Abstract—In this paper we analyze the average delay and total energy consumption of a hybrid automatic repeat request (ARQ) system taking into account the energy of the transmitter and receiver. We use the random coding bound and cutoff rate in our analysis. We demonstrate that there is an optimum code rate that achieves the minimum delay and energy consumed. To illustrate our analysis, we give examples using binary phase shift keying (BPSK) and 8PSK coherent detection over an additive white Gaussian noise (AWGN) channel. The results show that there is an optimum code rate that achieves a minimum delay and energy consumed. When the receiver’s energy is negligible compared to the transmitter’s energy, BPSK has a better performance than 8PSK. The case is reverse if receiver’s energy requirement dominates the total energy consumed.

I. INTRODUCTION

In an automatic repeat request (ARQ) protocol, the system detects frames with errors at the receiving data link control (DLC) module and then request the transmitting DLC module to repeat the information in those erroneous frames. When error correction codes are added to an ARQ protocol, the resultant schemes are known as hybrid ARQ protocols. These schemes are often employed to improve the throughput-delay performance and to increase network reliability [1]-[4]. One of the key performance metrics for hybrid-ARQ is the average delay, which is the average time needed for a successful transmission.

In [5], Lee analyzed the performance of a delay-constrained hybrid ARQ wireless system assuming the two-state Markov chain model for packet errors, using Bose Chaudhuri Hocquenghem (BCH) codes with hard decision decoding. Other research considering delay constraints and pure ARQ include [6], in which packet-dropping probability at the receiver due to excessive delay was obtained. Turin and Zorzi [7] generalized the result of [6] to include batch Markovian arrivals. Furthermore, Zorzi [8] analyzed three different packet-dropping models at the sender, including one for excessive delay. In [9], Zhou showed that by using adaptive subpacket transmission scheme for hybrid ARQ protocol, the system can provide higher throughput and smaller delay when convolutional coding is employed.

In our study, we consider two objectives. One objective is to minimize the average transmission power required to reliably transmit the data. In a wireless network, mobile users often rely on a battery with a limited amount of energy; minimizing the average transmission power leads to a more efficient utilization of battery energy. The second objective is to minimize the average delay incurred by the data. There are many aspects of the above description that need to be more precisely defined; this will be done in Section II.

In the study of a hybrid ARQ system, most research ignore power consumed by the receiver and use a specific coding scheme to evaluate performance [1]-[5]. In our paper, we take into account of the receiver’s power and use cutoff rate, $R_0$ [10]-[12] to analyze the performance of a hybrid ARQ protocol, taking into account the medium access control (MAC) and physical layers. The cutoff rate represents the practical maximum information rate of a coded communication system. It allows us to relate the performance to the packet length and code rate and appears in the union bound on the probability of error for coding and modulation systems thus, serves as a simple measure of reliability. It has long been used as a figure of merit for coding and modulation systems and this has been justified in well-known works, such as [13] and [14].

The rest of this paper is structured as follows. In Section II, we describe the system model used in our study. We analyze the performance of the system with the assumptions made. In Section III, we give examples for our analysis and Section IV concludes the paper.

II. SYSTEM MODEL AND ANALYSIS

We begin by formally describing the system model, stating the assumptions made. In our system, we encode a packet of $K$ data into $N$ coded symbols. The coded symbols are transmitted through an additive white Gaussian noise (AWGN) channel with power spectral density $N_0/2$. The code rate for this system is given by $R = K/N$. For example, if we use 8PSK modulation without any channel coding, the code rate of the system is 3 since $K = 3$ bits and $N = 1$ symbol per dimension. We assume that the packet is always received and the receiver sends an acknowledgement (ACK) or negative acknowledgement (NACK) to the transmitter via an error-free channel. We also assume that the packet transmission time is much longer than the receiver’s feedback time. Therefore, the delay is time taken for a
successful transmission. Since the packet transmission time is proportional to packet length, we normalized the delay to be the size of the packet length.

We use the random coding bound to compute the total energy consumed by the transmitter and receiver. Given a codeword of length $N$ and rate $R$, the average codeword error probability is bounded by [10]

$$P_E(\frac{E_c}{N_0}, N) < 2^{-N(R_0-R)} = 2^{-N R_0} 2^K$$

(1)

where $R_0$ is the cutoff rate and $\frac{E_c}{N_0}$ is the signal-to-noise ratio per coded bit.

Given a distance $d$ between the receiver and transmitter, the energy consumed at the transmitter per bit ($E_t$), with transmitter power amplifier efficiency $\eta$, is

$$E_t = \frac{N_0}{\eta f(d)} \frac{E_c}{RN_0}.$$  

(2)

Here $f(d)$ is the attenuation function with respect to distance. For example, when $d$ is large, $f(d) = \zeta / d^4$ where $\zeta$ is a constant related to the square of the product of height of the transmitting and receiving antenna [15].

At the receiver, we assume most of the energy is consumed at the front end. For a bandwidth $W$, the energy, $E_r$, consumed per information bit by the receiver with processing power $P_{rp}$ is [16]

$$E_r = \frac{NP_{rp}}{KW}.$$  

(3)

Therefore, the total consumed energy is given by

$$E_{tot} = E_t + E_r = \frac{N_0}{\eta f(d)} \frac{E_c}{RN_0} + \frac{NP_{rp}}{KW}.$$  

(4)

We normalized the total energy as follows

$$E_T = \eta f(d) E_{tot}.$$  

(5)

The normalized total signal-to-noise ratio is given by

$$\frac{E_T}{N_0} = \frac{E_c}{RN_0} + \frac{\gamma}{R}$$  

(6)

where $\gamma = \frac{\eta f(d) P_{rp}}{W N_0}$ is the normalized receiver’s energy [16].

We equate the packet error probability in (1) and obtain the following expression for the probability of a correct packet, $P_c$

$$P_c = 1 - P_E(\frac{E_c}{N_0}, N) = 1 - 2^{-N R_0} 2^K.$$  

(7)

Let $X$ be a random variable for the number of trials needed to get one successful transmission. Then the probability that $s$ trials are needed to get one successful transmission is

$$Pr(X = s) = (1 - P_c)^{s-1} P_c.$$  

(8)

Thus, for a data packet of length $N$ and error probability $P_E(\frac{E_c}{N_0}, N)$, the average delay is

$$\bar{D} = \frac{N}{1 - P_E(\frac{E_c}{N_0}, N)}$$  

(9)

and the total average energy consumed is given by

$$\bar{E}_T = \frac{E_c}{N_0} \frac{N}{1 - P_E(\frac{E_c}{N_0}, N)} = \frac{N \frac{E_c}{N_0} + \gamma}{1 - P_E(\frac{E_c}{N_0}, N)}.$$  

(10)

Our goal is to minimize the average delay (and average total energy consumed). We first consider $E_c$ fixed. Clearly, optimizing the delay over $N$ will also optimize the energy with respect to $N$. To compute the optimal $N$, we differentiate (9),

$$\frac{\partial \bar{D}}{\partial N} = \frac{1}{1 - P_E(\frac{E_c}{N_0}, N)} + \frac{N}{[1 - P_E(\frac{E_c}{N_0}, N)]^2} \frac{\partial P_E(\frac{E_c}{N_0}, N)}{\partial N}$$  

(11)

Here we treat $N$ as a continuous variable. Note that

$$\frac{\partial P_E(\frac{E_c}{N_0}, N)}{\partial N} = \frac{\partial[^{[2K}e^{-N R_0 \ln(2)}]}{\partial N}$$

$$= 2^{K-2}(-R_0 \ln(2))$$

$$= P_E(\frac{E_c}{N_0}, N)(-R_0 \ln(2))$$  

(12)

Thus

$$\frac{\partial \bar{D}}{\partial N} = \frac{1}{1 - P_E(\frac{E_c}{N_0}, N)}$$

$$+ \frac{N}{[1 - P_E(\frac{E_c}{N_0}, N)]^2} P_E(\frac{E_c}{N_0}, N)(-R_0 \ln(2))$$

$$= 1 - P_E(\frac{E_c}{N_0}, N) \ln(2^{-N R_0})$$

$$[1 - P_E(\frac{E_c}{N_0}, N)]^2$$  

(13)

The resulting packet error probability is

$$P_E(\frac{E_c}{N_0}, N^*) = \frac{1}{1 + N^* R_0 \ln 2},$$  

(14)

where $N^*$ is the optimal packet length. When $K$ is large, we can approximate (14) [17]

$$P_E(\frac{E_c}{N_0}, N^*) \approx \frac{1}{1 + K \ln 2}. $$  

(15)

Now we can solve for the approximate optimal packet length $N^*$ by solving

$$P_E(\frac{E_c}{N_0}, N^*) = 2^{K - 2}(-R_0).$$  

(16)

The approximate average minimum delay and average total energy consumed is

$$\min_N \bar{D} \approx \left[ K \frac{1}{R_0} + \log_2(K \ln(2)) \right]$$  

(17)

$$\min_N \bar{E}_T \approx \left( \frac{E_c}{N_0} + \gamma \right) \left[ \frac{1}{R_0} + \frac{\log_2(K \ln(2))}{K R_0} \right].$$  

(18)
and the approximated optimal code rate is
\[ R^* = \frac{K}{N^*} \approx R_0 + \frac{1}{N} \log_2 \left( 1 + K \ln(2) \right). \tag{19} \]

Note that we do not make any assumption here about what the channel might be (except that it is memoryless). Therefore, the above results apply to any channel. We just need to substitute the corresponding \( R_0 \) into the formulae.

In the next section, we give examples using the cutoff rate for binary phase shift keying (BPSK) and 8PSK with coherent detection.

III. ILLUSTRATIVE RESULTS

We consider the AWGN channel with coherent detection, normalized receiver power \( \gamma = 3 \) dB and 300 information bits (for 8PSK, it is 100 information symbols since 1 symbol consists of 3 bits). For binary antipodal signalling over an AWGN channel with coherent detection, the cutoff rate is \( R_0 = 1 - \log_2 \left( 1 + \exp \left( -\frac{E_c}{N_0} \right) \right) \) \( \tag{20} \)
with \( \frac{E_c}{N_0} = \frac{RE_b N_0}{2} \). In the case of 8PSK, the cutoff rate expression is \( R_0 = -\log_2 \left( \frac{1}{64} \sum_{l=1}^{8} \sum_{m=1}^{8} \exp \left( -\frac{d_{lm}^2}{4N_0} \right) \right) \) \( \tag{21} \)
where \( d_{lm} \) is the Euclidean distance between two symbols.

The relationship between total energy consumed and code rate for the case of BPSK signalling is shown in Figure 1 for \( \frac{E_c}{N_0} = 1, 2 \) and 3 dB. From the figure, we observe that there is an optimum code rate for every different value of \( \frac{E_c}{N_0} \). The optimum code rate increases as \( \frac{E_c}{N_0} \) increases. This is because with higher \( \frac{E_c}{N_0} \), each coded bit is more reliable and therefore the system is able to transmit at a higher rate. The variation of minimum average total energy consumed with code rate in is shown as the lower envelope curve in the figure. The optimum code rate is about 0.775 for \( K = 300 \) bits, \( \frac{E_c}{N_0} = 2.5 \) dB and \( \gamma = 3 \) dB.

In Figure 2, we present the relationship between delay and code rate. We observe that there is an optimum code rate for each different \( \frac{E_c}{N_0} \), similar to the the total energy - code rate relation. In addition, the lower envelope shows that the average minimum delay decreases monotonically with code rate. This is because when \( \frac{E_c}{N_0} \) is high, the optimum code rate is high as each coded bit is more reliable and the system is able to transmit at a higher rate to achieve a lower delay.

For 8PSK signalling with coherent detection, the energy and average delay performance is shown in Figure 3 and 4 respectively. We observe that the optimum code rate is greater than 1 as each symbol consists of 3 bits. Since there are 3 bits per symbol, the delay in Figure 4 (using 8PSK), is lower than the one in Figure 2 (using BPSK). Similar to the case of BPSK, the average minimum delay in decreases monotonically with code rate, as shown in Figure 4.

In Figure 5, we compare the performance of BPSK
and 8PSK using different $\gamma$ values. When the receiver requires a higher proportion of energy as compared to the transmitter, i.e., $\gamma \gg \frac{E_c}{N_0}$, 8PSK has a better performance than BPSK as the system performs better at higher rate. On the other hand, if the receiver’s power is negligible, BPSK outperforms 8PSK as a lower code rate enable a better error protection on the transmitted symbols.

IV. CONCLUSION

We analyzed the delay and energy consumed of a hybrid ARQ system, taking into account of the transmitter and receiver’s power. In our analysis, we employed the random coding bound and cutoff rate in our study. We show that there exists an optimum code rate for both the average delay and the total energy consumed. We illustrate our analysis using BPSK and 8PSK with coherent detection over an AWGN channel. The optimum code rate value is higher for 8PSK as compared to BPSK as 3 bits are used to represent a symbol in 8PSK. We demonstrated that the average minimum delay decreases monotonically with code rate and there is an optimum code rate for the average minimum total energy consumed. When the receiver’s processing energy is negligible as compare to the transmitter’s energy, BPSK has a better performance than 8PSK. However, the case is reverse if the transmitter’s energy is negligible as compared to the receiver’s energy.

REFERENCES