Erratum


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We provide a correction to the proof of the main result in Crupi and Tentori (2013). © 2014 Elsevier B.V. All rights reserved.

Michael Schippers (University of Oldenburg) pointed out to us in personal correspondence an error in the proof of the main result in Crupi and Tentori [1]. The flaw spotted by Schippers is that Lemma 2 (p. 369) does not hold in its original formulation: the scheme of assignment there defined does not guarantee that one ends up with a probabilistically coherent set of values. In order to amend and validate the proof, it is sufficient to replace Lemma 2 and the subsequent lines (up to Lemma 3) by the following.

Lemma 2 (Corrected). For any \( x, y_1, y_2 \) such that \( x \in [0,1], y_1, y_2 \in (0,1) \), there exist \( e, h_1, h_2 \in L_c \) and \( P'' \in \mathbb{P} \) such that \( P''(h_1 \mid e) / P''(h_1) = P''(h_2 \mid e) / P''(h_2) = x \), \( P''(h_1) = y_1 \), and \( P''(h_2) = y_2 \).

Proof [Corrected]. Let \( w \in (0,1) \) be given so that \( w < (1 - y_1) / (1 - xy_1) \), \( (1 - y_2) / (1 - xy_2) \) (as the latter quantities must all be positive, \( w \) exists). The equalities in Lemma 2 arise from the following scheme of probability assignments

\[
\begin{align*}
P''(h_1 \land h_2 \land e) &= x^2 y_1 y_2 w; \\
P''(-h_1 \land h_2 \land e) &= (1 - xw)^2 y_1 y_2 w; \\
P''(h_1 \land \neg h_2 \land e) &= x y_1 (1 - xy_2) w; \\
P''(-h_1 \land \neg h_2 \land e) &= (1 - xy_1) (1 - xy_2) w; \\
P''(h_1 \land \neg h_2 \land \neg e) &= (1 - xw) y_1 \left[1 - \frac{(1 - xw)y_2}{1 - w}\right]; \\
P''(-h_1 \land \neg h_2 \land \neg e) &= \left[1 - \frac{(1 - xw)y_1}{1 - w}\right]\left[1 - \frac{(1 - xw)y_2}{1 - w}\right] (1 - w). \\
\end{align*}
\]

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Suppose there exist \((x, y_1), (x, y_2) \in D_k\) such that \(k(x, y_1) \neq k(x, y_2)\). Then, by Lemma 2 \[Corrected\] and the definition of \(D_k\) (see Crupi and Tentori [1, p. 369]), there exist \(e, h_1, h_2 \in L_c\) and \(P'' \in \mathcal{P}\) such that \(P''(h_1|e)/P''(h_1) = P''(h_2|e)/P''(h_2) = x\), \(P''(h_1) = y_1\), \(P''(h_2) = y_2\), and \(P''(e) = w\). By the probability calculus, if the latter equalities hold, then
\[
P''(h_1 \land e) \leq P''(h_1) P''(e), \quad P''(h_2 \land e) \leq P''(h_2) P''(e),
\]
and moreover \(P''(e|h_1)/P''(e) = P''(e|h_2)/P''(e) = x\). Thus, there exist \(e, h_1, h_2 \in L_c\) and \(P'' \in \mathcal{P}\) such that either \(C_{P''}(h_1, e) = k(x, y_1) \neq k(x, w) = C_{P''}(e, h_1)\) even if \(P''(h_1 \land e) \leq P''(h_1) P''(e)\), or \(C_{P''}(h_2, e) = k(x, y_2) \neq k(x, w) = C_{P''}(e, h_2)\) even if \(P''(h_2 \land e) \leq P''(h_2) P''(e)\), contradicting axiom A2 (see Crupi and Tentori [1, p. 365]). Conversely, A2 implies that, for any \((x, y_1), (x, y_2) \in D_k\), \(k(x, y_1) = k(x, y_2)\). So, for A2 to hold, there must exist a function \(m\) such that, for any \(e, h \in L_c\) and \(P \in \mathcal{P}\), if \(P(h \land e) \leq P(h) P(e)\), then
\[
C_{P}(h, e) = m[P(h|e)/P(h)]
\]
and \(m(x) = k(x, y)\). We then posit \(m : [0, 1] \to \mathbb{R}\) and denote the domain of \(m\) as \(D_m\). \(\Box\)

Up to Lemma 2 \[Corrected\] and then again from Lemma 3 on, the proof proceeds unchanged.\(^1\)

References


\(^1\) Meanwhile, a self-contained and corrected version of the proof is available here: http://www.vincenzocrupi.com/website/wp-content/uploads/2014/02/proof_corrected.pdf.