UNKNOWN DEAD–ZONE COMPENSATION FOR NONLINEAR SYSTEMS USING ADAPTIVE $\mathcal{H}^\infty$ CONTROL METHOD

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Abstract: This paper deals with the adaptive dead–zone compensation strategy based on notion of $\mathcal{H}^\infty$ optimality. It is assumed that the dead–zone model can be divided into unknown parameters term and bounded disturbance term, an adaptive $\mathcal{H}^\infty$ control method is given. Proposed control strategy does not include the discontinuous function, therefore, it is effective for the practical applications. Moreover, in the closed–loop control system, the $\mathcal{L}_2$ gains from the disturbance to generalized outputs are made less than prescribed positive constants. The effectiveness of the proposed method is demonstrated by numerical simulations.

Keywords: Adaptive Control, Dead–zone Compensation, $\mathcal{H}^\infty$ control

1. INTRODUCTION

Some industrial motion control system has nonsmooth nonlinear characteristics, such as dead–zone, saturation, backlash, and so forth. These nonsmooth nonlinearities are often encountered in actuators. At the macro level, many factors affect nonlinearities such as lubrication, velocity, temperature and even the history of motion. For high or ultra precision positioning of nano–order scale production system, considering the nonsmooth nonlinearities is very important. Because, these phenomena will cause the serious disadvantage on control performances for a precision positioning control. Therefore, the research of nonsmooth nonlinearities has been great interest to control researchers for a long time.

Especially, dead–zone characteristics are often encountered in various areas of mechatronics. Dead–zone is most important nonsmooth nonlinearities arisen in actuators, such as DC servo motors and hydraulic servo valves, etc. When the expected accuracy of the motion system is high, we have to compensate the dead–zone phenomena. In general, the dead–zone parameters are poorly known, and it is hard to measure the output of the dead–zone. Therefore, an adaptive control method which based on the dead–zone inverse was proposed (Tao and Kokotović 1995). A perfect asymptotical adaptive cancellation of an unknown dead–zone method was shown (Cho and Bai 1998), but, unfortunately, it is assumed that the output of the dead–zone is measurable. A fuzzy logic controller for the systems with dead–zone (Kim, Lee, and Chong 1994, Zhang and Feng 1997, Lewis Tim, Wang, and Li 1999) and neural networks precompensator (Selmic and Lewis 2000) were also proposed.

The aforementioned researches are based on a dead–zone inverse method. Recently, new piece–

wise description of dead–zones has been proposed (Wang, Su, and Hong 2004). This description is organized as the linear for the dead–zone input term and bounded disturbance function term. Using this description, an approach for adaptive control of nonlinear system with dead–zones is proposed without using the dead–zone inverse method (Wang, Su, and Hong 2004). This method ensured that all the closed–loop signals are bounded and the state vector converges to prescribed region which depend on the design parameter ε. Besides, this control input and estimation strategies contain the saturated function term which reflects the component for compensation of the bounded function. It has already been noticed that if the controller design parameter ε is chosen too small, the linear region of saturated function will be too thin, which cause a risk of exciting high–frequency fluctuations. Moreover, if we choose the controller design parameter ε as nearly 0, then the controller contains a discontinuous structure, which may cause chattering phenomena. It means that the control strategy may have a performance limitation due to the discontinuous structure. For the practical applications, it is not suitable that there exist a trade–off between the design parameter and trajectory–following requirements.

In this paper, we propose a new adaptive control method without using the dead–zone inverse method and saturated functions. Based on inverse optimal control strategy (Freeman and Kokotović 1996, Miyasato 1999), an adaptive $\mathcal{H}^\infty$ control method is given which controller can be designed without solving the Hamilton–Jacobi–Isaacs equation. We can also show that the bounded disturbance can be compensated by the control input without using the saturated functions. Moreover, we can conclude that the $\mathcal{L}_2$ gin from the bounded disturbance to tracking error is prescribed by given constant, that is, the $\mathcal{H}^\infty$ control performance is attained adaptively for generalized output. Besides, it is shown that the overall system is bounded. As a consequence of the analysis, we can design the closed–loop system which can consider the trade–off between control performance and control input power. Numerical simulations will be given to show the effectiveness of our proposed method.

2. SYSTEM DESCRIPTION AND CONTROL OBJECTIVE

The nonlinear dynamic system preceded by actuators with dead–zone can be written as

$$x^{(n)}(t)+\sum_{i=1}^{r} a_i Y_i\left(x(t),\dot{x}(t),\cdots,x^{(n-1)}(t)\right)=bw(t) \tag{1}$$

where Y_i are known continuous, linear or nonlinear functions, a_i are unknown but constant parameters, and b is unknown but constant. Without loss of generality, we assume $b > 0$.

The dead–zone with input $u(t)$ and output $w(t)$, as shown in Fig. 1, is described by

$$\begin{aligned} w(t) &= \begin{cases} m_r(u(t)-b_r) & \text{for } u(t)\geq b_r \\ 0 & \text{for } b_l < u(t) < b_r \\ m_l(u(t)-b_l) & \text{for } u(t)\leq b_l \end{cases} \\ &= D(u(t)). \end{aligned} \tag{2}$$

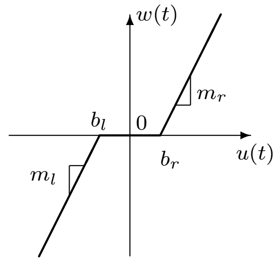

Fig. 1. Dead–zone model

We make the assumptions that the dead–zone has the following properties:

(A1) The dead–zone output $w(t)$ is not available for measurement.
(A2) The dead–zone slopes in positive and negative region are same, i. e., $m_r = m_l = m$.
(A3) The dead–zone parameters b_r, b_l, and m are unknown, but their signs are known: $b_r > 0$, $b_l < 0$, $m > 0$.
(A4) The dead–zone parameters b_r, b_l, and m are bounded, i. e., each lower and upper bounds are known and it can be described as follows:

$$b_r \in [b_{r\min}, b_{r\max}], \quad b_l \in [b_{l\min}, b_{l\max}],$$
$$m \in [m_{\min}, m_{\max}].$$

(A1) $\sim$ (A4) are satisfied in real plants. Then, model (2) can be rewritten as follows:

$$w(t) = D(u(t)) = mu(t) + d(u(t)) \tag{3}$$

where $d(u(t))$ can be described from (2) and (3) as

$$d(u(t)) = \begin{cases} -mb_r & \text{for } u(t)\geq b_r, \\ -mu(t) & \text{for } b_l < u(t) < b_r, \\ -mb_l & \text{for } u(t)\leq b_l. \end{cases} \tag{4}$$

From (A2) and (A4), we can evaluate $d(u(t))$ as follows:

$$|d(u(t))| \leq \rho \tag{5}$$

where ρ is upper–bound of $d(u(t))$, which can be chosen as

$$\rho = \max\{m_{\max} b_{r\max}, -m_{\max} b_{l\min}\}, \qquad (6)$$

where $b_{l\min}$ is negative values.

The control objective is to design a control strategy for $u(t)$ in (3) to let the plant state vector, $\boldsymbol{x}(t) = \left[x(t), \dot{x}(t), \cdots, x^{(n-1)}(t)\right]^{\mathrm{T}}$, follow a specified desired trajectory,

$$\boldsymbol{x}_d(t) = \left[x_d(t), \dot{x}_d(t), \cdots, x_d^{(n-1)}(t)\right]^{\mathrm{T}},$$

i. e., which satisfies as follows:

$$\lim_{t\to\infty} \boldsymbol{x}(t) = \boldsymbol{x}_d(t).$$

We make the following assumption for the desired trajectory.

(A5) The desired trajectory

$$\boldsymbol{x}(t) = [x(t), \dot{x}(t), \cdots, x^{(n-1)}(t)]^{\mathrm{T}}$$

is continuous and available. Besides

$$[\boldsymbol{x}_d^{\mathrm{T}}(t), x_d^{(n)}(t)]^{\mathrm{T}} \in \Omega_d \subset \mathbb{R}^{n+1}$$

with Ω_d being a compact set.

Due to the analysis of an adaptive control strategy, a filtered tracking error is defined as

$$s(t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} \tilde{x}(t), \quad \lambda > 0. \qquad (7)$$

(7) can be rewritten as follows:

$$s(t) = \Lambda^{\mathrm{T}} \tilde{x}(t),$$

with

$$\Lambda^{\mathrm{T}} = [\lambda^{n-1}, (n-1)\lambda^{n-2}, \cdots, 1],$$
$$\tilde{\boldsymbol{x}}(t) = \boldsymbol{x}(t) - \boldsymbol{x}_d(t).$$

Moreover, we define as

$$\Lambda_v^{\mathrm{T}} = [0, \lambda^{n-1}, (n-1)\lambda^{n-2}, \cdots, (n-1)\lambda],$$

then it follows:

$$\begin{aligned}\dot{s}(t) &= \Lambda_v^{\mathrm{T}} \boldsymbol{x}(t) + \tilde{x}^n(t),\\ &= \Lambda_v^{\mathrm{T}} \boldsymbol{x}(t) - \sum_{i=1}^{r} a_i Y_i(\boldsymbol{x}(t)) + bmu(t),\\ &\quad + bd(u(t)) - x_d^{(n)}(t). \end{aligned} \qquad (8)$$

In the controller design, system parameters a_i, b, and m which are in (8) are unknown and a robust adaptive control strategy should be considered.

To present the adaptive control strategy, we define an unknown parameter vector θ and a constant ϕ as follows:

$$\theta \equiv [a_1/bm, \cdots, a_r/bm]^{\mathrm{T}} \in \mathbb{R}^r, \quad \phi \equiv 1/bm.$$

Then, we can define the estimated errors as

$$\tilde{\theta} = \hat{\theta} - \theta, \quad \tilde{\phi} = \hat{\phi} - \phi,$$

where $\hat{\theta}$ and $\hat{\phi}$ are estimate values of θ and ϕ, respectively.

3. PREVIOUS RESULT

In this section, we shall introduce the previous research (Wang, Su, and Hong 2004) which is based on adaptive control without constructing the inverse of the dead–zone.

Based on the error equation (8), the following adaptive control strategy is given:

$$u(t) = -k_d s(t) + \hat{\phi} u_{fd}(t) + Y^{\mathrm{T}}(\boldsymbol{x})\hat{\theta} - k^* \operatorname{sat}\left(\frac{s(t)}{\epsilon}\right), \qquad (9)$$

$$\dot{\hat{\theta}}_i = -\gamma Y_i(\boldsymbol{x}) s_\epsilon, \quad \dot{\hat{\phi}}_i = -\eta u_{fd} s_\epsilon, \qquad (10)$$

where

$$\operatorname{sat}(s(t)/\epsilon) = \begin{cases} 1 & \text{for } s(t)/\epsilon \geq 1,\\ s(t)/\epsilon & \text{for } -1 < s(t)/\epsilon < 1,\\ -1 & \text{for } s(t)/\epsilon \leq -1, \end{cases} \qquad (11)$$

and

$$s_\epsilon = s(t) - \epsilon \operatorname{sat}\left(\frac{s(t)}{\epsilon}\right),$$
$$u_{fd}(t) = x_d^{(n)}(t) - \Lambda_v^{\mathrm{T}} \tilde{\boldsymbol{x}}(t),$$
$$Y \equiv [Y_1, \cdots, Y_r]^{\mathrm{T}} \in \mathbb{R}^r.$$

γ and η are positive constants and k^* is a control gain which satisfying

$$k^* \geq \rho/m_{\min},$$

where ρ is defined in (6).

It has already shown that the stability of the closed–loop system described by (1), (2), (9), and (10) is ensured. Besides, all the closed–loop signal are bounded and the state vector $\boldsymbol{x}(t)$ converges to $\Omega_\epsilon = \left\{\boldsymbol{x}(t) \,\middle|\, |\tilde{\boldsymbol{x}}_i| \leq 2^{i-1}\lambda^{i-n}\epsilon, i = 1, \cdots, n\right\}$ for $\forall\, t \geq t_0$.

Remark 1.

From the definition of Ω_ϵ, the convergence region of $\boldsymbol{x}(t)$ is depend on ϵ. The control input (9) and (10) contains the term $\operatorname{sat}(s(t)/\epsilon)$ which reflects the component for compensation of the bounded function $d(u(t))$. It has already been noticed that if ϵ is chosen too small, the linear region of function $\operatorname{sat}(s(t)/\epsilon)$ will be too thin, which cause a risk of exciting high–frequency fluctuations (Wang, Su, and Hong 2004). Besides, if we choose the design parameter ϵ as nearly 0, then the controller contains a discontinuous structure, which may cause chattering phenomena. It means that a control strategy (9) may have a performance limitation due to the discontinuous structure. Moreover, for the practical applications, it is not suitable that there exist a trade–off between the controller parameter ϵ and trajectory–following requirements.

In the next section, we will introduce a different adaptive control approach. Based on the inverse optimality method, an adaptive $\mathcal{H}^\infty$ control method without including the saturated function is proposed.

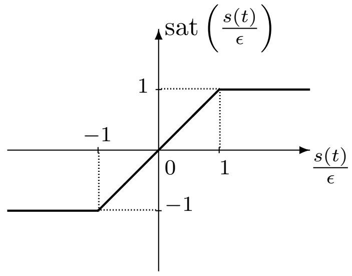

Fig. 2. Saturated function: sat $\left(\frac{s(t)}{\epsilon}\right)$

4. ADAPTIVE $\mathcal{H}^\infty$ CONTROL METHOD

To solve the problems of the previous researches, we have proposed another adaptive control method which can compensate the bounded function $d(u(t))$ (i. e., disturbance) without using a saturated function. According to this method (Freeman and Kokotović 1996, Miyasato 1999), the resulting control system is shown to be sub–optimal to the cost functionals which prescribe the $\mathcal{L}_2$–gains from the disturbance to the tracking error.

First of all, we give the virtual process as follows:

$$\dot{s}(t) = f(s(t)) + g_1\frac{d(u(t))}{m} + g_2 v(t),$$
$$f(s(t)) = -k_d s(t), g_1 = 1, g_2 = 1,$$

where k_d is positive constant and $v(t)$ is a new control input which will be given in the following. For the virtual system, we can derive positive functions h, δ satisfying the following Hamilton–Jacobi–Isaacs (HJI) equation and define the positive function $\tilde{V} = \frac{1}{2}s^2(t)$,

$$\frac{\partial\tilde{V}}{\partial s}f(s(t)) + \frac{1}{4}\left(\frac{g_1^2}{\gamma_\infty^2} - \frac{g_2^2}{\delta}\right)\left(\frac{\partial\tilde{V}}{\partial s}\right)^2 + hs^2(t) \le 0. \tag{12}$$

Substitute $\tilde{V}$ into (12), then we have

$$-k_d s^2(t) + \left(\frac{1}{\gamma_\infty^2} - \frac{1}{\delta}\right)\frac{s^2(t)}{4} + hs^2(t) \le 0. \tag{13}$$

By utilizing the aforementioned, the adaptive control method for (1) and (2) is constructed in the following way.

$$u(t) = -k_d s(t) + \hat{\phi}u_{fd}(t) + Y^{\mathrm{T}}(\boldsymbol{x})\hat{\theta} + v(t), \tag{14}$$

$$\dot{\hat{\theta}}_i(t) = -\gamma Y_i(\boldsymbol{x})s(t), \tag{15}$$

$$\dot{\hat{\phi}}_i(t) = -\eta u_{fd}s(t). \tag{16}$$

It should be noted that the right hand side of the estimation strategies $\hat{\theta}$ and $\hat{\phi}$ do not contain the saturated function s_ϵ, it only uses the signal $s(t)$.

Then, the following theorem is given.

Theorem 1.

The adaptive controller (14) and estimation strategies (15) and (16) applied to the plant (1) with dead–zone (2), and set the new control input $v(t)$ as follows:

$$v(t) = -\frac{1}{2\delta}g_2\frac{\partial\tilde{V}}{\partial s} = -\frac{1}{2\delta}s(t), \tag{17}$$

then the control system is sub–optimal in the sense that it minimizes the upper bound on the quadratic cost functional J defined by

$$J = \sup_{\frac{d(u(t))}{m}\in\mathcal{L}_2}\left\{\int_0^t \left(hs^2(\tau) + \delta v^2(\tau) - \gamma_\infty^2\left(\frac{d(u(\tau))}{m}\right)^2\right)\mathrm{d}\tau + V(t)\right\}, \tag{18}$$

for any $t \le \infty$, h and γ are positive constants, and $V(t)$ is positive function defined as

$$V(t) = \frac{1}{2}\left[\frac{1}{bm}s^2(t) + \frac{1}{\gamma}\tilde{\theta}^{\mathrm{T}}(t)\tilde{\theta}(t) + \frac{1}{\eta}\tilde{\phi}^2(t)\right]. \tag{19}$$

Proof

Taking the time derivative of (19) along the trajectory of the (8), then we have

$$\begin{aligned}\dot{V} &= \frac{1}{bm}s\dot{s} + \frac{1}{\gamma}\tilde{\theta}^{\mathrm{T}}\dot{\hat{\theta}} + \frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}} \\ &= -k_d s^2 + s\tilde{\phi}u_{fd}(t) + sY^{\mathrm{T}}(\boldsymbol{x}(t))\tilde{\theta} + sv(t) \\ &\quad + s\frac{d(u(t))}{m} + \frac{1}{\gamma}\tilde{\theta}^{\mathrm{T}}\dot{\hat{\theta}} + \frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}}\end{aligned} \tag{20}$$

We apply the estimation strategy (15) and (16), substitute (13) into (20), then

$$\begin{aligned}\dot{V} &\le -\frac{1}{4}\left(\frac{1}{\gamma_\infty} - \frac{1}{\delta}\right)s^2 - hs^2 + v(t)s + \frac{d(u(t))}{m}s \\ &= \delta\left(\frac{1}{2\delta}s + v(t)\right)^2 - \delta v^2(t) - hs^2 \\ &\quad - \gamma_\infty^2\left(\frac{d(u(t))}{m} - \frac{1}{2\gamma_\infty^2}s\right)^2 + \gamma_\infty^2\left(\frac{d(u(t))}{m}\right)^2\end{aligned} \tag{21}$$

Choosing $v(t)$ as (17) and integrating both sides, then we can conclude that the all signals in the closed–loop system are bounded and the $v(t)$ is a sub–optimal control input which minimize the upper bound on the cost functional (18). □

Remark 2.

Since the above arguments, the bounded disturbance $d(u(t))/m$ can be compensated by the control input (14) and (17). Theorem 1 shows that the $\mathcal{L}_2$ gin from the bounded disturbance

$d(u(t))/m$ to tracking error $s(t)$ is prescribed by given constant γ_∞, that is, the $\mathcal{H}^\infty$ control performance is attained adaptively for generalized output $\sqrt{hs^2(t) + \delta v^2(t)}$. Besides, it is shown that the overall system is bounded.

Remark 3.

As a result, the proposed method is one of the high–gain output feedback control method, but the control gain is determined by notion of $\mathcal{H}^\infty$ control method. Moreover, we can design the closed–loop system which can take into account of the trade–off between control performance and control input power.

5. SIMULATION STUDIES

To show the effectiveness of the proposed method, we apply the proposed adaptive controller to a nonlinear system with dead–zone. In this paper, we consider the plant which described as (Zhang and Feng 1997, Wang, Su, and Hong 2004)

$$\ddot{x}(t) = a_1 \frac{1 - e^{-x(t)}}{1 + e^{-x(t)}} - a_2(\dot{x}^2(t) + 2x(t)) \sin \dot{x}(t) - 0.5a_3 x(t) \sin 3t + bw(t) \tag{22}$$

where $w(t)$ is an output of a dead–zone.

In the following, all of the plant and dead–zone parameters are same which was used in the previous work (Wang, Su, and Hong 2004). The parameters are $a_1 = a_2 = a_3 = 1$, $b = 1$, $b_r = 0.5$, $b_l = -0.6$, and $m = 1$. The control objective is to track the system state $[x(t), \dot{x}(t)]^{\mathrm{T}}$ to the desired trajectory $[x_d(t), \dot{x}_d(t)]^{\mathrm{T}}$. Lower and upper bounds of the dead–zone which was shown in (A4) are selected as $b_{l\min} = -0.7$, $b_{l\max} = -0.1$, $b_{r\min} = 0.1$, $b_{r\max} = 0.6$, $m_{\min} = 0.85$, and $m_{\max} = 1.25$. The desired trajectory is given as $x_d(t) = 2.5 \sin t$ and plant initial values are selected as $x(0) = [-2.5, 3.5]^{\mathrm{T}}$. The design parameters common to (9), (14), (10), (15), and (16) are selected as $k_d = 10$, $\gamma = 0.5$, $\eta = 0.5$, and $\lambda = 10$. For each control strategies (9) and (14), the design parameters are choosen as $\epsilon = 0.1$, $k^* = 2.5$, and $\delta = 0.01$. In the simulation, we set the initial values for the estimation strategies as:

Case 1: $\phi(0) = \theta_1(0) = \theta_2(0) = \theta_3(0) = 0.85$,

Case 2: $\phi(0) = \theta_1(0) = \theta_2(0) = \theta_3(0) = 0.35$.

Figs. 3 and 4 show the position tracking error. From Fig. 3, it clearly shows that the proposed adaptive controller results in good tracking performance compared with the previous method using (9) and (10).

Fig. 5 shows the position tracking error using (9) and (10) with $\epsilon = 0.1$ and $\epsilon = 0.01$.

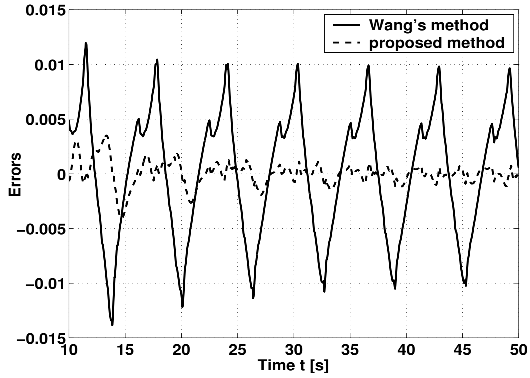

Fig. 3. Error signals of Case 1.

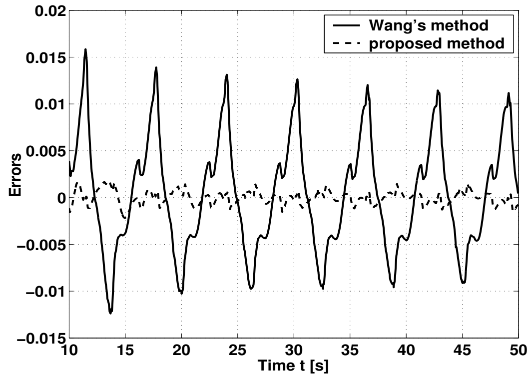

Fig. 4. Error signals of Case 2.

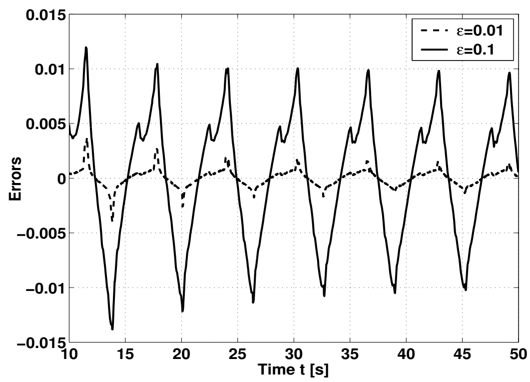

Fig. 5. Error signals for $\epsilon = 0.1$ and 0.01.

As we have shown in remark 1, from Fig. 5, it clearly shows that the design parameter ϵ selected as 0.01, then the control performance is improved compared with $\epsilon = 0.1$. Figs. 6 and 7 show the time history of the signal $s(t)/\epsilon$ which is input for the saturated function (11). Comparing Figs. 6 and 7, in the case of $\epsilon = 0.01$, the signal $s(t)/\epsilon$ involves high–frequency fluctuations between 1 and -1. This means that the output of the function $\mathrm{sat}(s(t)/\epsilon)$ eventually becomes discontinuous, therefore, it is not appropriate for the practical applications. Fig. 8 depicts the position tracking error when the design parameter δ is changed with the control method (14) and estimation mechanism (15) and (16). This figure shows that the control performance is improved when the design parameter δ is selected as 0.01. This suggests

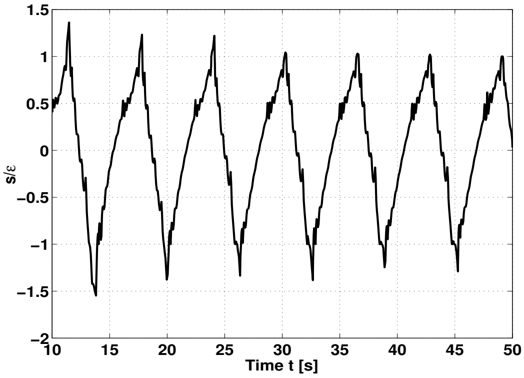

Fig. 6. $s(t)/\epsilon$ with $\epsilon = 0.1$.

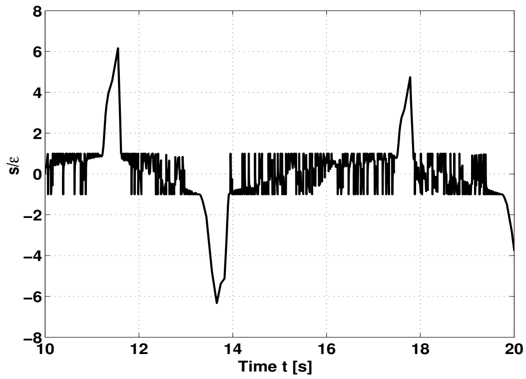

Fig. 7. $s(t)/\epsilon$ with $\epsilon = 0.01$.

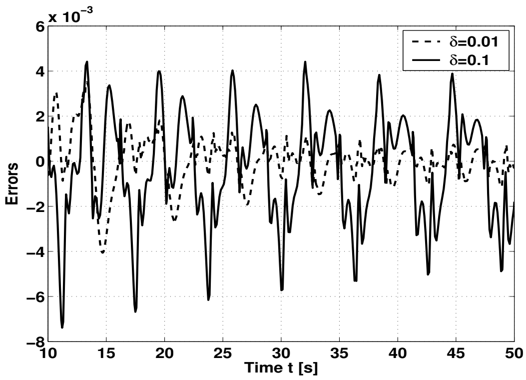

Fig. 8. Error signals for $\delta = 0.1$ and 0.01.

that if the design parameter δ is chosen small, then the effect of the bounded disturbance can be suppressed.

6. CONCLUSIONS

In practical applications dead–zones with unknown physical parameters must be taken into account for the precise control performance. In this paper, an adaptive $\mathcal{H}^\infty$ control strategy was proposed for a class of continuous–time nonlinear dynamic system with dead–zone. It is assumed that the dead–zone model can be described by the unknown parameters part and bounded disturbance part, the adaptive $\mathcal{H}^\infty$ control method was proposed without constructing a dead–zone inverse. Besides, it was also shown that the bounded disturbance can be suppressed by the proposed control method without using the saturated functions, and it is effective for the practical applications. The proposed control strategy achieves both stabilization and good tracking performance compared with the previous method using saturated functions. Simulations results showed the effectiveness of our proposed method.

REFERENCES

J. H. Kim, J. H. Park, S. W. Lee, and E. K. P. Chong: A two–layered fuzzy logic controller for systems with dead–zones, *IEEE Trans. on Idustrial Electronics*, Vol. 41, No. 2, pp. 155–161, 1994.

G. Tao and P. V. Kokotović: Adaptive control of plants with unknown dead–zones, *IEEE Trans. AC.*, Vol. 39, No. 1, pp. 59–68, 1995.

R. Freeman and P. Kokotović: Robust Nonlinear Control Design (State–space and Lyapunov Techniques), Brikhäuser, 1996.

T. P. Zhang and C. B. Feng: Adaptive fuzzy sliding mode control for a class of nonlinear systems, *Acta Automatica Sinica*, Vol. 23, pp. 361–369, 1997.

H. Y. Cho and E. W. Bai: Convergence results for an adaptive dead zone inverse, *International Journal of Adaptive Control and Signal Processing*, Vol. 12, pp. 451–466, 1998.

F. L. Lewis, W. K. Tim, L. Z. Wang, and Z. X. Li: Dead–zone compensation in motion control systems using adaptive fuzzy logic control, *IEEE Trans. on Control System Technology*, Vol. 7, No. 6, pp. 731–741, 1999.

Y. Miyasato: Redesign of Adaptive Control Systems Based on the Notion of Optimality, *Procs. of the 38th IEEE Conference on Decision and Control*, pp. 3315–3320, 1999.

R. R. Selmic and F. L. Lewis: Dead–zone compensation in motion control systems using neural network, *IEEE Trans. AC.*, Vol. 45, No. 4, pp. 602–613, 2000.

X. S. Wang, C. Y. Su and H. Hong: Robust adaptive control of a class of nonlinear systems with unknown dead–zone, *Automatica*, Vol. 40, pp. 407–413, 2004.