Precoding Vector Distribution under Spatial Correlated Channel and Nonuniform Codebook Design

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Abstract—In this paper, influence of spatial correlation on codebook-based precoding technology is analyzed. Optimum precoder distribution under spatial correlated channel is achieved at first, which is no longer isotropical on the unitary matrix (vector) set. Selection probability of the matrix (vector) in codebook is not uniform too. Then, a novel codebook design is presented for spatial correlated channel, which is termed as nonuniform codebook. Codebook is designed as function of spatial correlation characteristic of channel and presents a nonuniform quantization of unitary vector set. Smaller quantization error and higher spectral efficiency can be achieved by proposed codebook design than that of traditional fixed uniform codebook design, with the cost of slight feedback overhead increasing.

I. INTRODUCTION

Theoretical work has shown that in sufficiently rich scattering environments, multiple-input-multiple-output (MIMO) systems hold the potential for enormous spectral efficiency improvements relative to single-input-single-output systems [1]. Capacity of MIMO system is derived in [1] [2]. On the assumption that full channel state information (CSI) is achievable both at transmitter and receiver, channel capacity can be approached with singular value decomposition based channel diagonalization and water-filling power allocation [3] ∼ [5]. Right singular vector matrix is multiplied on the transmit signal before antenna mapping, which is the operation of linear precoding. Unfortunately, full CSI at transmitter is difficult in practical application especially in frequency division duplex (FDD) systems. Research on limited feedback precoding is done by Love et al. [6] ∼ [8]. It’s proved that an optimum generalized precoder is the matrix constructed by the first several right singular vectors [7]. The usage of codebook allows system designer to constrain the feedback overhead of precoding to a reasonable limit. Distortion metric is defined in [7] as cost function for codebook evaluation. But the research of [6] ∼ [8] is limited on the situation that the number of data streams is smaller than the number of transmit antennas.

In 3GPP long-time-evolution (LTE) and IEEE 802.16 systems, codebook-based precoding is an important adaptive MIMO technology. Codebook designs for practical application are proposed by many companies [9] ∼ [11]. Currently most of the codebook designs focus on the situation of independent and identically distributed (i.i.d) MIMO channel, but we focus on the spatial correlated channel, in which right singular vector distribution is quite different. Traditional codebook design is presented as uniform quantization of isotropically distributed right singular vector. In this paper, a novel codebook design is presented as a quantization of non-isotropically distributed one, which is termed as nonuniform codebook.

This paper is organized as follows. System model is described in Section II. Traditional codebook design is reviewed in Section III. Distribution of right singular vector under i.i.d MIMO channel and spatial correlated channel is analyzed in Section IV. A so-called nonuniform codebook is presented in Section V. Section VI gives some numerical results and Section VII concludes this paper. $A^T$, $A^*$, $A^H$ and $A^\dagger$ denote the transposition, conjugate, conjugate transposition and pseudo-inverse of matrix $A$ respectively.

II. SYSTEM MODEL

MIMO system with $N_T$ transmit antennas and $N_R$ receive antennas could be described as follow:

$$y = Hx + n$$  \hspace{1cm} (1)

where $x$ ($N_T \times 1$) denotes the transmit signal vector, $y$ ($N_R \times 1$) denotes the receive signal vector, $H$ ($N_R \times N_T$) denotes channel transfer matrix. If linear precoding is used, (1) is rewritten as:

$$y = HF_s + n$$  \hspace{1cm} (2)
where \( \mathbf{F} (N_T \times N_S) \) denotes the precoding matrix, \( \mathbf{s} (N_S \times 1) \) denotes the original data vector, \( N_S \) is the number of data stream. System model of codebook-based precoding in FDD system is shown in Fig.1. Precoding matrix selection is done in receiver and its index in codebook is sent back to transmitter.

### III. CODEBOOK DESIGN

#### A. Theoretical Codebook Design

Codebook design and cost function for its evaluation are presented in [6] ~ [8]. It’s proved that an optimum precoding matrix \( \mathbf{V} \) is constructed by the right singular vectors of \( \mathbf{H} \):

\[
\mathbf{H} = \mathbf{USV}^H
\]

(3)
or the first several columns of \( \mathbf{V} \):

\[
\mathbf{V}_{NS} = \mathbf{V} \cdot [\mathbf{I}_{NS} \ 0_{N_S,N_T-N_S}]^T
\]

(4)
where \( \mathbf{I}_{NS} \) denotes the identity matrix with \( N_S \) rows and \( N_S \) columns, \( 0_{N_S,N_T-N_S} \) denotes an all-zero matrix with \( N_S \) rows and \( N_T-N_S \) columns. Mean minimum squared chordal distance is used in [7] for codebook evaluation:

\[
EV_{NS} \left[ \min_{i \in \{1,2,\cdots,N\}} \frac{1}{2} \left\| \mathbf{V}_{NS} \mathbf{V}^H_{NS} - \mathbf{F}_i \mathbf{F}_i^H \right\|_F^2 \right]
\]

(5)
where \( N \) is the size of codebook, \( \| \cdot \|_F \) denotes Frobenius norm, \( E[\cdot] \) denotes mathematical expectation. Meanwhile, it’s implied in (5) that the precoder selection criteria is the minimum of subspace chordal distance. It’s notable that chordal distance between two subspaces used in [7] is useless for square unitary matrix: if \( N_S = N_T \),

\[
\mathbf{V}_{NS} \mathbf{V}^H_{NS} = \mathbf{F}_i \mathbf{F}_i^H = \mathbf{I}_{NS}, \forall i \in \{1,2,\cdots,N\}
\]

(6)
Considering a special case where \( N_S = 1 \), a new cost function was defined as:

\[
EV_{NS} \left[ \max_{i \in \{1,2,\cdots,N\}} \left| \mathbf{V}^H_{NS} \cdot \mathbf{F}_i \right|^2 \right]
\]

(7)
where the precoder selection criteria is maximization of inner product [8]. Due to

\[
1 - \frac{1}{2} \left\| \mathbf{V}_{NS} \mathbf{V}^H_{NS} - \mathbf{F}_i \mathbf{F}_i^H \right\|_F^2 = \left\| \mathbf{V}^H_{NS} \cdot \mathbf{F}_i \right\|^2
\]

(8)
expression (7) is equivalent to (5).

It is shown in [13] that for i.i.d MIMO Rayleigh fading channels, right singular vector matrix \( \mathbf{V} \) is isotropically distributed on the set of unitary matrices \( \mathcal{U}(N_T,N_T) \). Codebook design is equivalent to complex Grassmannian space packing problem. Finding good Grassmannian packing for arbitrary \( N_T \) and \( N \) is actually quite difficult. Iterative optimization algorithms or random searches are suggested in [7].

Research of [6] ~ [8] can be summarized as follow:

1) An optimum precoder is constructed from the first several right singular vectors.

2) For i.i.d MIMO Rayleigh fading channels, the right singular vector matrix is isotropically distributed on the set of unitary matrices.

3) Codebook presents a quantization of right singular vector matrix. Mean quantization error is defined as (5) and (7).

#### B. Practical Codebook Design

Random searches codebook design presented in [7] is difficult for practical application. Regular and uniform codebook design is preferred for its advantage on storage and calculation complexity. According to the description in [10], square unitary matrix with \( N_T \) rows and \( N_T \) columns is equivalent to the product of \( N_T(N_T-1)/2 \) Givens matrices. Especially \( 2 \times 2 \) Givens matrix is expressed as following:

\[
\mathbf{G} = \begin{bmatrix} \cos \theta_1 & \sin \theta_1 e^{j\theta_2} \\ -\sin \theta_1 e^{j\theta_2} & \cos \theta_1 e^{j(\theta_2 + \pi)} \end{bmatrix}
\]

\[
\theta_1 \in \left[ 0, \frac{\pi}{2} \right), \theta_2 \in [0, 2\pi)
\]

(9)
which is equivalent to \( 2 \times 2 \) unitary matrix. Independent quantization of angle parameters \( \theta_1, \theta_2 \) can also solve the Grassmannian space packing problem. In practical application, many codebook design use this approach or its equivalent scheme. In 3GPP LTE system, codebook design for \( N_T = 2 \) is defined in [12]. Identity matrix is included in codebook to guarantee that the system performance using precoding technology should be better than that without precoding. The other two matrices are shown as follow:

\[
\begin{bmatrix} 1 & 1 \\ \sqrt{2} & -1 \end{bmatrix} = \begin{bmatrix} \cos \frac{\pi}{4} & \sin \frac{\pi}{4} \\ \sin \frac{\pi}{4} e^{j0} & \cos \frac{\pi}{4} e^{j(0+\pi)} \end{bmatrix}
\]

\[
\begin{bmatrix} 1 & 1 \\ \sqrt{2} & -j \end{bmatrix} = \begin{bmatrix} \cos \frac{\pi}{4} & \sin \frac{\pi}{4} \\ \sin \frac{\pi}{4} e^{j\pi} & \cos \frac{\pi}{4} e^{j(\pi+\pi)} \end{bmatrix}
\]

(10)
where \( \theta_1 \) is quantized with 1 level (\( \theta_1 = \pi/4 \)), \( \theta_2 \) is quantized with 2 levels, \( \theta_2 \in \{0, \pi/2\} \).

### IV. SPATIAL CORRELATION AND DISTRIBUTION OF RIGHT SINGULAR VECTOR

#### A. Distribution of Right Singular Vector under i.i.d MIMO Channel

When \( N_T = 2 \), \( \mathbf{V} \) is equivalent to a Givens matrix as shown in (9) (It’s acceptable that vector \( \mathbf{v} \) is equivalent to \( a \mathbf{v} \), where \( |a| = 1 \)). Two angle parameters defined in (9) related to \( \mathbf{V} \) can be calculated as follow:

\[
\cos^2(\theta_1) = \frac{1}{2} + \frac{\|\mathbf{H}_{i,1}\|_2^2 - \|\mathbf{H}_{i,2}\|_2^2}{2\sqrt{\|\mathbf{H}\|_F^2 - 4|\det(\mathbf{H})|^2}}
\]

(11)
\[
e^{j\theta_2} = \frac{\mathbf{H}_{i,1}^H \cdot \mathbf{H}_{i,2}}{\|\mathbf{H}_{i,1}^H\|_2 \cdot \|\mathbf{H}_{i,2}\|_2}
\]

(12)
where \( \mathbf{H}_{i, \cdot} \) denotes the \( i \)-th column of \( \mathbf{H} \). Probability density function (PDF) of \( \theta_1 \) and \( \theta_2 \) under i.i.d MIMO channel can be achieved according to (11) and (12). We can qualitatively analyze their PDF at first. \( \mathbf{H}_{i,1} \) and \( \mathbf{H}_{i,2} \) have the same distribution, and entries in the denominator of (11) are symmetrical to each other, therefore \( \cos^2(\theta_1) = 1/2 \) is the symmetrical axis of its PDF, which is demonstrated by numerical result in Fig.2.

Accurate PDF can be achieved using numerical integration or random sample simulation, which is shown in Fig.2 with black line. Distribution of \( \cos^2(\theta_1) \) in region \([0,1]\) is uniform. Distribution of \( \theta_2 \) in region \([0,2\pi]\) is uniform too.
Correlated MIMO channel can be generated using following coefficient among receive (transmit) antennas correspondingly. The symmetrical axis of its PDF is $\pi/2$ larger.

$$H_{\alpha}$$ situations [14]. Parameter assumption is validated by measurements for non-line-of-sight antennas is independent to that among receive antennas. This cient among receive antennas is shown in Fig. 2. We can summarize it as follow: antennas’ correlation matrix:

$$H = C_R^H \cdot H_W \cdot C_T$$

where $H_W$ denotes i.i.d Rayleigh channel matrix, $C_R$ ($C_T$) denotes the Cholesky decomposition of receive (transmit) antennas’ correlation matrix:

$$C_R^H \cdot C_R = R_R = \begin{bmatrix} 1 & \alpha \\ \alpha^* & 1 \end{bmatrix}$$

$$C_T^H \cdot C_T = R_T^* = \begin{bmatrix} 1 & \beta^* \\ \beta & 1 \end{bmatrix}$$

Slight difference between $C_R$ and $C_T$ is notable.

Combining (13) with (11) and (12), PDF of $\theta_1$ and $\theta_2$ under spatial correlated channel can be achieved. Correlation coefficient among receive antennas $\alpha$ does not affect the distribution. Influence of correlation coefficient among transmit antennas $\beta$ is shown in Fig. 2. We can summarize it as follow:

1. PDF of $\theta_1 (\cos^2(\theta_1))$ gets more concentative as $|\beta|$ gets larger. $\pi/4$ (1/2) is always the symmetrical axis of its PDF. $angle(\beta)$ does not affect the distribution of $\theta_1 (\cos^2(\theta_1))$.

2. PDF of $\theta_2$ gets more concentative as $|\beta|$ gets larger too. The symmetrical axis of its PDF is $\theta_2 = angle(\beta)$.

### C. Influence of Spatial Correlation on Codebook-based Precoding

The PDF of angle parameter is no longer uniform under spatial correlated channel. If codebook-based precoding is used, selection probability of matrix(vector) in codebook is affected by the module and angle of $\beta$. Take codebook in [12] as an example. Identity matrix is excluded from codebook temporarily for analysis simplification. $\theta_1$ is quantized with 1 level, so the selection of two matrices in (10) depends on $\theta_2$.

The optimum precoder is selected as follow:

$$\theta_2 = -angle(H_{1,1}^*, H_{1,2} + H_{2,1}^*, H_{2,2})$$

$$F_{opt} = \begin{cases} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, & \theta_2 \in (-\frac{\pi}{4}, \frac{3\pi}{4}) \\ \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & -j \end{bmatrix}, & \theta_2 \in (\frac{\pi}{4}, \frac{3\pi}{4}) \end{cases}$$

Under i.i.d Rayleigh channel, distribution of $\theta_2$ in region $[0, 2\pi]$ is uniform. Therefore, the selection probabilities of the two matrices is equal:

$$P[\theta_2 \in (-\frac{\pi}{4}, \frac{3\pi}{4})] = P[\theta_2 \in (\frac{\pi}{4}, \frac{3\pi}{4})]$$

Under spatial correlated channel, for example $\beta = 0.5e^{-j\frac{\pi}{2}}$, selection probability of the second matrix is larger:

$$P[\theta_2 \in (\frac{\pi}{4}, \frac{3\pi}{4})] > P[\theta_2 \in (-\frac{\pi}{4}, \frac{3\pi}{4})]$$

An extreme situation can be considered, $|\beta| = 1$, then (12) can be rewritten as:

$$e^{j\theta_2} = \frac{|H_{1,1}^* \cdot H_{1,1} \beta^* + H_{2,1}^* \cdot H_{2,1} \beta^*|}{H_{1,1}^* \cdot H_{1,1} \beta^* + H_{2,1}^* \cdot H_{2,1} \beta^*} = \beta$$

PDF of $\theta_2$ is a Dirac delta function $\delta(\theta_2 - angle(\beta))$. The second matrix will be selected for every channel realization. The first matrix in the codebook is useless now. Spatial coefficient among transmit antennas affects the distribution of V, therefore, affects the selection probability of precoder in codebook. This phenomenon is useful for codebook design under correlated channel.

### V. NONUNIFORM CODEBOOK DESIGN

Codebook presents a quantization of $V(V_{NT})$. Under spatial correlated channel, distributions of the two angle parameters $\cos^2(\theta_1)$ and $\theta_2$ are no longer uniform. Nonuniform quantization may be used for codebook design. $\theta_1$ is distributed around $angle(\beta)$ whose potential region is $[-\pi, \pi]$. Different nonuniform quantization should be utilized for various correlation coefficient, which is the origin of nonuniform codebook design. In nonuniform codebook design, current codebook is the function of correlation coefficient $\beta$. Oppositely, traditional codebook is termed as uniform and fixed codebook. An example of nonuniform codebook for $N_T = 2$ is described as follow. We can define

$$agl = \begin{cases} angle(\beta), & angle(\beta) \geq 0 \\ angle(\beta) + \pi, & angle(\beta) < 0 \end{cases}$$
According to $agl$, nonuniform quantization of $\theta_2$ can be summarized as follow:

$$\theta_2 \in \begin{cases} 
\{0, \pi/4\}, & agl \in [0, \pi/2) \\
\{\pi/4, \pi/2\}, & agl \in [\pi/2, \pi/4) \\
\{\pi/2, \pi\}, & agl \in [\pi/4, \pi/2) \\
\{0, \pi\}, & agl \in [\pi, \pi/4)
\end{cases} \quad (21)$$

When $angle(\beta) \in [0, \pi/4)$ ($angle(\beta) \in [\pi, 5\pi/4)$), for example $angle(\beta) = \pi/8$ ($9\pi/8$), distribution density of $\theta_2$ around $\pi/8$ ($9\pi/8$) is larger than any other region. Nonuniform quantization $\theta_2 \in \{0, \pi/4\} (\theta_2 \in \{\pi, 5\pi/4\})$ is utilized. When $\theta_2 = \pi/4$, the periods of $\theta_2$ is reduced to $\pi$, quantization $\theta_2 \in \{0, \pi/4\}$ is equivalent to $\theta_2 \in \{\pi, 5\pi/4\}$.

The random variable in (7) can be rewritten as:

$$\max_{i \in \{1, \ldots, N\}} \left| \mathbf{V}^H \mathbf{F}_i \cdot \mathbf{F}_i \right|^2 = \frac{1}{4} + \frac{1}{2} \cos \theta_1 \sin \theta_1 \cos(\theta_{F2} - \theta_2) \quad (22)$$

where $\theta_{F2}$ denotes the optimum quantization angle of $\theta_2$. For uniform codebook utilization,

$$\theta_{F2} = \begin{cases} 
0, & \theta_2 \in (0, \pi/4) \\
\pi/2, & \theta_2 \in (\pi/4, \pi/2) \\
\pi, & \theta_2 \in (\pi/2, \pi/4) \\
\pi/4, & \theta_2 \in (\pi/4, \pi)
\end{cases} \quad (23)$$

Take the below expression instead of (7) as the quantization error cost function for comparison of uniform and nonuniform codebook:

$$E[\cos(\theta_{F2} - \theta_2)] \quad (24)$$

The comparison of uniform codebook and nonuniform codebook is plotted in Fig.3. When $|\beta| > 0.7$, mean quantization error performance of nonuniform codebook is better than that of uniform codebook. Both quantization error curves of uniform and nonuniform codebook are periodical functions of $angle(\beta)$. For uniform codebook, the larger $|\beta|$, the larger oscillation amplitude of the quantization error curve. For nonuniform codebook, oscillation amplitude is smaller than that of uniform one. The larger $|\beta|$, the smaller quantization error of nonuniform codebook. The utilization situations of nonuniform codebook is also implied in Fig.3. For the situation $|\beta| > 0.7$, nonuniform codebook according to (21) is better than uniform and fixed codebook.
mean AOD is the key factor of $\beta$, which basically depends on the position of user in the cell. Changing speed of mean AOD is slower than that of mean angle of arrival in downlink transmission. Therefore, achievement of correlation coefficient in receiver and feedback to transmitter are feasible in the downlink transmission of FDD system. The feedback periods of correlation information or codebook index can be hundreds times of the feedback periods of matrix index and MCS information. Take the nonuniform codebook defined in (21) as an example, only 2 bits is used for feedback of codebook index. Therefore, less than 0.02 bit increasing on feedback information overhead is brought up by utilizing nonuniform codebook. In a word, utilization of nonuniform codebook results in higher spectral efficiency with the cost of slight feedback overhead increasing.

VII. CONCLUSION

In this paper, the influence of spatial correlation on precoding technology is analyzed. The distribution of right singular vector keeps close to the angle of correlation coefficient among transmit antennas. According to the analysis result, nonuniform codebook design is present, in which codebook is function of correlation coefficient among transmit antennas and presents a nonuniform quantization of unitary vector set. Under spatial correlated MIMO channel, nonuniform codebook results in smaller quantization error and higher spectral efficiency than traditional uniform and fixed codebook does, with the cost of slight feedback overhead increasing.

REFERENCES


