We first investigate the notion of fuzzy entire sequence space with a suitable example. Also we deal with the properties of the space of fuzzy entire sequences. The concepts of subset and superset of the fuzzy entire sequence spaces are introduced and their properties are discussed.

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1. Introduction

The concepts of fuzzy sets and fuzzy set operations were first introduced by Zadeh [1] and subsequently several authors discussed various aspects of the theory and applications of fuzzy sets such as fuzzy topological spaces, fuzzy functional analysis, and fuzzy sequence spaces. Bounded and convergent sequences of fuzzy numbers were introduced by Matloka [2] where every convergent sequence is bounded. Nanda [3] has studied the spaces of bounded and convergent sequences of fuzzy numbers and has shown that they are complete metric spaces. The space $\Gamma$ of entire sequences was studied by Ganapathy Iyer [4]. In this paper, we introduce fuzzy topology into $\Gamma$. Then $\Gamma$ turns out to be a fuzzy sequence spaces. Firstly we define the metric for the fuzzy sequence spaces and we try to introduce the notion of the fuzzy entire sequence spaces. Also we deal with topological properties of the space of fuzzy entire sequences. We first prove that the space of fuzzy entire sequence is a complete fuzzy metric space and also is a fuzzy linear metric space. The concepts of subset and superset of the fuzzy entire sequence space are introduced and their properties are discussed.
2. Preliminaries

At first, we recall some definitions and results about fuzzy numbers. A fuzzy number space is a fuzzy set on the real axis, that is, denote $F(R) = \{ \tilde{a} \mid \tilde{a} : R^n \to [0, 1], \tilde{a} \hat{\otimes} \}$ has the following properties (a)–(d):

(a) $\tilde{a}$ is normal, that is, there exists an $x_0 \in R^n$ such that $\tilde{a}(x_0) = 1$;
(b) $\tilde{a}$ is convex, that is, $\tilde{a}(\lambda x + (1 - \lambda)y) \geq \min\{\tilde{a}(x), \tilde{a}(y)\}$ whenever $x, y \in R^n$ and $0 \leq \lambda \leq 1$;
(c) $\tilde{a}(x)$ is upper semicontinuous;
(d) $[\tilde{a}]^0 = \text{cl}\{x \in R^n : \tilde{a}(x) > 0\}$ is a compact set.

As obtained by Zadeh, $\tilde{a}$ is convex if and only if each of its $\alpha$-level sets $\tilde{a}_\alpha$, where $\tilde{a}_\alpha = \{x \in R^n : \tilde{a}(x) \geq \alpha\}$ for each $\alpha \in (0, 1]$, is a nonempty compact convex subset of $R^n$ with compact support. The $\alpha$-level set of an upper semicontinuous convex normal fuzzy number is a closed interval $[\tilde{a}_\alpha^-, \tilde{a}_\alpha^+]$, where the values $\tilde{a}_\alpha^- = -\infty$ and $\tilde{a}_\alpha^+ = +\infty$ are admissible. Since each $x \in R^n$ can be considered as a fuzzy number $\tilde{a}$, defined by (2.1), the real number can be embedded in $F^+(R)$. A fuzzy number $\tilde{a}$ is called nonnegative if $\tilde{a}(x) = 0$ for all $x < 0$. The set of all nonnegative fuzzy numbers of $F^+(R)$ is denoted by $F(R)$. Let $k \in F(R)$ and $k = \bigcup_{\lambda \in [0, 1]} \lambda[k,k]$, 

$$\tilde{a}(x) = \begin{cases} 1 & \text{for } x = k, k \in R^n, \\ 0 & \text{for } x \neq k, k \in R^n. \end{cases}$$

(2.1)

For any $\tilde{a} \in F(R)$, $\tilde{a}$ is called fuzzy number and $F(R)$ is called a fuzzy number space. For $\tilde{a}, \tilde{b} \in F(R)$, we define $\tilde{a} \leq \tilde{b}$ if and only if $[\tilde{a}]_\lambda = [\tilde{a}_\lambda^-, \tilde{a}_\lambda^+] \subseteq [\tilde{b}]_\lambda = [\tilde{b}_\lambda^-, \tilde{b}_\lambda^+]$ and $[\tilde{a}]_\lambda \subseteq [\tilde{b}]_\lambda$ if and only if $\tilde{a}_\lambda^- \leq \tilde{b}_\lambda^-$ and $\tilde{a}_\lambda^+ \leq \tilde{b}_\lambda^+$ for any $\lambda \in [0, 1]$.

Theorem 2.1 (representation theorem). For $\tilde{a} \in F(R)$,

1. $\tilde{a}_\lambda^-$ is a bounded left continuous nondecreasing function on $(0, 1]$;
2. $\tilde{a}_\lambda^+$ is a bounded left continuous nonincreasing function on $(0, 1]$;
3. $\tilde{a}_\lambda^-$ and $\tilde{a}_\lambda^+$ is right continuous at $\lambda = 0$;
4. $\tilde{a}_\lambda^- \leq \tilde{a}_\lambda^+$.

Moreover, if the pair of functions $a(\lambda)$ and $b(\lambda)$ satisfies (1)–(4), then there exists a unique $\tilde{a} \in F(R)$ such that $\tilde{a}_\lambda = [a(\lambda), b(\lambda)]$ for each $\lambda \in [0, 1]$.

Define $F(R) \times F(R) \to R$ by $d(\tilde{a}, \tilde{b}) = \sup_{0 \leq \lambda \leq 1} \delta_\infty(\tilde{a}_\lambda, \tilde{b}_\lambda)$ for $\tilde{a}, \tilde{b} \in F(R)$ and $\delta_\infty(A, B) = \max\{\sup_{a \in A} \inf_{b \in B} \|a - b\|, \sup_{b \in B} \inf_{a \in A} \|a - b\|\}$. It is known that $(F(R), d)$ is a complete metric space.

Let $\tilde{0}$ and $\tilde{1}$ be defined by 

$$\tilde{0}(x) = \begin{cases} 1 & \text{for } x = 0, \\ 0 & \text{for } x \neq 0, \end{cases} \quad \tilde{1}(x) = \begin{cases} 1 & \text{for } x = 1, \\ 0 & \text{for } x \neq 1. \end{cases}$$

(2.2)

The absolute value $|\tilde{a}|$ of $\tilde{a} \in F(R)$ is defined by

$$|\tilde{a}|(t) = \begin{cases} \max\{\tilde{a}(t), \tilde{a}(-t)\}, & t > 0, \\ 0, & t < 0. \end{cases}$$

(2.3)
Definition 2.2 [5]. A sequence $\tilde{x} = (\tilde{x}_k)$ of fuzzy numbers is said to be a Cauchy sequence to a fuzzy number $(\tilde{x}_m)$, written as $\lim_{n\to k} \tilde{x}_k = \tilde{x}_m$, if for every $\epsilon > 0$ there exists a positive integer $N_0$ such that $\rho(\tilde{x}_k, \tilde{x}_m) < \epsilon$ for $k, m > N_0$.

3. Fuzzy entire sequence $\Gamma(R)$

Now we introduce the new fuzzy sequence space and we show that this is a complete metric space.

For each fixed $n$, define the fuzzy metric

$$\tilde{\rho}(\tilde{x}_n, \tilde{y}_n) = \bigcup_{\lambda \in [0,1]} \lambda \left\{ \left( (x_n)_1 - (y_n)_1 \right)^{1/n}, \sup_{\lambda \leq \eta \leq 1} \left\{ \left| (x_n)_\eta - (y_n)_\eta \right|^{1/n} \lor \left| (x_n)_\eta + (y_n)_\eta \right|^{1/n} \right\} \right\}.$$  

(3.1)

Let $\tilde{x} = \{\tilde{x}_n\}$ and $\tilde{y} = \{\tilde{y}_n\}$ be sequences of fuzzy real numbers. Define their distance by

$$\tilde{\theta}(\tilde{x}, \tilde{y}) = \bigcup_{\lambda \in [0,1]} \lambda \sup_{(n)} \left\{ \left( (x_n)_1 - (y_n)_1 \right)^{1/n}, \sup_{(n)} \left\{ \sup_{\lambda \leq \eta \leq 1} \left\{ \left| (x_n)_\eta - (y_n)_\eta \right|^{1/n} \lor \left| (x_n)_\eta + (y_n)_\eta \right|^{1/n} \right\} \right\} \right\}.$$  

(3.2)

Clearly, $(F(R), \tilde{\rho})$ is a complete metric space [6].

Definition 3.1. $\tilde{x} = \{\tilde{x}_n\}$ is called a fuzzy entire sequence if $\tilde{\rho}(\lim_{n\to \infty} \tilde{x}_n) = \tilde{0}$. In other words, given $\epsilon > 0$ there exists a positive integer $N$ such that $\tilde{\rho}(\tilde{x}_n, \tilde{0}) < \epsilon$ for all $n \in N$. Let $\Gamma(R) = \{\text{all fuzzy entire sequences}\}$.

Example 3.2. Let $\tilde{a}_n = 1/n!$ for $n = 1, 2, \ldots$ be a sequence fuzzy numbers. Now we have $\tilde{a} = (\tilde{a}_n) = (1/n!)$, where $n \in F(R)$.

$$\tilde{\rho}(\tilde{a}_n, \tilde{0}) = \bigcup_{\lambda \in [0,1]} \lambda \sup_{(n)} \left\{ \left| \left( \frac{1}{n!} \right)_\eta - 0 \right|^{1/n}, \sup_{(n)} \left\{ \sup_{\lambda \leq \eta \leq 1} \left\{ \left| \left( \frac{1}{n!} \right)_\eta - 0 \right|^{1/n} \lor \left| \left( \frac{1}{n!} \right)_\eta + 0 \right|^{1/n} \right\} \right\} \right\} = \tilde{0}.$$  

(3.3)

This implies that $\tilde{a} = (\tilde{a}_n) = (1/n!) \in \Gamma(R)$. Hence, $\tilde{a} = (\tilde{a}_n)$ is a fuzzy entire sequence.

Theorem 3.3. $(\Gamma(R), \tilde{\theta})$ is a fuzzy complete metric space.
Proof. It can be seen that \( \tilde{\theta} \) is a metric for \( \Gamma(R) \). Let \( \{x^{(i)}\} \) be any fuzzy Cauchy sequence in \( \Gamma(R) \). Then for every \( \epsilon > 0 \), there exist a positive integer \( N \) such that

\[
\tilde{\theta}(\tilde{x}^{(i)}, \tilde{x}^{(j)}) = \sqrt{\frac{1}{n}} \max_{\lambda \in [0,1]} \left\{ \sup_{n} \left| x_{n,-}^{(i)} - x_{n,-}^{(j)} \right| \right\} \leq \epsilon \quad \text{for} \quad i, j > N.
\]

This implies that \( \{x^{(i)}\}_{i=1}^{\infty} \) is a Cauchy sequence in \( F(R) \) for each fixed \( n \).

(3.4)

\( (F(R), \tilde{\rho}) \) is a complete metric space; hence the Cauchy sequence \( \{x^{(i)}(n)\}_{i=1}^{\infty} \) converges to \( \tilde{x}_{n} \), that is \( \rho(\tilde{x}^{(i)}_{n}, \tilde{x}_{n}) = 0 \) as \( i \to \infty \) for each fixed \( n \).

\[
\Rightarrow \lim_{i \to \infty} \sup_{\lambda \in [0,1]} \left\{ \left| x_{n,-}^{(i)} - x_{n,-}^{(j)} \right| \right\} = 0 \quad \forall n
\]

\[
\Rightarrow \lim_{i \to \infty} \sup_{\lambda \in [0,1]} \left\{ \left| x_{n,-}^{(i)} - x_{n,-}^{(j)} \right| \right\} = 0 \quad \forall n
\]

\[
\Rightarrow \lim_{i \to \infty} \tilde{\theta}(\tilde{x}^{(i)}, \tilde{x}) = 0, \quad \text{where} \quad \tilde{x} \in (\tilde{x}_{n}).
\]

(3.5)

Now we will show that \( x \in \Gamma(R) \). In (3.4), letting \( j \to \infty \), we get \( \tilde{\rho}(\tilde{x}^{(i)}, \tilde{x}) < \epsilon/5 \), since \( \{x^{(i)}\} \) is a Cauchy sequence for each \( n \).

This implies that \( \tilde{\rho}(\tilde{x}^{(i)}_{n}, \tilde{x}_{k}) < \epsilon/5 \) for \( n, k \in N \).

Similarly, \( \tilde{\rho}(\tilde{x}^{(i)}_{n}, \tilde{x}^{(j)}_{k}) < \epsilon/5 \) for \( n, k \in N \). For each fixed \( j \), put \( N = \max\{N_{n}, N_{i}, N_{j}\} \).

Then for given \( \epsilon > 0 \), there exist \( x^{(i)}, x^{(j)} \in F(R) \) in connection with (3.4) such that

\[
\tilde{\rho}(\tilde{x}^{(i)}_{n}, \tilde{x}^{(j)}_{n}) < \epsilon/5, \quad \tilde{\rho}(\tilde{x}^{(i)}_{n}, \tilde{x}^{(j)}_{n}) < \epsilon/5
\]

\[
\Rightarrow \tilde{\rho}(\tilde{x}^{(i)}_{n}, \tilde{x}^{(j)}_{n}) < \epsilon/5 + \epsilon/5 = \frac{3\epsilon}{5}
\]

\[
\Rightarrow \{x^{(i)}_{n}\}_{i=1}^{\infty} \text{ is a fuzzy Cauchy sequence in } F(R)
\]

(3.6)

by the completeness of \( F(R) \), \( \tilde{\rho}(\tilde{x}^{(i)}_{n}, \tilde{x}_{k}) < \frac{3\epsilon}{5} \) for some \( \tilde{x}_{k} \in F(R) \).

\[
\Rightarrow \tilde{\rho}(\tilde{x}^{(i)}_{n}, \tilde{x}_{k}) < \tilde{\rho}(\tilde{x}^{(i)}_{n}, \tilde{x}^{(j)}_{n}) + \tilde{\rho}(\tilde{x}^{(j)}_{n}, \tilde{x}^{(j)}_{n}) + \tilde{\rho}(\tilde{x}^{(j)}_{n}, \tilde{x}_{k}) < \frac{3\epsilon}{5} + \frac{3\epsilon}{5} = \epsilon
\]

for each \( n, k \geq N \).

\[
\Rightarrow \{\tilde{x}_{n}\} \text{ is a Cauchy sequence with respect to } \tilde{\theta}.
\]

This implies that \( \tilde{x} = \{\tilde{x}_{n}\} \in \Gamma(R) \). Hence, \( (\Gamma(R), \tilde{\theta}) \) is a fuzzy complete metric space. \( \square \)
Theorem 3.4. The space of the fuzzy entire sequences is a fuzzy linear space.

Proof. Let \( \tilde{\alpha}, \tilde{\beta} \in \Gamma(R) \) and let \( \tilde{\alpha} = (\tilde{a}_n) \) and \( \tilde{\beta} = (\tilde{b}_n) \), where \( \tilde{\rho}_{\lim_{n \to \infty}} \tilde{a}_n = \tilde{0} \) and \( \tilde{\rho}_{\lim_{n \to \infty}} \tilde{b}_n = \tilde{0} \). To prove \( a \tilde{\alpha} + b \tilde{\beta} \in \Gamma(R) \) and \( a \tilde{\alpha} + b \tilde{\beta} = a(\tilde{a}_n) + b(\tilde{b}_n) \). It is enough to prove that 

\[
\tilde{\rho}_{\lim_{n \to \infty}} (a \tilde{a}_n + b \tilde{b}_n) = 0.
\]

Since \( \tilde{\rho}_{\lim_{n \to \infty}} \tilde{a}_n = \tilde{0} \), given \( \epsilon > 0 \), there exists a positive integer \( N_1 \) such that \( \tilde{\rho}(\tilde{a}_n, \tilde{0}) < \epsilon/|a| \) for all \( n \geq N_1 \). Since \( \tilde{\rho}_{\lim_{n \to \infty}} \tilde{b}_n = \tilde{0} \), given \( \epsilon > 0 \), there exists a positive integer \( N_2 \) such that \( \tilde{\rho}(\tilde{b}_n, \tilde{0}) < \epsilon/|b| \) for all \( n \geq N_2 \).

Let \( N = \max\{N_1, N_2\} \). Then

\[
\tilde{\rho}_{\lim_{n \to \infty}} (a \tilde{a}_n + b \tilde{b}_n) = \tilde{\rho}(a \tilde{a}_n + b \tilde{b}_n, \tilde{0}) = a \tilde{\rho}(\tilde{a}_n, \tilde{0}) + b \tilde{\rho}(\tilde{b}_n, \tilde{0}) = a \frac{|a|}{\epsilon} + b \frac{|b|}{\epsilon} = \epsilon + \epsilon < \epsilon.
\]

This implies that \( \tilde{\rho}_{\lim_{n \to \infty}} (a \tilde{a}_n + b \tilde{b}_n) = 0 \). Therefore, \( a \tilde{\alpha} + b \tilde{\beta} \in \Gamma(R) \). Hence, \( \Gamma(R) \) is a fuzzy linear space.

Theorem 3.5. The space of the fuzzy entire sequence is a fuzzy linear metric space.

Proof. \( \Gamma(R) \) is a fuzzy metric space if we define the metric \( \tilde{\rho} \) by

\[
\tilde{\rho}(\tilde{\alpha}, \tilde{\beta}) = \bigcup_{\lambda \in [0,1]} \lambda \left\{ \sup_{\eta \leq \lambda} |(a_n)_{\eta} - (b_n)_{\eta}|^{1/n}, \sup_{\lambda < \eta \leq 1} \left| (a_n)^+_{\eta} - (b_n)^+_{\eta} \right|^{1/n} \right\},
\]

where \( \tilde{\alpha}, \tilde{\beta} \in \Gamma(R) \) and \( \tilde{\alpha} = (\tilde{a}_n) \) and \( \tilde{\beta} = (\tilde{b}_n) \). To prove that \( \Gamma(R) \) is a fuzzy linear metric space, it is enough to prove \( \tilde{\alpha} + \tilde{\beta} \), \( k \tilde{\alpha} \) where \( \tilde{\alpha}, \tilde{\beta} \in \Gamma(R) \) and \( k \in R^+ \). That \( \tilde{\alpha} + \tilde{\beta} \) is fuzzy continuous follows from the property \( |\tilde{\alpha} + \tilde{\beta}| \leq |\tilde{\alpha}| \oplus |\tilde{\beta}| \). To prove that \( k \tilde{\alpha} \) is continuous; it is enough to prove that \( \tilde{\alpha}_n \to \tilde{\alpha} \in \Gamma(R) \) and \( k \tilde{\alpha}_n \to k \tilde{\alpha} \) for each \( \tilde{\alpha} \in \Gamma(R) \).

Case 1. Let \( \tilde{\alpha}_n \to \tilde{\alpha} \) in \( \Gamma(R) \). To prove that \( k \tilde{\alpha}_n \to k \tilde{\alpha} \) since \( \tilde{\alpha}_n \to \tilde{\alpha} \), given \( \epsilon > 0 \), there exists \( N \) such that \( |\tilde{\rho}(\tilde{\alpha}_n, \tilde{\alpha})| < \epsilon/|k|^n \) for all \( n \geq N \).

\[
\tilde{\rho}(k \tilde{\alpha}_n, k \tilde{\alpha}) \leq \bigcup_{\lambda \in [0,1]} \left\{ \sup_{\eta \leq \lambda} |k \tilde{\alpha}_n - k \tilde{\alpha}|^{1/n}, \sup_{\lambda < \eta \leq 1} \left| k \tilde{\alpha}_n^+ - k \tilde{\alpha}^+ \right|^{1/n} \right\},
\]

Therefore, \( k \tilde{\alpha} \to k \tilde{\alpha} \) as \( n \to \infty \) that is \( \tilde{\rho}_{\lim_{n \to \infty}} \tilde{a}_n = \tilde{\alpha} \).
From the fuzzy metric space [6], obviously we get a positive integer $N$

**Definition 4.1.**

Let $\tilde{x}$ be an element of $\Gamma(R)$, where $\tilde{y} = \lim_{n \to \infty} \tilde{x}_n = \tilde{0}$. Since $\tilde{y}$ is a proper subset of $\tilde{x}$, given $\epsilon > 0$, there exists $N_1$ such that $\tilde{y}(\tilde{x}_n, \tilde{0}) < \epsilon$ for all $n \geq N_1$. From the fuzzy metric space [6], obviously we get $|k_n\tilde{x}| \leq \epsilon$ for $n \geq N_1$. That is, $k_n\tilde{x} \to \tilde{0}$ as $n \to \infty$. This proves that $\Gamma(R)$ is a fuzzy linear metric space.

**Theorem 3.6.** A fuzzy entire sequence space is separable.

**Proof.** Let $C = \{x_1, x_2, x_3, \ldots, x_n, 0, 0, 0, \ldots\}$ be a countable subset of $\Gamma(R)$ and $x_i \in Q \subset \Gamma(R)$, where $Q = \{\text{all fuzzy rational numbers}\}$. Hence, $\Gamma(R)$ is separable. 

**4. A subset of $\Gamma(R)$**

In fact, $\chi$ is a subset of $\Gamma$. For each fixed $n$, define the fuzzy metric

$$\tilde{d}(\tilde{x}, \tilde{y}) = \bigcup_{\lambda \in [0,1]} \lambda \left\{ \sup_{(n)} \left( \frac{1}{n} \right) \sup_{\eta \leq 1} \left\{ |n(x_n)_\eta - n(y_n)_\eta|^{1/n} \right\}, \sup_{\lambda \leq 1} \left\{ |n(x_n)_\lambda - n(y_n)_\lambda|^{1/n} \right\} \right\}. 

$$

Let $\tilde{x} = \{\tilde{x}_n\}$ and $\tilde{y} = \{\tilde{y}_n\}$ be sequence of fuzzy real numbers. Define their distance by

$$\tilde{d}(\tilde{x}, \tilde{y}) = \bigcup_{\lambda \in [0,1]} \lambda \left\{ \sup_{(n)} \left( \frac{1}{n} \right) \sup_{\eta \leq 1} \left\{ |n(x_n)_\eta - n(y_n)_\eta|^{1/n} \right\}, \sup_{\lambda \leq 1} \left\{ |n(x_n)_\lambda - n(y_n)_\lambda|^{1/n} \right\} \right\}. 

$$

**Definition 4.1.** $\tilde{x} = \{\tilde{x}_n\} \in \chi$ if $\tilde{d}(\lim_{n \to \infty} \tilde{x}_n, \tilde{0}) = \tilde{0}$. In other words, given $\epsilon > 0$ there exists a positive integer $N$ such that $\tilde{d}(\tilde{x}_n, \tilde{0}) < \epsilon$ for all $n \in N$. The set of all fuzzy subsets of $\Gamma(R)$ is denoted by $\chi(R)$. Note that $\chi(R) \subset \Gamma(R)$.

**Proposition 4.2.** $\chi(R)$ is a proper subset of $\Gamma(R)$.

**Proof.** Let $|x_n| \leq \angle n|x_n|$. This implies that $|x_n|^{1/n} \leq (\angle n|x_n|)^{1/n}$. Let $(x_n) \in \chi(R) \Rightarrow (\angle n|x_n|)^{1/n} < \epsilon \Rightarrow |x_n|^{1/n} < \epsilon$ for all $n \geq n_0 \Rightarrow (x_n) \in \Gamma(R) \Rightarrow \chi(R) \subset \Gamma(R)$. 

**Theorem 4.3.** $\chi(R)$ is a proper closed subspace of $\Gamma(R)$. 6 International Journal of Mathematics and Mathematical Sciences
Proof

Step 1. \((1/n) \notin \chi(R)\) But \((1/n) \in \Gamma(R)\). This implies that \(\chi(R)\) is a proper subspace of \(\Gamma(R)\).

Step 2. Suppose that \(a^{(p)} \to a \in \chi(R)\) \(\Rightarrow |a_{0}^{(p)} - a_{0}|, [\angle n|a_{n}^{(p)} - a_{n}|]^{1/n} < \varepsilon\) for \(p \geq p_{0}\),

\[
\Rightarrow [\angle n|a_{n}|]^{1/n} \leq [\angle n|a_{n}^{(p)}|]^{1/n} + [\angle n|a_{n}^{(p)} - a_{n}|]^{1/n}
< [\angle n|a_{n}^{(p)}|]^{1/n} + (\varepsilon^{n})^{1/n} \varepsilon/n \text{ for } p \geq p_{0} \tag{4.3}
\]

\[
\Rightarrow [\angle n|a_{n}|]^{1/n} < \varepsilon + k\varepsilon, \text{ for sufficiently large } n.
\]

From Stirling's formula, where \(\varepsilon \to 0\) as \(p \to \infty\) and \(k\) is a positive constant

\[
\Rightarrow \tilde{\mu}([\angle n|a_{n}|]^{1/n}) < \varepsilon \quad \forall n \geq n_{0} \Rightarrow a = (a_{n}) \in \chi(R) \tag{4.4}
\]

\[\square\]

Theorem 4.4. A subset \(\chi(R)\) of \(\Gamma(R)\) is a fuzzy complete.

Proof. Let \(\{a_{p} : p \geq 1\}\) be a fuzzy Cauchy sequence in \(\chi(R)\), where \(\angle n|a_{n}^{(p)}|^{1/n} \to 0\) as \(n \to \infty\), for each \(p \geq 1\).

Let \(\varepsilon > 0\), there exists \(N = N(\varepsilon)\) such that

\[
\sup \{ |a_{0}^{(p)} - a_{0}^{(m)}|, [\angle n|a_{n}^{(p)} - a_{n}^{(m)}|]^{1/n}, n \geq 1 \} < \varepsilon \quad \text{for } m, p \geq N \Rightarrow |a_{0}^{(p)} - a_{0}^{(m)}| < \varepsilon, [\angle n|a_{n}^{(p)} - a_{n}^{(m)}|]^{1/n} < \varepsilon \quad \text{for } m, p \geq N \quad (n = 1, 2, \ldots) \tag{4.5}
\]

\[
\Rightarrow \{\angle n|a_{n}^{(p)}|^{1/n}\} \text{ and so } \{a_{n}^{(p)}\} \text{ is a fuzzy Cauchy sequence in the complex plane for each } n \geq 1.
\]

\[
\Rightarrow a_{n}^{(p)} \to a_{n} \quad \text{as } p \to \infty \quad \forall n \geq 1 \Rightarrow \angle n|a_{n}^{(p)} \to \angle n|a_{n} \quad \text{as } p \to \infty. \tag{4.6}
\]

Now for \(n \geq 1, p \geq 1\), \(\angle n|a_{n}|^{1/n} \leq [\angle n|a_{n} - a_{n}^{(p)}|]^{1/n} + \{\angle n|a_{n}^{(p)}|\}^{1/n}.

Let \(\varepsilon > 0\), then there exists \(p_{0}\) such that \(\angle n|a_{n}^{(p_{0})} - a_{n}|]^{1/n} < \varepsilon\) and so this holds in particular for \(p = p_{0}\).

Consequently,

\[
\angle n|a_{n}|^{1/n} < \varepsilon + [\angle n|a_{n}^{(p_{0})}|]^{1/n} \quad \text{for } n \geq 1. \quad \text{Now } |\angle n|a_{n}^{(p_{0})}|^{1/n} \to 0 \quad \text{as } n \to \infty. \tag{4.7}
\]
Thus,
\[
\tilde{\mu}(\lim_{n \to \infty} \angle n |a_n|^\frac{1}{n}) \leq \varepsilon, \Rightarrow \lim_{n \to \infty} \tilde{\mu}[\angle n |a_n|^\frac{1}{n}] = 0, \Rightarrow a = (a_n) \in \chi(R). 
\] (4.8)

Also (4.5) and (4.6), \(|a_n^{(p)} - a_0| < \varepsilon; [\angle n |a_n^{(p)} - a_n|^\frac{1}{n}] < \varepsilon\) for \(p \geq N(n = 1, 2, \ldots) \Rightarrow |a_p - a| \to 0\) as \(p \to \infty \Rightarrow \chi(R)\) is a fuzzy complete of \(\Gamma(R)\).

**Corollary 4.5.** \((\chi(R), \tilde{\theta})\) is a complete fuzzy metric space.

**Proof.** From Proposition 4.2 and Theorem 3.3, it follows that \(\chi(R)\) is a closed subspace of the complete fuzzy metric space of \(\Gamma(R)\). Hence, \(\chi(R)\) is complete fuzzy metric space. □

5. A superset of \(\Gamma(R)\)

In fact, \(\Lambda\) is a superset of \(\Gamma\).

**Definition 5.1.** \(\tilde{x} = \{\tilde{x}_n\} \in \Lambda\) is called a fuzzy analytic sequence \(\Lambda\), if \(\tilde{\rho}(\tilde{x}_n, \tilde{0}) \leq M,\) for all \(n\) and some constant \(M > 0\). The set of all fuzzy analytic sequences is denoted by \(\Lambda(R)\). Note that \(\Gamma(R) \subset \Lambda(R)\).

Without proof, we state the following theorems.

**Theorem 5.2.** \((\Lambda(R), \tilde{\theta})\) is a complete fuzzy metric space.

**Proof.** It is similar to the proof of Theorem 3.3. □

**Theorem 5.3.** \(\Gamma(R)\) is a closed subspace of \(\Lambda(R)\).

**Proof.** It is similar to the proof of Theorem 4.3. □

6. Conclusion

In this paper, we try to introduce the concept of fuzzy entire sequence spaces and we deal with some topological properties also. Many known sequence spaces can be fuzzified. There is considerable scope for further research in this area like in matrix transformation.

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Call for Papers

Virtually every atmospheric scientist agrees that climate change—most of it anthropogenic—is occurring rapidly. This includes, but is not limited to, global warming. Other variables include changes in rainfall, weather-related natural hazards, and humidity. The Intergovernmental Panel on Climate Change (IPCC) issued a major report earlier this year establishing, without a doubt, that global warming is occurring, and that it is due to human activities.

Beginning about two decades ago, scientists began studying (and speculating) how global warming might affect the distribution of infectious disease, with almost total emphasis on vector-borne diseases. Much of the speculation was based upon the prediction that if mean temperatures increase over time with greater distance from the equator, there would be a northward and southward movement of vectors, and therefore the prevalence of vector-borne diseases would increase in temperate zones. The reality has been more elusive, and predictive epidemiology has not yet allowed us to come to conclusive predictions that have been tested concerning the relationship between climate change and infectious disease. The impact of climate change on infectious disease is not limited to vector-borne disease, or to infections directly impacting human health. Climate change may affect patterns of disease among plants and animals, impacting the human food supply, or indirectly affecting human disease patterns as the host range for disease reservoirs change.

In this special issue, Interdisciplinary Perspectives on Infectious Diseases is soliciting cross-cutting, interdisciplinary articles that take new and broad perspectives ranging from what we might learn from previous climate changes on disease spread to integrating evolutionary and ecologic theory with epidemiologic evidence in order to identify key areas for study in order to predict the impact of ongoing climate change on the spread of infectious diseases. We especially encourage papers addressing broad questions like the following. How do the dynamics of the drivers of climate change affect downstream patterns of disease in human, other animals, and plants? Is climate change an evolutionary pressure for pathogens? Can climate change and infectious disease be integrated in a systems framework? What are the relationships between climate change at the macro level and microbes at the micro level?

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The smallest primitive employed for describing an image is the pixel. However, analyzing an image as an ensemble of patches (i.e., spatially adjacent pixels/descriptors which are treated collectively as a single primitive), rather than individual pixels/descriptors, has some inherent advantages (i.e., computation, generalization, context, etc.) for numerous image and video content extraction applications (e.g., matching, correspondence, tracking, rendering, etc.). Common descriptors in literature, other than pixels, have been contours, shape, flow, and so forth.

Recently, many inroads have been made into novel tasks in image and video content extraction through the employment of patch-based representations with machine learning and pattern recognition techniques. Some of these novel areas include (but are not limited to):

- Object recognition/detection/tracking
- Event recognition/detection
- Structure from motion/multiview

In this special issue, we are soliciting papers from the image/video processing, computer vision, and pattern recognition communities that expand and explore the boundaries of patch representations in image and video content extraction.

Relevant topics to the issue include (but are not limited to):

- Novel methods for identifying (e.g., SIFT, DoGs, Harris detector) and employing salient patches
- Techniques that explore criteria for deciding the size and shape of a patch based on image content and the application
- Approaches that explore the employment of multiple and/or heterogeneous patch sizes and shapes during the analysis of an image
- Applications that explore how important relative patch position is, and whether there are advantages in allowing those patches to move freely or in a constrained fashion
- Novel methods that explore and extend the concept of patches to video (e.g. space-time patches/volumes)
- Approaches that draw upon previous work in structural pattern recognition in order to improve current patch-based algorithms
- Novel applications that extend the concept of patch-based analysis to other, hitherto, nonconventional areas of image and video processing, computer vision, and pattern recognition
- Novel techniques for estimating dependencies between patches in the same image (e.g., 3D rotations) to improve matching/correspondence algorithmic performance

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Distributed source coding (DSC) is a new paradigm based on two information theory theorems: Slepian-Wolf and Wyner-Ziv. Basically, the Slepian-Wolf theorem states that, in the lossless case, the optimal rate achieved when performing joint encoding and decoding of two or more correlated sources can theoretically be reached by doing separate encoding and joint decoding. The Wyner-Ziv theorem extends this result to lossy coding. Based on this paradigm, a new video coding model is defined, referred to as distributed video coding (DVC), which relies on a new statistical framework, instead of the deterministic approach of conventional coding techniques such as MPEG standards.

DVC offers a number of potential advantages. It first allows for a flexible partitioning of the complexity between the encoder and decoder. Furthermore, due to its intrinsic joint source-channel coding framework, DVC is robust to channel errors. Because it does no longer rely on a prediction loop, DVC provides codec independent scalability. Finally, DVC is well suited for multiview coding by exploiting correlation between views without requiring communications between the cameras.

High-quality original papers are solicited for this special issue. Topics of interest include (but are not limited to):

- Architecture of DVC codec
- Coding efficiency improvement
- Side information generation
- Channel statistical modeling and channel coding
- Joint source-channel coding
- DVC for error resilience
- DVC-based scalable coding
- Multiview DVC
- Complexity analysis and reduction
- DSC principles applied to other applications such as encryption, authentication, biometrics, device forensics, query, and retrieval

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Special Issue on
Social Image and Video Content Analysis

Call for Papers
The performance of image and video analysis algorithms for content understanding has improved considerably over the last decade and their practical applications are already appearing in large-scale professional multimedia databases. However, the emergence and growing popularity of social networks and Web 2.0 applications, coupled with the ubiquity of affordable media capture, has recently stimulated huge growth in the amount of personal content available. This content brings very different challenges compared to professionally authored content: it is unstructured (i.e., it needs not conform to a generally accepted high-level syntax), typically complementary sources are available when it is captured or published, and it features the ‘user-in-the-loop’ at all stages of the content life-cycle (capture, editing, publishing, and sharing). To date, user provided metadata, tagging, rating and so on are typically used to index content in such environments. Automated analysis has not been widely deployed yet, as research is needed to adapt existing approaches to address these new challenges.

Research directions such as multimodal fusion, collaborative computing, using location or acquisition metadata, personal and social context, tags, and other contextual information, are currently being explored in such environments. As the Web has become a massive source of multimedia content, the research community responded by developing automated methods that collect and organize ground truth collections of content, vocabularies, and so on, and similar initiatives are now required for social content. The challenge will be to demonstrate that such methods can provide a more powerful experience for the user, generate awareness, and pave the way for innovative future applications.

This issue calls for high quality, original contributions focusing on image and video analysis in large scale, distributed, social networking, and web environments. We particularly welcome papers that explore information fusion, collaborative techniques, or context analysis.

Topics of interest include, but are not limited to:
- Knowledge-driven analysis and reasoning in social network environments
- Classification, structuring, and abstraction of large-scale, heterogeneous visual content
- Multimodal person detection and behavior analysis for individuals and groups
- Collaborative visual content annotation and ground truth generation using analysis tools
- User profile modeling in social network environments and personalized visual search
- Visual content analysis employing social interaction and community behavior models
- Using folksonomies, tagging, and social navigation for visual analysis

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EURASIP Journal on Image and Video Processing

http://www.hindawi.com
Special Issue on
Human-Centric Applications of Distributed Camera Networks

Call for Papers

In a camera network, access to multiple sources of visual data often allows for making more comprehensive interpretation of events and activities. Vision-based sensing fits well within the notion of pervasive sensing and computing environments, enabling novel user-centric applications. In such applications, the actions of the users and their interactions with the environment are detected and interpreted by the network of cameras, and proper services or responses are offered based on the context.

Gesture recognition problems have been extensively studied in human computer interactions (HCIs), where often a set of predefined gestures is used for delivering instructions to machines. However, passive gestures predominate in behavior descriptions in many applications. Some traditional application examples include surveillance and security applications, while novel application classes arise in emergency detection in elderly care and assisted living, video conferencing, creating human models for gaming and virtual environments, and biomechanics applications analyzing human movements. Through pervasive visual sensing and collaborative processing, distributed camera networks offer the potential of a generalized HCI environment, in which the network reacts to various intentional gestures of the users stated, for example, via hand movements or gazing at a region of interest, as well as to unintentional posture changes caused by events such as accidental falls in assisted living applications.

Application development based on visual information obtained via multiple cameras requires new methodologies to efficiently fuse the data in the network. In a multi camera network, the option to employ local processing of acquired video at the source camera facilitates operation of scalable vision networks by avoiding transfer of raw images. Embedded processing utilizes the increasingly available computing power at the source to extract features from the images, which are exchanged with other cameras. Additional motivation for distributed processing stems from an effort to preserve privacy of the network users while offering services in applications such as assisted living. In a distributed processing framework, data fusion can occur across the three dimensions of 3D space (multiple views), time, and feature levels.

The goal of this special issue is to provide a coverage of the various approaches to human-centric application development in a multi camera setting. In particular, approaches based on the distributed processing of acquired video sequences, model-based approaches for human behavior monitoring, vision-based information fusion and collaborative decision making, and interfaces between the vision network and high-level reasoning modules that provide interpretative deductions will fit well within the scope of the special issue. The special issue also aims to provide insight into algorithm and system development topics pertaining to real-world application design for smart environments.

Original papers, previously unpublished and not currently under review by another journal, are solicited to cover one or more of the following topics:

- Distributed and collaborative vision algorithms for human-centric applications
- Model-based human gesture recognition in camera networks
- Motion analysis and spatial reasoning for behavior models
- Spatiotemporal data and estimate fusion techniques
- High-level interpretation of human activities
- Activity monitoring in crowds
- Environment discovery
- Detection of abnormal behavior
- Applications in smart homes and other environments

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Special Issue on
The Human Microbiome and Infectious Diseases: Beyond Koch

Call for Papers
A century after Robert Koch linked individual cultured microbes to specific diseases (Koch’s postulates), it is increasingly apparent that the complex community of microorganisms associated with the human body (the “microbiome”) plays a key role in health and disease. The National Institute of Health (NIH) recently announced the Human Microbiome Project and among its goals is to understand the relationship between host-associated microbial communities and disease. Many physicians and researchers, however, have only passing familiarity with the concepts involved in the study and therapeutic manipulation of complex microbial communities. The aims of this special issue are (1) to familiarize the readers with the concepts and methods for the study of complex microbial communities, (2) to demonstrate how changes in the indigenous microbial community can play a role in diseases such as antibiotic-associated diarrhea, bacterial vaginosis, and cystic fibrosis, and (3) to review how probiotics may hold promise for the therapeutic manipulation of the indigenous microbiota. Review articles and original research papers are being sought for this special issue.

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