assortative along age, height and education.

Let us assume that $m$ men and $n$ women are registered to the website with their details related to $t$ attributes (e.g. education/occupation, wealth, personality). Let $D_1, D_2, D_3, ..., D_t$ represent the customer’s attribute-wise requirements in the form of fuzzy sets as $\{(D_1, \mu_{D_1}), (D_2, \mu_{D_2}), ..., (D_n, \mu_{D_n})\}$, where $\mu_{D_t}\forall D=1, 2, 3, ..., t$ represents the membership values of the customer’s specifications on attribute $t$.

**Step 2 Pairwise comparison and calculating score of each profile**

A typical AHP procedure is conducted, which consists of pairwise comparison of attributes at respective hierarchy to calculate weights for respective criteria. It leads to calculate the aggregated profile score where
weight of each attribute is multiplied by its respective fuzzy membership function and then added.

Ranking, using this profile score, will not suffice the purpose of online matchmaking because of the following reasons:

- Matchmaking is a two-sided utility maximization process.
- Every member of the website does rate his or her prospective spouse’s profile.
- It may happen that a person rates the prospective spouse as the best match; on the contrary, that prospective spouse may rate the person as the worst match and vice versa.

**Step 3 Two-sided matching and sorting**

Marriage matching on Internet-based system is a typical two-way preferential matching problem. Authors of experimental, empirical, theoretical and computational studies of two-sided matching markets have recognized the importance of correlated preferences. Celik and Knoblauch (2007) developed a general method for the study of the effect of correlation of preferences on the outcomes generated by two-sided matching mechanisms. Preferences of both sides are important in two-sided matching. Every member of the website has his or her own attribute requirements and priorities to choose a life partner. Simultaneously, a male member’s profile is rated by respective prospective partner member’s set factors and subsequent priorities.

Here, we propose a CI-based method of two-sided matching to emphasize a both-ways matching intent. There are two objectives in this composition. First, the sum of these two scores in a pair has to be maximized. Second, the difference of the opinion about each other in terms of aggregate score should be at minimum. To meet these objectives, a CI is defined as follows:

\[
CI = \frac{1}{3} \left( A_{m,n} + B_{n,m} \right) \left[ 1 + 0 \left( \frac{5}{\sqrt{(A_{m,n} - B_{n,m})^2 + 1}} \right) \right]
\]

where

- \( A_{m,n} \) = aggregated score of \( n \)th female profile as per \( m \)th male’s preferences;
- \( B_{n,m} \) = aggregated score of \( m \)th male profile as per \( n \)th female’s preferences.

Final sorting in descending order has to be carried out based on this index CI.

**Step 4 Stable matching based on Gale-Shapley algorithm**

Stable matching problems consist of a set of agents, each of whom submits a preference list ranking a subset of the other agents in order of preference. The problem is to form a matching \( M \) of the agents such that no two agents would prefer each other to their assignment in \( M \) (Abraham, 2003). The stable matching problem is to find such a match between pair of agents so that neither of the pair finds any other match better than the allocated match.

An instance of the stable marriage problem consists of \( N \) men, \( N \) women and each person’s preference list. A preference list is a totally ordered list including all members of the opposite sex depending on his or her preference. For a matching \( M \) between men and women, a pair of a man \( m \) and a woman \( w \) is called a blocking pair if both prefer each other to their current partners. A matching with no blocking pair is called stable. Gale and Shapley showed that every instance admits at least one stable matching and proposed a polynomial time algorithm to find one, which is known as the Gale-Shapley algorithm (Gale & Shapley, 1962). Teo et al. (2001) studied the matching mechanism used by the Ministry of Education in the placement of primary six students in secondary schools and discussed why the current method has limited success in accommodating the preferences of the students and the specific needs of the schools (in terms of the ‘mix’ of admitted students). They showed that stable matching mechanisms are more appropriate in this matching market and explained why the strategic behaviour of the students...
need not be a major concern. The final outcome of the process is a stable match profile and the list of most relevant profiles. The Gale-Shapley framework is not just a seminal theoretical benchmark in the economic analysis of marriage markets; it also provides an approximation to the match outcomes from a realistic search and matching model that resembles the environment of an online dating site (Adachi, 2003).

An instance of a stable marriage problem may be specified by the male and female ranking matrices. Relative to arbitrary but fixed numberings of men and women, these are defined by \( mr(i, k) = j \), if woman \( k \) is the \( j \)th choice of man \( i \); \( wr(i, k) = j \) if man \( k \) is the \( j \)th choice of woman \( i \). The problem of how to find a stable marriage maximizing total satisfaction was unsolved until Irving et al. (1987) used the egalitarian measure of optimality under which total satisfaction is maximized.

Suppose that for a given stable marriage instance, \( S = \{(m_1, w_1) \ldots (m_n, w_n)\} \), is a stable matching. They defined the value \( c(S) \) of \( S \) by

\[
c(S) = \sum^n_{i=1} mr(m_i, w_i) + \sum^n_{i=1} wr(w_i, m_i)
\]

and they said that a stable matching \( S \) is optimal if it has minimum possible value \( c(S) \). In real online system, there are \( m \) male and \( n \) female profiles as mentioned.

Figure 2. Search process with fuzzy analytical hierarchy process and stable matching.

Figure 3. Analytical hierarchy process.
Figure 4. Customer’s attribute-wise requirements in the form of fuzzy sets. Please see Table VI for abbreviations used.
earlier. Every customer will receive a suggested stable matched profile and also his or her own preference list based on ‘compatibility index’. This enhances the probability of getting a positive response from recipient person.

4. ILLUSTRATED EXAMPLE

To validate the proposed method, we chose an advanced search process from a Web portal known as Jeevansathi.com. In this illustration, we have assumed case of a man who is looking for a woman as prospective partner for marriage. The customer is interested in various profiles with following requirements that are arranged in hierarchical manner as shown in Figure 3.

Usually in India, partner search is based upon three broad criteria of prospective partner, viz. education/occupation, wealth, personality. An education criterion is related to qualification like Undergraduate, Masters or PhD etc. in diverse disciplines like arts, commerce, engineering and medicine etc. Occupation criteria means a person is an engineer, a doctor, a chartered accountant or a professor etc. A Wealth criterion consists of family status (low class, middle class, upper class), earnings (annual income) and location (semi-urban, urban, metro city). A personality criterion consists of age (years), height (cm), complexion (dark, whitish brown, brown, fair), diet (vegetarian, eggitarian, non-vegetarian) and body type (slim, athletic, heavy).

**Step 1 Defining attributes in terms of fuzzy sets**

The male customer who is looking for a spouse, i.e. a woman with various attributes with hierarchy as stated in Figure 3. Assume that the customer is looking for a ‘very fair’ girl in case of attribute type ‘complexion’. The customer can define the acceptability with respect to complexion as fuzzy set, for example 0.8. This means that his satisfaction level with that particular attribute is 80%. Similarly for every attribute under consideration, he defines his acceptability level, i.e. membership level is asked and fuzzily defined requirement is generated as shown in Figure 4.

**Step 2 Pairwise comparison and calculating aggregate score of each profile**

A classic AHP procedure is conducted which consists of a pairwise comparison of attributes at respective

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Table I. Pairwise comparison of education and occupation

<table>
<thead>
<tr>
<th></th>
<th>Education</th>
<th>Occupation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Education</td>
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<td>4</td>
</tr>
<tr>
<td>Occupation</td>
<td>1/4</td>
<td>1</td>
</tr>
</tbody>
</table>

Table II. Pairwise comparison of family status, earning and location

<table>
<thead>
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<th></th>
<th>Family status</th>
<th>Earning</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1/4</td>
<td>1/7</td>
</tr>
<tr>
<td>Earning</td>
<td>4</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>Location</td>
<td>7</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Table III. Pairwise comparison of height, age, complexion, diet and body type

<table>
<thead>
<tr>
<th></th>
<th>Height</th>
<th>Age</th>
<th>Complexion</th>
<th>Diet</th>
<th>Body type</th>
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</thead>
<tbody>
<tr>
<td>Height</td>
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<td>8</td>
<td>9</td>
<td>5</td>
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<tr>
<td>Age</td>
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<td>1</td>
<td>7</td>
<td>1/5</td>
<td>1/10</td>
</tr>
<tr>
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<td>1/7</td>
<td>1</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>Diet</td>
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<td>5</td>
<td>1/7</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Body type</td>
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<td>1/2</td>
<td>1/5</td>
<td>1</td>
</tr>
</tbody>
</table>

Table IV. Pairwise comparison of education, wealth and personality

<table>
<thead>
<tr>
<th></th>
<th>Education</th>
<th>Wealth</th>
<th>Personality</th>
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<tbody>
<tr>
<td>Education</td>
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<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Wealth</td>
<td>1/3</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>Personality</td>
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<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Table V. The weight of attributes

<table>
<thead>
<tr>
<th>Attribute</th>
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<th>Height</th>
<th>Occupation</th>
<th>Location</th>
<th>Body type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (%)</td>
<td>39.45</td>
<td>15.13</td>
<td>9.86</td>
<td>8.11</td>
<td>6.21</td>
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<tr>
<td>Rank</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Attribute</td>
<td>Diet</td>
<td>Complexion</td>
<td>Earning</td>
<td>Age</td>
<td>Family status</td>
</tr>
<tr>
<td>Weight (%)</td>
<td>5.87</td>
<td>5.36</td>
<td>4.46</td>
<td>4.42</td>
<td>1.13</td>
</tr>
<tr>
<td>Rank</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
</tbody>
</table>
hierarchy to calculate respective weight for each attribute (Tables I–IV). In this case, customer rates each attribute with respect to other attribute at the same level and under the same group. From the pairwise comparisons, we get relative weights of attributes as shown in Table V.

In our example, we consider 10 female member’s data with different attributes as shown in Table VI. There can be thousands of the profiles in the database of any website available. Let us calculate profile score for profile no. 2. This profile has attributes like 27 years old, 134 cm height, complexion as whitish brown, qualification as chartered accountant (CA), belongs to Rich class, annual income 22 lac, living in Metro, Eating habit as Vegetarian and body type is Athletic etc.

For every profile according to fuzzily defined requirements, membership value can be calculated. Comparing the membership values from respective fuzzy sets and attributes of the profile, we get various membership values as Education is CA and respective satisfaction level is 1; therefore, Education = 1. Likewise, remaining membership values are Occupation = 0.44, Family status = 0, Earning = 1, Location = 0.7, Height = 0, Age = 0.6, Complexion = 0.7, Diet = 1 and Body type = 0.9.

A profile score is calculated by multiplying this membership value with respective weight from Table V and aggregating for all attributes. For example, for education, the weight is 0.394.

Profile score \( (A_{1,2}) = 0.3945(1) + 0.0986(0.44) + 0.0113(0) + 0.0446(1) + 0.0811(0.7) + 0.1513(0) + 0.0442(0.6) + 0.0536(0.7) + 0.0587(1) + 0.0621(0.9) = 0.7179. \)

Thus, the profile score is 0.7179. Similarly, profile scores as per nomenclature described earlier are as shown in Tables VII and VIII. As mentioned earlier, in this mutual matching problem, we must consider what each woman is looking for. Every woman also has defined her requirements in terms of fuzzy sets and done pairwise comparison. Thus, profile score of this male customer who is looking for female partner can be calculated. To calculate the compatibility score, we use formula defined in step 3.

For example, in case of pair male1 and female5, i.e. \( A_{1,5} \) and \( B_{5,1} \).

\[
CI = 1/3 \times \left[ (0.8372 + 0.5839) + (0.8372 + 0.5839) + (0.8372 + 0.5839) \right] = 0.6963.
\]

Thus, CI based on male no.1’s preference with each female are listed in Table VIII. Similarly, for every possible matching pair, CI is calculated as recorded in Table IX.
Based on compatibility scores, profiles get shortlisted and ranked in descending order. Furthermore, the customer has liberty to re-rank the profiles after looking at each of the top 10 profiles. Subsequently, if the customer re-ranks the profiles, the preferences are taken as input for Gale-Shapley algorithm. Otherwise, profiles ranked based on CI are treated with respect to their rank. To demonstrate Gale-Shapley algorithm in this context, we assume that there are five male and five female profiles with their preferences as mentioned in the following table:

Gale-Shapley algorithm generates the following matches:

Male1 is paired with female5; male2 is paired with female3; male3 is paired with female4; male4 is paired with female2; and male5 is paired with female1.

This is an add-on facility for a customer, where he or she will receive a suggested stable matched profile. If they contact this suggested profile as well as a few profiles listed based on their CI, then the probability of getting positive response increases.

5. CONCLUSIONS

In today’s Internet era, services seeking efficiency is of paramount importance. The approach presented in this paper attempts to exploit current IT-enabled partner search for marriage through Web portals.

Salient features of this proposed method are as follows:

- Integrated way to quantify the online profiles with implicit needs.
- Two-phase short listing, i.e. FAHP and stable matching algorithm.
- Reducing customer’s effort to find their mate online according to their implicit needs (defined fuzzily).
- Enhancing the probability of getting positive response and matchmaking.

A sorting based on CI in descending order enhances the probability of matchmaking. It thus leads to reduction in lead time of waiting of the positive or negative reply from the opposite party. This operational viewpoint has been presented in this paper with
introduction of a CI. This index helps user to maximize their requirements while mutual matching.

Future studies might cover an enhancement in rating parameters. Few Web portals have incorporated matchmaking based on personality behaviour. More qualitative and quantitative factors can be included with involvement of ratings by parents and relatives. Such study will lead to multi-criteria group decision-making kind of problem.

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