Performance Analysis of Coded Cooperation with Hierarchical Modulation

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Abstract—In order to achieve the improved diversity over the original coded cooperation in fast fading channel, the novel cooperation strategy with hierarchical modulation is proposed in this paper, where users send both their own as well as their partners’ parity bits with hierarchical error protection during the cooperation frame. Only the frame-level synchronization is necessary when using the proposed strategy, which facilitates the implementation on the uplink. The bounds for the block-error rate (BLER) of the proposed strategy are analyzed and the simulation results given.

I. INTRODUCTION

Recently cooperative communication provides a new way of introducing spatial diversity in the wireless systems where the mobile stations may not be able to support multiple antennas due to size or other constraints. By creating a “virtual array” through cooperation, cooperative diversity emerges as a special form of spatial diversity [1]. Some repetition-based cooperative diversity algorithms such as amplify-and-forward (AF) and decode-and-forward (DF) are developed to fully exploit the spectral diversity for reducing the outage probability at the cost of decreasing spectral efficiency [2]. The authors in [3] have proposed several coded cooperation techniques for increasing spectral efficiency while maintaining the diversity gain. In the original coded cooperation strategy, users transmit only their partners’ data in the cooperation frame whenever possible, which can achieve full diversity and give significant performance improvement for the case of slow fading. But this strategy can not provide the added benefit from diversity gain and suffers from cooperation imbalances between users operating in fast fading environment. Then, the space-time cooperation is proposed to achieve improved diversity over the original coded cooperation and reduce the impact of cooperation imbalances [4]. However, synchronization for space-time cooperation especially in time division multiple access (TDMA) systems becomes a serious problem because the symbol-level synchronization between users in the uplink can’t be guaranteed.

On the other hand, the hierarchical transmission scheme with multi-level coding and multi-resolution modulation has already been used for the broadcast services, which provides different service quality for the mobile station with different channel condition. It can offer a mechanism to provide multiple error protection in the same transmission, where the higher priority data provided by a high-priority (HP) base layer and lower priority data provided by a low-priority (LP) enhancement layer. Recently, the hierarchical modulation has already been used with DF relaying to deal with the signal-to-noise-ratio (SNR) difference between the source to relay and the source to destination [5][6]. Now let us consider the situation in the cooperative network instead of relaying. The user’s own bits and its partner’s bits can be regarded as two prioritized data streams from the perspective of the user itself. Then, it is also expected to use the hierarchical transmission as the extension of coded cooperation.

Motivated by these considerations, this paper presents the novel cooperation strategy with hierarchical modulation, called as hierarchical cooperation communication, where users send both their own as well as their partners’ parity bits with hierarchical error protection during the cooperation frame. Compared with the original coded cooperation strategy, our proposed method can provide the added benefit of diversity gain and reduce the impact of cooperation imbalances. Only frame-level synchronization is necessary when using this proposed strategy in uplink which facilitates the implementation.

II. SYSTEM MODEL

Let us consider the cellular system where two users are communicating with a base station (BS). The channels between each user and the BS are independent of each other and independent of the channel between users (inter-user channel). All channels are assumed to be flat fading for the sake of simplification. The data transmitted from users to BS is split into packets and protected against transmission errors by a channel encoder before being mapped to a particular modulated signal constellation. At time instant $t$, the modulator output of user $i \in \{1, 2\}$ is $s_t^{(i)}$. The corresponding signal received by user $j \in \{0, 1, 2\}$ ($j \neq i$, and $j = 0$ denotes the BS) is

$$y_t^{(i,j)} = \sqrt{E_s} \alpha_t^{(i,j)} s_t^{(i)} + z_t^{(i)}$$

(1)

where $E_s$ is the transmitted energy per symbol for user $i$, $\alpha_t^{(i,j)}$ is the independent complex valued Rayleigh fading coefficient between user $i$ and $j$, while $z_t^{(i)}$ accounts for the
noise plus interference at the receiver. For slow fading (quasi-
static) cases, the fading coefficients are constant during the
transmission of each source block. For fast fading cases, they
change independently for each transmitted frame. The noise
term $z_{t}^{(i,j)}$ is modeled as independent and identically distributed
(i.i.d.) zero-mean complex Gaussian noise with variance $\sigma^2_{i,j}$.
We use the idealized simplifying assumption that the channel
state information is perfectly known and coherent detection is
applied at the receiver. Then, we can define the instantaneous
SNR at each link as
\begin{equation}
\gamma_{t}^{(i,j)} = \frac{E_{s}|\alpha_{t}^{(i,j)}|^2}{\sigma^2_{i,j}} \tag{2}
\end{equation}
where $\gamma_{t}^{(i,j)}$ has an exponential distribution with a mean of
$\bar{\gamma}_{t}^{(i,j)} = \frac{1}{\frac{\gamma_{t}^{(i,j)}}{\sigma^2_{i,j}}} = \frac{1}{\frac{\alpha_{t}^{(i,j)}}{\sigma^2_{i,j}}}$.

III. PROPOSED HIERARCHICAL CODED COOPERATION
A. Original Coded Cooperation Strategy

To demonstrate the motivation behind the proposed tech-
nique, let us first take a close look at the performance of the
original coded cooperation strategy. Without loss of generality,
two users are assumed to be allocated orthogonal channels
by TDMA scheme as illustrated in Fig.1(a). The source bits of
each user are segmented into blocks with size of $K$ bits
including cyclic redundancy check (CRC) bits. Each source
block is encoded into a overall rate $R$ codeword with $N = \frac{K}{R}$
bits. In the first frame, user $i$ transmits the first $N_1$ bits of its
codeword with rate $R_1 = \frac{K}{N_1}$ to BS and user $j$. If user $j$ is able to decode it correctly (which can be
checked by using CRC), it then transmits the remaining $N_2$
of user $i$’s codeword to the base station in the second frame
($N_1 + N_2 = N$). For example, if the first frame was obtained
via puncturing, these $N_2$ bits could be the puncture bits left out
of the first frame. Otherwise, it will switch to non-cooperative
mode by sending its own $N_2$ remaining coded bits. The level
of cooperation is defined as $\rho = \frac{N_2}{N}$. All of these are done
automatically through code design, and there is no need for
each user knowing whether his partner successfully decoded
their data.

At each receiver, the soft reliability value expressed in the
form of log-likelihood-ratio (LLR) can be computed from
the received signal for each bit. For QPSK modulation, the
channel’s output LLRs for the odd and even bits of the first
$N_1$ and the remaining $N_2$ bits are given by
\begin{align}
L_{odd}^{(i,j)} &= \log \frac{P(x = 1|y_t^{(i,j)})}{P(x = 0|y_t^{(i,j)})} = 2\sqrt{2}\gamma_{t}^{(i,j)} Re\{\frac{y_t^{(i,j)}}{\alpha_t}\} \tag{3} \\
L_{even}^{(i,j)} &= \log \frac{P(x = 1|y_t^{(i,j)})}{P(x = 0|y_t^{(i,j)})} = 2\sqrt{2}\gamma_{t}^{(i,j)} Im\{\frac{y_t^{(i,j)}}{\alpha_t}\} \tag{4}
\end{align}
where $Re\{\}$ and $Im\{\}$ denote the real and imaginary part,
respectively.

B. Novel Hierarchical Coded Cooperation Strategy

The modulation in the hierarchical transmission is based
on multi-resolution modulation which allows one to design
a hierarchical protection scheme. Its constellation consists of
cluster of points spaced by different distances. Each cluster
may itself has sub-clusters, and so on. The minimal distance
between two clusters is larger than the minimal distance
between two sub-clusters. Then, the basic idea is to assign the
HP bits to the clusters and LP bits to the sub-clusters. Let us
take the case of multi-resolution modulation with 16 points as
a simple example. The hierarchical transmission system maps
the data onto 16 points in such a way that there is effectively
a QPSK stream embedded within QSPK stream. Furthermore,
the hierarchical ratio $\lambda = D_2/D_1$ can be adjusted to protect
the HP stream at the expense of the LP stream, where $D_1$ is
the distance between center of HP symbols and $D_2$ is the
distance between center of the LP symbols.

By introducing the hierarchical modulation into coded co-
operation, we propose a new cooperation strategy as illustrated
in Fig.1(b). In the first frame, user $i$ transmits the first $N_1$ bits
of its codeword with rate $R_1 = K/N_1$ by using a low-order
common modulation (e.g. QPSK). If user $j$, $j \neq i$ is able to
decode it correctly, it then transmits the second $N_2$ bits not
only of user $i$’s codeword but also of its own codeword to
BS in its second frame by high-order hierarchical modulation
(e.g. hierarchical 16QAM) while keeping same throughput as
the original coded cooperation. Usually, the $N_2$ bits of its own
codeword as HP stream is put on the basement bit position of
hierarchical constellation while the $N_2$ bits of the partner’s
codeword as LP stream is on the enhancement bit position.
If the inter-user channel is not reliable, the user will change
to non-cooperation mode and continue the transmission of its
own $N_2$ remaining coded bits with low-order modulation (e.g.
QPSK).

When hierarchical coded cooperation is used, the calculation of
channel LLRs for the first $N_1$ bits is similar to that with the
original coded cooperation because of the same transmission
scheme. If the cooperation between users is successful, the
remaining $N_2$ bits of user $i$ may be transmitted twice, i.e. as
HP bits in the second frame using its own channel and as
LP bits in the second frame through the partner’s channel.
Therefore, the corresponding channel LLR for the odd bits
of these $N_2$ bits can be computed through maximum ratio
combining (MRC) of those two packets:
When codeword part instead of real part of received complex signals, the channel LLR for the even bits can be given by using the imaginary value

\[ \text{PEP conditioned on the set of instantaneous received SNR} \]

coherent detection and maximum-likelihood decoding, the general, for a system of binary coded with QPSK modulation, the error probabilities are determined by using weight enumerating functions. The pairwise error probability (PEP) is developed by the tools and techniques in our framework, the terminated convolutional codes with a finite uncoded block length \( K \) and \( N \), can be obtained by using well-known enumerating techniques.

\( \gamma_{1,0} \) and \( \gamma_{2,0} \) denote the fading coefficient of channel from user \( i \in \{1, 2\} \) to BS in the first frame and the second frame, respectively. According to (2), \( \gamma_{1,0} \) and \( \gamma_{2,0} \) represent the instantaneous SNRs of these links correspondingly.

Without loss of generality, the performance of user 1 will be analyzed first. And the same results can be generated in case of user 2. As described in Section III, in the hierarchical coded cooperation transmission, user 1 transmits its first \( N_1 \) bits to BS and its partner, i.e. user 2, by QPSK modulation in the first frame. Then, if cooperation by user 2 is happened successfully, the second \( N_2 \) bits of user 1 are modulated using the hierarchical 16QAM modulation and possibly transmitted twice through two users’ channels, i.e. put not only on the basement bit position as HP stream in the hierarchical constellation of itself in the second frame through user 1’s channel, but also on the enhancement bit position as LP stream in the hierarchical constellation of its partner in the second frame through user 2’s channel. Since these twice-transmitted \( N_2 \) bits are modulated by high-order hierarchical 16QAM, the data throughput can be kept same as that in the original coded cooperation with low-order QPSK modulation. Meanwhile, the reliability of each bit in the \( N_2 \) bits of user 1 is same, which is similar to the case by QPSK modulation. Therefore, the hierarchical 16QAM modulation with coded cooperation transmission can be regarded as the virtual QPSK modulation. On the other hand, the second \( N_2 \) bits of user 1 are transmitted by QPSK modulation if no cooperation. In summary, the first \( N_1 \) bits of user 1 are transmitted by using QPSK modulation and its remaining \( N_2 \) bits may be transmitted by QPSK or virtual QPSK modulation. Then, we can rewrite (7) for user 1’s codeword as

\[ P(d|\gamma_{1,0}, \gamma_{eq}) = Q(\sqrt{d_1 \gamma_{1,0} + d_2 \gamma_{eq}}) \]

where \( d_1 \) and \( d_2 \) are the numbers of bits in the Hamming weight \( d \) that are transmitted through only user 1’s channel and both users’ channels respectively with \( d_1 + d_2 = d \), and \( \gamma_{eq} \) represents the equivalent SNR for transmitting the second \( N_2 \) bits by QPSK or virtual QPSK modulation. It is noted that \( d_1 \) and \( d_2 \) are independent on \( \gamma_{1,0} \) and \( \gamma_{eq} \). Furthermore, the computation of \( \gamma_{eq} \) depends on the four possible happened events as shown in Fig.2, also it is the function of \( \gamma_{1,0} \) and \( \gamma_{2,0} \).

B. Block Error Rate (BLER)

The union bounds for the BLER as a function of the PEP can be obtained by using well-known enumerating techniques. In our framework, the terminated convolutional codes with a finite uncoded block length \( K \) and coded block length \( N \) are considered. Then, the bounds for the BLER using the weight enumerating function of the equivalent block code can be
calculated as

$$P_{\text{block}}(\gamma) \leq \sum_{d, d_f} \sum_{w=1}^{K} a_{w,d} P(d|\gamma)$$

where $d_f$ is the free distance of the code, $a_{w,d}$ is the multiplicity of codewords corresponding to input weight $w$ and output weight $d$. This expression employs a standard technique used for ideally interleaved fading channels and yields simple and tractable results but very loose. Then, we can use the limit-before-average technique with the appropriate conditional PEP expressions to obtain tight bounds for fast fading channels [8]. Then, the average block error probability becomes

$$P_{\text{block}}(\gamma) \leq \min\left(1, \sum_{d, d_f} \sum_{w=1}^{K} a_{w,d} P(d|\gamma)\right)$$

To obtain the BLER of Case $m$, we must take the expected values of (8) over the distributions of fading coefficients:

$$P_{\text{block}}(\text{Case } m) \leq \int_0^\infty \int_0^\infty \int_0^\infty P_{\text{block}}(\gamma) p(\gamma_{1,0}) p(\gamma_{2,0}) d\gamma_{1,0} d\gamma_{2,0}$$

where $p(x)$ is the probability density function (pdf) of random variable $x$. Under the Rayleigh fading channel, the fading coefficients $\gamma_{1,0}$ have an exponential distribution. So the BLER of user 1 can be rewritten as

$$P_{\text{block}}(\text{Case } m) \leq \frac{1}{\sum_{d_f}^{2} - \sum_{d_f}^{1} \sum_{d_f}^{2} \min(1, \text{BLER}_{\text{union}}) \frac{\gamma_{1,0}^{-1} d\gamma_{1,0}^{-2} d\gamma_{2,0}}{\gamma_{1,0}^{-1} d\gamma_{1,0}^{-2} d\gamma_{2,0}}$$

where $\gamma_{1,0}^{(1,0)}$ and $\gamma_{2,0}^{(2,0)}$ stand for the average SNR of the channels between user 1 and user 2 to BS respectively, and

$$\text{BLER}_{\text{union}} = \sum_{d_f}^{w} \sum_{w=1}^{K} a_{w,d} \left( \gamma_{eq}^{(1,0)} (d_f) + (d - d_f) \gamma_{eq}^{(2,0)} (1,0) \right)$$

### C. Overall Block Error Rate

The overall unconditional BLER is equal to the average of the BLERs over four possible transmission cases discussed before:

$$P_{\text{block}} = \sum_{m=1}^{4} P_{\text{block}}(\text{Case } m) P(\text{Case } m)$$

where $P(\text{Case } m)$ is the probability of occurrence if Case $m$, whose bound for each of the four cases is obtained from the BLER as (12). The calculation of $P(\text{Case } m)$ is omitted here but can be found in [9]. Finally we can obtain the overall BLER based on (14).

### V. Simulations and Numerical Results

In order to evaluate the performance of hierarchical coded cooperation transmission, we employ the family of rate-compatible punctured convolutional (RCPC) codes with memory $M = 7$, rate $R = 1/2$ mother code, and generator polynomials $G(171,133)$. The source block has $K = 200$ bits and $N = 400$ while $R_2$ depends on the cooperation level $\rho$. In our simulation and analysis, $R_1 = 5/6$ with $N_1 = 240$ and $\rho = 40\%$ is assumed under the fast fading channel. All comparisons between systems are with the equal information rate and equal code rate. The distance spectra $a_{w,d}$ is computed via computer enumeration for our analysis. QPSK is used in original coded cooperation (CC) transmission while hierarchical 16QAM in hierarchical coded cooperation (HCC) transmission. We consider the scenarios in which the average received SNRs for two users’ uplink channel are equal (statistically similar channels) and unequal (statistically dissimilar channels). This statistically similar or dissimilar due to the relative proximity of the base station, which is called as symmetric or asymmetric scenario.

In Fig.3, we first examine the analytical bound and simulation results of block error rate (BLER) performance by CC or HCC with the hierarchical ratio $\lambda = 1/2$, when both users have the similar received SNRs (i.e. symmetric scenario) and the SNR of inter-user channel is assumed to be 15dB. The proposed HCC achieves better performance than the original CC because of more diversity exploited. For instance, at BLER=10^{-2}, HCC achieves about 2dB SNR gain than CC, which demonstrates the advantage of the proposed strategy. The union bounds match well to the simulation results.

In the asymmetric scenario, the uplink average SNR of user 1 varies from 0 to 18 dB whereas the average SNR of user 2 is fixed at 18dB. Fig. 4 and Fig. 5 compare two users’ BLER performances of CC and HCC with $\lambda = 1/2$ respectively, where...
the inter-user channel average SNR is 15dB. Compared with non-cooperation, CC transmission offers no more diversity gain. It is because the channel states in the neighboring frames can be regarded as independent under fast fading channel in non-cooperation. When CC is applied, the BLER performance of user 1 with the poor channel (i.e. SNR smaller than 12dB) is improved only at the cost of the performance of user 2 with the better channel. However, in the proposed HCC, the second $N_2$ bits of users experience two independent channel before the combination at the receiver so that the performance of both users can be improved by exploiting more diversity gain in time-spatial channels. As shown in Fig.4, user 1 can achieve the gain of more than 1dB if the target BLER is $10^{-2}$ compared with the case of non-cooperation. Meanwhile, in Fig.5, the performance of user 2 with better channel can also be improved a little. These union bounds also match well to the simulation results.

VI. CONCLUSION

In this paper, we have presented the novel coded cooperation strategy with hierarchical modulation, termed as hierarchical coded cooperation strategy, which can provide the improved diversity gain over the original coded cooperation. Our numerical and simulation results demonstrate the BLER performance gain of the systems with the proposed scheme over those with the conventional coded cooperation scheme under fast fading channels.

ACKNOWLEDGMENT

This work is supported by the National Basic Research Program of China (973 Program) under Grant 2007CB310602.

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