Low-Complexity Signal Detection by Multi-Dimensional Search for Correlated MIMO Channels

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Abstract—This paper proposes a low-complexity signal detection algorithm for spatially correlated multiple-input multiple-output (MIMO) channels. The proposed algorithm sets a minimum mean-square error (MMSE) detection result to the starting point, and searches for signal candidates in multi-dimensions of the noise enhancement from which the MMSE detection suffers. The multi-dimensional search is needed because the number of dominant directions of the noise enhancement is likely to be more than one over the correlated MIMO channels. To reduce the computational complexity of the multi-dimensional search, the proposed algorithm limits the number of signal candidates to $O(N_T)$ where $N_T$ is the number of transmit antennas. Specifically, the signal candidates, which are unquantized, are obtained as the solution of a minimization problem under a constraint that a stream of the candidates should be equal to a constellation point. Finally, the detected signal is selected from hard decisions of both the MMSE detection result and unquantized signal candidates on the basis of the log likelihood function. For reducing the complexity of this process, the proposed algorithm decreases the number of calculations of the log likelihood functions for the quantized signal candidates. Computer simulations under a correlated MIMO channel condition demonstrate that the proposed scheme provides an excellent trade-off between BER performance and complexity, and that it is superior to conventional one-dimensional search algorithms in BER performance while requiring less complexity than that of the conventional algorithms.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) mobile communications have attracted much attention because MIMO can increase system capacity and data-rate without expanding frequency bands [1]. The optimal signal detection for the MIMO system is the maximum likelihood detection (MLD), which can achieve the minimum bit error rate (BER) [2]. Unfortunately, the computational complexity of MLD is prohibitive because it increases exponentially with the number of data streams. Thus, suboptimal detection algorithms that can reduce the complexity are required [3]-[7].

The minimum mean-square error (MMSE) detection, which is such a suboptimal detection scheme, needs a very low level of complexity but exhibits poor BER performance owing to the noise enhancement. To alleviate the degradation caused by the noise enhancement, a one-dimensional search algorithm has been proposed [4], [5]. This search algorithm sets an MMSE detection result to a starting point, and searches for signal candidates in one dominant direction of the noise enhancement. The detected signal is selected from the signal candidates and the quantized MMSE detection result on the basis of the log likelihood function [5]. The conventional algorithm shows almost the same BER performance as MLD over uncorrelated MIMO channels. However, it suffers severe degradation of BER performance over spatially correlated MIMO channels, because plural dominant directions of the noise enhancement are likely to appear.

Multi-dimensional search algorithms are expected to solve the above problem. As a multi-dimensional search algorithm, geometric decoding (GD) has been proposed in [6]. GD searches for signal candidates in the vicinity of a multi-dimensional affine set, but has two drawbacks. One drawback is that it is difficult to extend GD to more than two-dimensional search. The other is that GD still requires a large amount of computational complexity because multi-dimensional search needs to examine more signal candidates.

To reduce the complexity of the multi-dimensional search, this paper proposes a low-complexity algorithm that limits the number of signal candidates to $O(N_T)$ where $N_T$ is the number of transmit antennas. Specifically, the signal candidates, which are unquantized, are obtained as the solution of a minimization problem under a constraint that a stream of the candidate should be equal to a constellation point. Therefore, the proposed algorithm can be easily extended into more than two-dimensional search, in contrast with GD.

II. SYSTEM MODEL

A. Signal Model

Fig. 1 shows a MIMO system with $N_T$ transmit antennas and $N_R$ ($N_R \geq N_T$) receive antennas. The channel is assumed to be quasi-static and time-invariant flat fading during one frame, and let $h_{lk}$ denote the channel impulse response between the $l$-th ($1 \leq l \leq N_R$) receive antenna and the $k$-th ($1 \leq k \leq N_T$) transmit antenna. Also, let $T$ and $s_k(i)$ be the symbol duration and the transmitted signal from the $k$-th transmit antenna at discrete time $iT$, respectively. Thus, the signal received by the $l$-th receive antenna at time $iT$, $y_l(i)$, can be expressed as

$$y_l(i) = \sum_{k=1}^{N_T} h_{lk} s_k(i) + n_l(i),$$

where $n_l(i)$ is additive white Gaussian noise at the $l$-th receive antenna. $n_l(i)$ is statistically independent with respect to indices $i$ and $l$, which is given by

$$\langle n_{1i}^* (i_1) n_{2i} (i_2) \rangle = \sigma_n^2 \delta_{i_1 i_2} \delta_{1i_2}.\tag{2}$$

Here $\langle \rangle$ and the asterisk denote the statistical expectation operator and complex conjugation, respectively. In addition, $\sigma_n^2$ is the average power of the noise and $\delta_{1i_2}$ is Kronecker’s delta.

$s_k(i)$ is statistically independent of $n_l(i)$, and let us assume that $\langle s_k^* (i_1) s_{k2} (i_2) \rangle = \delta_{k1k2} \delta_{i1i2}.\tag{3}$
Fig. 1. MIMO system

For simplicity, (1) is rewritten in a vector form as
\[ \mathbf{y}(i) = \mathbf{Hs}(i) + \mathbf{n}(i), \]
where the \(N_R\)-by-1 received signal vector \( \mathbf{y}(i) \), the \(N_R\)-by-\(N_T\) channel matrix \( \mathbf{H} \), the \(N_T\)-by-1 transmitted signal vector \( \mathbf{s}(i) \), and the \(N_R\)-by-1 noise vector \( \mathbf{n}(i) \) are defined as
\[ \mathbf{y}(i) = [y_1(i), y_2(i), \ldots, y_{N_R}(i)], \]
\[ \mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \ldots & h_{1N_T} \\ h_{21} & h_{22} & \ldots & h_{2N_T} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_R1} & h_{N_R2} & \ldots & h_{N_RN_T} \end{bmatrix}, \]
\[ \mathbf{s}(i) = [s_1(i), s_2(i), \ldots, s_{N_T}(i)], \]
\[ \mathbf{n}(i) = [n_1(i), n_2(i), \ldots, n_{N_R}(i)]. \]
\( \mathbf{n} \) is an \(N_R\)-by-1 vector and the superscript \(^H\) denotes Hermitian transposition.

The channel estimator in Fig. 1 performs channel estimation by using both training signals and \( y_1(i) \), and provides estimates of the channel impulse responses for the signal detector. The detector performs signal detection of \( \mathbf{s}(i) \), which will be detailed below. From now on, the ideal channel estimation is assumed.

B. Spatially Correlated Channel Model

In this paper, downlink transmission in a cellular system is considered. Since mobile stations (MSs) are generally surrounded with many local scatterers, channel correlation between receive antennas is negligible when the spacing between the receive antennas is large enough. Transmit antennas of a base station (BS) are commonly located on a high building, which causes a small angular spread. Therefore, channel correlation between the transmit antennas is considerable even when the spacing between the transmit antennas is large. Considering such correlation, the following Kronecker channel model is assumed as the correlated channel [8]:
\[ \mathbf{H} = \mathbf{H}_w \mathbf{R}_t^{1/2}, \]
where \( \mathbf{H}_w \) is an \(N_R\)-by-\(N_T\) matrix of which elements are independent and identically-distributed complex Gaussian with zero mean and unit variance. The \(N_T\)-by-\(N_T\) matrix, \( \mathbf{R}_t \), represents the channel correlation at the transmitter, and is defined as \( \mathbf{R}_t = (\mathbf{H}^H \mathbf{H}) \).

III. CONVENTIONAL SIGNAL DETECTION

A. Maximum Likelihood Detection (MLD)

Let us consider MLD of \( \mathbf{s}(i) \). The likelihood function \( p(\mathbf{y}(i)|\mathbf{H}, \mathbf{s}(i)) \) is derived from (2) and (4) as
\[ p(\mathbf{y}(i)|\mathbf{H}, \mathbf{s}(i)) = \frac{1}{(\pi \sigma_n^2)^{N_R}} e^{-\|\mathbf{y}(i) - \mathbf{Hs}(i)\|^2/\sigma_n^2}, \]
where \( \| \mathbf{u} \| \) is the 2-norm of a vector \( \mathbf{u} \), and \( \| \mathbf{u} \| = (\mathbf{u}^H \mathbf{u})^{1/2} \).

Since the maximization of \( p(\mathbf{y}(i)|\mathbf{H}, \mathbf{s}(i)) \) is equivalent to the minimization of \( \| \mathbf{y}(i) - \mathbf{Hs}(i)\|^2 \), MLD searches the candidate of \( \mathbf{s}(i) \) that minimizes the log likelihood function \( L(\mathbf{s}(i)) \) defined as
\[ L(\mathbf{s}(i)) = \| \mathbf{y}(i) - \mathbf{Hs}(i)\|^2. \]

MLD is the optimal detection and can achieve the best BER performance. However, its computational complexity grows exponentially with the number of transmitting antennas \( N_T \). This is because the complexity is proportional to the number of signal candidates, which is equal to \(M^{N_T}\) with \( M \) being the modulation order.

B. MMSE Detection

The MMSE detection multiplies \( \mathbf{y}(i) \) by a weight matrix and the detection result \( \hat{\mathbf{x}}(i) \) is given by
\[ \hat{\mathbf{x}}(i) = \mathbf{P}^H \mathbf{y}(i), \]
\[ \mathbf{P} = (\mathbf{H}^H \mathbf{H} + \sigma_n^2 \mathbf{I}_{N_T})^{-1}, \]
where \( \mathbf{P} \) and \( \mathbf{I}_{N_T} \) is an \(N_T\)-by-\(N_T\) identity matrix and \(N_T\)-by-\(N_T\) identity matrix, respectively.

The complexity of the MMSE detection is approximately proportional to \( N_T^2 \) and is much less than that of MLD. However, the noise is enhanced in the directions of the eigenvectors having large eigenvalues of \( \mathbf{P} \), which degrades BER performance of the MMSE detection.

IV. PROPOSED SIGNAL DETECTION

A. Error Analysis of MMSE Detection

Since the proposed algorithm is based on the MMSE detection, this subsection analyzes the error of the MMSE detection.

First, substituting (4) into (13) yields
\[ \hat{\mathbf{x}}(i) = \mathbf{P} (\mathbf{H}^H \mathbf{H} + \sigma_n^2 \mathbf{I}_{N_T})^{-1} \mathbf{s}(i) - \sigma_n^2 \mathbf{I}_{N_T} \mathbf{s}(i) + \mathbf{PH} \mathbf{n}(i) \]
\[ = \mathbf{s}(i) - \sigma_n^2 \mathbf{I}_{N_T} \mathbf{s}(i) + \mathbf{PH} \mathbf{n}(i). \]

The autocorrelation matrix of the difference between \( \hat{\mathbf{x}}(i) \) and \( \mathbf{s}(i) \) is given by
\[ (\mathbf{H}^H \mathbf{I}_{N_T} \mathbf{H}) = \sigma_n^2 \mathbf{P}^2 + \sigma_n^2 \mathbf{PH} \mathbf{H} \mathbf{P} \]
\[ = \mathbf{P} (\sigma_n^2 \mathbf{I}_{N_T} + \mathbf{PH} \mathbf{H} \mathbf{P}) = \sigma_n^2 \mathbf{P}, \]
where the statistical properties of \( \mathbf{s}(i) \) and \( \mathbf{n}(i) \) were used. The first term on the right hand side of (16) is \( O(\sigma_n^4) \) and comes from \( -\sigma_n^2 \mathbf{Ps}(i) \), whereas the second term is \( O(\sigma_n^2) \) and comes from \( \mathbf{PH} \mathbf{n}(i) \). When the signal-to-noise ratio (SNR) is high, \( \sigma_n^2 \) is negligible and \( \mathbf{PH} \mathbf{n}(i) \) is much more dominant than \( -\sigma_n^2 \mathbf{Ps}(i) \). Thus, \( \hat{\mathbf{x}}(i) - \mathbf{s}(i) \) can be approximated as \( \mathbf{PH} \mathbf{n}(i) \) and follows the multivariate Gaussian distribution.

Considering such an approximation and (17), the proposed algorithm uses the following signal model:
\[ \mathbf{s}(i) = \hat{\mathbf{x}}(i) + \mathbf{P}^{1/2} \mathbf{n}(i), \]
where \( \mathbf{n}(i) \) is an \(N_T\)-by-1 hypothetical noise vector having complex Gaussian random variables as its elements. The statistical properties of \( \mathbf{n}(i) \) are given by
\[ \langle \mathbf{n}(i) \mathbf{n}(i)^H \rangle = \mathbf{P}, \]
\[ \langle \mathbf{n}(i) \rangle = \mathbf{0}_{N_T}, \]
where \( \| \mathbf{u} \| \) is the 2-norm of a vector \( \mathbf{u} \), and \( \| \mathbf{u} \| = (\mathbf{u}^H \mathbf{u})^{1/2} \).
where $0_{k' \times k'}$ is the $l'$-by-$k'$ null matrix. Therefore, the probability density function (PDF) of $\mathbf{n}(i)$, $p[\mathbf{n}(i)]$, is given by

$$p[\mathbf{n}(i)] = \frac{1}{(\pi \sigma_n^2)^N} e^{-\mathbf{n}^H(i)\mathbf{n}(i)/\sigma_n^2}. \quad (21)$$

Applying the eigenvalue decomposition to $\mathbf{P}$ yields

$$\mathbf{P} = \mathbf{VDV}^H,$$  

(22)

where $\mathbf{V}$ and $\mathbf{D}$ are $N_T$-by-$N_T$ unitary and diagonal matrices, respectively. $\mathbf{V}$ can be expressed as

$$\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_{N_T}],$$  

(23)

where $\mathbf{v}_k$ is an $N_T$-by-1 orthonormal eigenvector of $\mathbf{P}$. $\mathbf{D}$ can be rewritten as

$$\mathbf{D} = \text{diag} [\lambda_1, \lambda_2, \ldots, \lambda_{N_T}],$$  

(24)

where diag $[\cdot]$ denotes the diagonal matrix having the arguments as its diagonal elements, and $\lambda_k$ is the $k$-th eigenvalue of $\mathbf{P}$. $\lambda_k > 0$ $(1 \le k \le N_T)$ because $\mathbf{P}$ is a positive-definite matrix. Without loss of generality, $\lambda_1 \ge \lambda_2 \ge \ldots \ge \lambda_{N_T} > 0$ is assumed.

Since $\mathbf{P}^{1/2}$ can be obtained as $\mathbf{VDV}^{1/2}$ from (22), $\mathbf{P}^{1/2}\mathbf{n}(i)$ is given by

$$\mathbf{P}^{1/2}\mathbf{n}(i) = \mathbf{VDV}^{1/2}\mathbf{a}(i),$$  

(25)

$$\mathbf{a}(i) = \mathbf{V}^{H}\mathbf{n}(i),$$  

(26)

where $\mathbf{a}(i)$ is an $N_T$-by-1 vector.

Since $\mathbf{V}$ is a unitary matrix, the PDF of $\mathbf{a}(i)$, $p[\mathbf{a}(i)]$, is obtained from (21) as

$$p[\mathbf{a}(i)] = \frac{1}{(\pi \sigma_a^2)^N} e^{-\mathbf{a}^H(i)\mathbf{a}(i)/\sigma_a^2}. \quad (27)$$

Let $a_k(i)$ denote the $k$-th element of $\mathbf{a}(i)$. Using (23)-(25), $\mathbf{P}^{1/2}\mathbf{n}(i)$ can be rewritten as

$$\mathbf{P}^{1/2}\mathbf{n}(i) = \sum_{k=1}^{N_T} a_k(i)\lambda_k^{1/2}\mathbf{v}_k,$$  

(28)

Finally, substituting (28) into (18) yields

$$\mathbf{s}(i) = \tilde{\mathbf{x}}(i) + \sum_{k=1}^{N_T} a_k(i)\lambda_k^{1/2}\mathbf{v}_k.$$  

(29)

B. Multi-Dimensional Search Algorithm

Suppose that the number of dominant $\lambda_k^{1/2}$s is $N_P$ $(1 \le N_P \le N_T)$. Thus, (29) can be approximated as

$$\mathbf{s}(i) \simeq \tilde{\mathbf{x}}(i) + \sum_{k=1}^{N_P} a_k(i)\lambda_k^{1/2}\mathbf{v}_k.$$  

(30)

Let us assume that the $k$-th element of $\mathbf{s}(i)$ is equal to $b(m)$, where $m$ $(1 \le m \le M)$ is an integer and $b(m)$ is one of constellation points. Using (30), the above assumption can be expressed as

$$\sum_{k=1}^{N_P} a_k(i)\lambda_k^{1/2}(\mathbf{v}_k)_k = b(m) - [\tilde{\mathbf{x}}(i)]_k,$$  

(31)

where $(\cdot)_k$ denotes the $k$-th element of a vector. The equation can be rewritten in a vector format as

$$e(m, k) = b(m) - [\tilde{\mathbf{x}}(i)]_k = \mathbf{c}_k^H\mathbf{a}(i),$$  

(32)

$$\mathbf{c}_k^H = [\lambda_1^{1/2}(\mathbf{v}_1)_k, \lambda_2^{1/2}(\mathbf{v}_2)_k, \ldots, \lambda_{N_T}^{1/2}(\mathbf{v}_{N_T})_k],$$  

(33)

$$\mathbf{a}(i) = [a_1^*(i), a_2^*(i), \ldots, a_{N_P}^*(i)],$$  

(34)

where $\mathbf{a}(i)$ and $\mathbf{c}_k$ are $N_P$-by-1 vectors.

The proposed algorithm performs the maximum likelihood estimation of $\tilde{\mathbf{a}}(i)$ for obtaining candidates of $\mathbf{s}(i)$. The log likelihood function of $\tilde{\mathbf{a}}(i)$ is obtained as $\tilde{\mathbf{a}}^H(i)\tilde{\mathbf{a}}(i)$ from (27). The minimization of $\tilde{\mathbf{a}}^H(i)\tilde{\mathbf{a}}(i)$ under the constraint of (32) can be solved by the method of Lagrange multipliers. Thus, the estimation becomes equivalent to finding $\hat{\mathbf{a}}(i)$ that minimizes the following cost function $f[\hat{\mathbf{a}}(i)]$:

$$f[\hat{\mathbf{a}}(i)] = \mathbf{a}^H(i)\hat{\mathbf{a}}(i) + \lambda|e(m, k) - \mathbf{c}_k^H\hat{\mathbf{a}}(i)|^2$$  

$$+ \lambda^*|e^*(m, k) - \mathbf{c}_k^H\hat{\mathbf{a}}(i)^*|^2,$$  

(35)

where $\lambda$ is a complex Lagrange multiplier.

The desired $\hat{\mathbf{a}}(i)$ should simultaneously satisfy the following equations:

$$\frac{\partial f[\hat{\mathbf{a}}(i)]}{\partial \mathbf{a}(i)} = \hat{\mathbf{a}}(i) - \lambda\mathbf{c}_k$$  

(36)

$$\mathbf{c}_k^H\hat{\mathbf{a}}(i) = e(m, k).$$  

(37)

Let $\hat{\mathbf{a}}(i)$ be $\hat{\mathbf{a}}(i)$, where $1 \le k_1 \le N_P$. Replacing $a_{k_1}(i)$ in (30) by $\hat{a}_{k_1}(i)$ yields $\hat{s}(i, m, k)$ that is given by

$$\hat{s}(i, m, k) = \mathbf{x}(i) + \sum_{k_1=1}^{N_P} \hat{a}_{k_1}(i)\lambda_{k_1}^{1/2}\mathbf{v}_{k_1}.$$  

(39)

Let $C$ be the set of $\text{Dec}[\hat{s}(i, m, k)]$ plus $\text{Dec}[\tilde{\mathbf{x}}(i)]$, where $\text{Dec}[\cdot]$ denotes the hard decision or quantization operation. Therefore, the cardinality $|C|$ is less than or equal to $MN_T + 1$. The finally detected signal $\hat{s}(i)$ is selected as the element of $C$ that minimizes the log likelihood function, that is

$$\hat{s}(i) = \arg \min_{\mathbf{s}(i, m, k) \in C} \| \mathbf{y}(i) - \mathbf{H}\mathbf{s}(i) \|^2.$$  

(40)

Evidently, the proposed algorithm can be easily extended into more than two-dimensional search by increasing the parameter $N_P$, in contrast with GD [6].

C. Low-Complexity Calculation of Log Likelihood Function

This subsection discusses a low-complexity algorithm for the log likelihood function, because the procedure of (40) still requires a considerable amount of computational complexity.

When SNR is high, the log likelihood function $L[\hat{s}(i, m, k)]$ can be approximated as

$$L[\hat{s}(i, m, k)] \simeq L[\tilde{\mathbf{x}}(i)] + ||\hat{\mathbf{a}}(i)||^2.$$  

(41)

The derivation of (41) is detailed in Appendix A. Using (38), an approximated log likelihood function $L[\hat{s}(i, m, k)]$ is defined as

$$L[\text{Dec}[\hat{s}(i, m, k)]] = ||\hat{\mathbf{a}}(i)||^2 = |e(m, k)|^2 ||\mathbf{c}_k||^2.$$  

(42)

$$L[\text{Dec}[\hat{s}(i, m, k)]]$$ and $L[\hat{s}(i, m, k)]$ satisfy the following inequality:

$$L[\text{Dec}[\hat{s}(i, m, k)]] \ge L[\hat{s}(i, m, k)].$$  

(43)

The proof is given below. The $k$-th element of $\text{Dec}[\hat{s}(i, m, k)]$ is also equal to $b(m)$. Considering the minimization of $||\hat{\mathbf{a}}(i)||^2$ under the constraint of (32), $||\hat{\mathbf{a}}(i)||^2$ of $\text{Dec}[\hat{s}(i, m, k)]$ is greater than or equal to that of $\hat{s}(i, m, k)$.

The low complexity algorithm takes advantage of (42) and (43) as follows. First, the elements of $C$ are sorted on the basis of (42) of $\hat{s}(i, m, k)$ in ascending order. The result is expressed as $\{\tilde{s}_0(i), \tilde{s}_1(i), \ldots, \tilde{s}_{M_{NP}}(i)\}$, where $\tilde{s}_0(i) = \text{Dec}[\tilde{\mathbf{x}}(i)]$. Next, index $c$ is set to 1, and $s = \tilde{s}_0(i)$. $L[\tilde{s}_c(i)]$ is compared with $L[\tilde{s}].$ If $L[\tilde{s}_c(i)] < L[\tilde{s}], s$ is replaced by $\tilde{s}_c(i)$,
increases by one, and the process is repeated. Otherwise, $\hat{s}$ is selected as $\hat{s}(i)$ in (40), and the process stops. The reason why $\hat{s}(i) = \hat{s}$ is that $L[\hat{s}(i)] < L[\hat{s}]$ for $c' > c$ is not likely to occur owing to the above sorting and (43). In this manner, the number of calculations for the log likelihood function of (12) can be reduced.

**D. Summary of Proposed Signal Detection**

The procedure of the proposed algorithm is summarized in Table I, where $R_P$ is the maximum number of iteration for the power method [7]. The power method is iteratively applied to the eigenvalue decomposition of $P$ for reducing the complexity [7].

**E. Relation between Proposed and PM Algorithms**

When $N_P$ is set to 1, (39) is equivalent to

$$\hat{s}(i,m,k) = \hat{s}(i) + \frac{c(m,k)}{(v_1)_k}v_1,$$

(44)

Note that (44) coincides with the one-dimensional search in [5], which is referred to as the projection method (PM).

PM can be extended into plural one-dimensional search, plural PM [7] as

$$\hat{s}(i,m,k) = \hat{s}(i) + \frac{c(m,k)}{(v_d)_k}v_d,$$

(45)

Table I

**PROPOSED ALGORITHM PROCEDURE**

<table>
<thead>
<tr>
<th>Constant Parameters: $N_P$, $H_P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inputs: $H$, $\sigma_H^2$, $y(i)$</td>
</tr>
<tr>
<td>Outputs: $\hat{s}(i)$</td>
</tr>
</tbody>
</table>

1. Compute $P = (HH^H + \sigma_H^2 I_{N_P})^{-1}$ and $PHH^H$.
2. Estimate $v_1$, $\ldots$, $v_{N_P}$ and $x_1$, $\ldots$, $x_{N_P}$ by the power method iteration.
3. Calculate $s_k$ of (33) and $1/|\hat{c}_k|^2$.

**Succeeding Processing per Symbol**

1. Compute $\hat{s}(i) = PH^Hy(i)$.
2. for $k = 1$ to $N_T$
3. for $m = 1$ to $M$
4. Calculate $c(m,k) = b(m) - |\hat{s}(i)|k$ and $|c(m,k)|^2$.
5. end for
6. end for
7. Sort $s_k(i)$ ($1 \leq c \leq MN_T$) on the basis of (42) of $\hat{s}(i,m,k)$ in ascending order.
8. $\hat{s} = \text{Dec} [\hat{k}(i)]$
9. for $c = 1$ to $MN_T$
10. Calculate $\hat{a}(i)$ by (38).
11. Calculate $\hat{s}_c(i)$ as hard decision of (39).
12. if $L[\hat{s}_c(i)] < L[\hat{s}]$
13. $\hat{s} = \hat{s}_c(i)$
14. else
15. $\hat{s}(i) = \hat{s}$
16. end if
17. end if
18. end for

Table II

**SIMULATION CONDITIONS**

<table>
<thead>
<tr>
<th>Modulation</th>
<th>QPSK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of transmit antennas, $N_T$</td>
<td>8</td>
</tr>
<tr>
<td>Number of receive antennas, $N_R$</td>
<td>8</td>
</tr>
<tr>
<td>Frame duration, $N_S$ (Symbol)</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Channel</th>
<th>correlated Rayleigh flat fading channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of departure (AOD)</td>
<td>Gaussian distribution (standard deviation $\sigma_o$: $4^\circ$)</td>
</tr>
<tr>
<td>Transmit antenna spacing</td>
<td>$4 \times$ wavelength</td>
</tr>
</tbody>
</table>

where $1 \leq d \leq N_P$. In total, $N_PMN_T + 1$ candidates are examined.

Note that PM and plural PM do not use the low complexity algorithm in section IV. C. Therefore, the proposed algorithm can be superior to them in complexity.

**V. COMPUTER SIMULATIONS**

A. Simulation Conditions

Computer simulations were conducted to clarify the performance of the proposed algorithm. The simulation parameters are listed in Table II. As conventional algorithms, MMSE, PM [5], plural PM [7], GD [6], sphere decoding (SD) [3], and MLD were also investigated.

B. BER Performance vs. $E_b/N_0$

Fig. 2 shows the BER performance of the proposed and conventional algorithms with QPSK modulation and $N_T = N_R = 8$ on the correlated Rayleigh fading channel. The performance of MLD can be considered the lower bound. SD can maintain almost the same performance as that of MLD, while GD can not. The proposed algorithm is a little inferior to GD but outperforms plural PM and PM. Therefore, the proposed scheme is more robust against the correlated MIMO channel than plural PM and PM.

C. Computational Complexity

Fig. 3 shows the average number of complex multiplications which the proposed and conventional algorithms require.
during one frame. The proposed algorithm needs much less computational complexity than MLD. When average $E_b/N_0$ is equal to 30 dB, the proposed algorithm can reduce the complexity to about 8.5% of that required by GD, 12.5% of that required by PM, and 5.9% of that required by plural PM. The reason why the proposed algorithm is superior to PM in complexity is that the proposed algorithm can drastically reduce complexity of (40) by the algorithm in section IV. C. SD requires more computational complexity than the other suboptimal algorithms. Note that the complexity of the proposed algorithm increases monotonically with average $E_b/N_0$ when average $E_b/N_0 \leq 5$ dB. This is because the approximation of (41) and the sorting of $\hat{s}_i(i)$ based on (42) become more inaccurate under such a low $E_b/N_0$ condition and thus the process of (40) does not properly work. If it properly operates, the complexity of the proposed algorithm should decrease monotonically with average $E_b/N_0$. It is also noteworthy that MLD, SD, and GD which outperform the proposed algorithm require much more complexity than the proposed algorithm.

VI. Conclusions

This paper has proposed a low-complexity signal detection algorithm for spatially correlated MIMO channels. The proposed algorithm sets an MMSE detection result to a starting point, and searches for unquantized signal candidates in multi-dimensions of the noise enhancement from which MMSE detection suffers. The multi-dimensional search is needed because the number of dominant directions of the noise enhancement is likely to be more than one over the correlated MIMO channels. The detected signal is selected from hard decisions of both the MMSE detection result and the unquantized signal candidates on the basis of the log likelihood function. Computer simulations under a correlated Rayleigh flat fading channel have shown that the proposed scheme provides an excellent trade-off between BER performance and complexity, and that it outperforms conventional one-dimensional search algorithms, plural PM [7] and PM [5] while requiring much less complexity than the conventional algorithms, when the number of receive or transmit antennas is equal to 8 and QPSK is used as the modulation scheme.

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References


APPENDIX

A. Derivation of (41)

$L[s(i)]$ of (12) is transformed into

\[ L[s(i)] = \| y(i) - H\hat{x}(i) \|^2 \]

\[ = L[\hat{x}(i)] + [s(i) - \hat{x}(i)]^H H^H H [s(i) - \hat{x}(i)] \]

\[ - [s(i) - \hat{x}(i)]^H [H^H y(i) - H^H \hat{x}(i)] \]

Using (13) and (14), $H^H y(i) - H^H \hat{x}(i)$ is rewritten as

\[ H^H y(i) - H^H \hat{x}(i) = (I_{N_T} - H^H H) H^H y(i) = (P - 1) PH^H y(i) = \sigma_n^2 \hat{x}(i). \]

Substituting (47) into (46) yields

\[ L[s(i)] = L[\hat{x}(i)] + \sigma_n^2 \| \hat{x}(i) \|^2 - \| s(i) \|^2 \]

\[ + \sum_{k_1=1}^{N_T} a_{k_1}^*(i) \lambda^{1/2}_{k_1} v_{k_1} D^{-1} v_{k_1} \sum_{k_1=1}^{N_T} a_{k_1} (i) \lambda^{1/2}_{k_2} v_{k_2} \]

\[ + \sum_{k_1=1}^{N_T} \sum_{k_2=1}^{N_T} a_{k_1}^*(i) a_{k_2} (i) (\lambda_{k_1} \lambda_{k_2})^{1/2} e_{k_1}^T D^{-1} e_{k_2}, \]

where $e_k$ is the $N_T$-by-$1$ vector with $(e_k)_k = 1$ and $(e_k)_{k'} = 0$ for $k' \neq k$. The derivation used the property that $v_k$’s are orthonormal vectors. Since $D^{-1}$ is the diagonal matrix having $\lambda^{-1}_k$ as the $(k, k)$-th element, (49) is given by

\[ L[s(i)] = L[\hat{x}(i)] + \sigma_n^2 \| \hat{x}(i) \|^2 - \| s(i) \|^2 + \| \Delta(i) \|^2. \]

Furthermore, applying the approximation of (30) to (50) yields

\[ L[s(i)] \approx L[\hat{x}(i)] + \sigma_n^2 \| \hat{x}(i) \|^2 - \| s(i) \|^2 + \| \Delta(i) \|^2. \]

From (18), the average second term on the right side of (51) is given by

\[ \sigma_n^2 \langle \| \hat{x}(i) \|^2 - \| s(i) \|^2 \rangle = \sigma_n^4 \text{tr}\{P\}, \]

where $\text{tr}\{\}$ denotes the trace of a square matrix. Evidently from (26), the third term on the right hand side of (51) is $O(\sigma_n^2)$. When SNR is high, therefore, the second term can be neglected. Thus

\[ L[\hat{s}(i, m, k)] \approx L[\hat{x}(i)] + \| \Delta(i) \|^2. \]