The Acoustic Vector-Sensor’s Near-Field Array-Manifold
Yue Ivan Wu, Kaimain Thomas Wong, and Siu-Kit Lau

Abstract—The acoustic vector-sensor is a practical and versatile sound-measurement system, for applications in-room, open-air, or underwater. Its far-field measurement model has been introduced into signal processing over a decade ago; and many direction-finding algorithms have since been developed for acoustic vector-sensors, but only for far-field sources. Missing in the literature is a near-field measurement model for the acoustic vector-sensor. This correspondence fills this literature gap.

Index Terms—Acoustic arrays, acoustic position measurement, acoustic signal processing, array signal processing, direction of arrival estimation, underwater acoustic arrays.

I. INTRODUCTION

An acoustic vector-sensor (a.k.a. vector-hydrophone) consists of three identical, but orthogonally oriented, acoustic velocity-sensors, plus an acoustic pressure-sensor—all spatially co-located in a point-like geometry. Each acoustic velocity-sensor measures one Cartesian component of the incident acoustic particle-field vector. The entire acoustic vector-sensor thus distinctly measures all three Cartesian components of the particle-velocity vector plus the pressure scalar. This contrasts with a customary microphone or hydrophone, measuring only the acoustic pressure.

More precisely: for a point-source incident with unit-power from the far field, an acoustic vector-sensor (located at the Cartesian coordinates' origin) has this array manifold, [3], [5],

\[ a_{i,s} \text{ def } = \begin{bmatrix} u(\psi_s, \phi_s) \\ v(\psi_s, \phi_s) \\ w(\psi_s, \phi_s) \end{bmatrix} \text{ def } = \begin{bmatrix} \sin \psi_s \cos \phi_s \\ \sin \psi_s \sin \phi_s \\ \cos \psi_s \end{bmatrix} \]

(1)

where \( 0 \leq \psi_s \leq \pi \) symbolizes the elevation-angle measured from the vertical \( z \)-axis, \( 0 \leq \phi_s < 2\pi \) denotes the azimuth-angle measured from the positive \( x \)-axis, \( u(\psi_s, \phi_s) \) refers to the direction-cosine along the \( x \)-axis, \( v(\psi_s, \phi_s) \) refers to the direction-cosine along the \( y \)-axis, and \( w(\psi_s, \phi_s) \) refers to the direction-cosine along the \( z \)-axis. Specifically, the first, second, and third components in (1) correspond to the acoustic velocity-sensors aligned along the \( x \)-axis, the \( y \)-axis, and the \( z \)-axis, respectively. These three Cartesian components of particle-velocity field-vector has a Euclidean norm (i.e., \( [u(\psi_s, \phi_s)]^2 + [v(\psi_s, \phi_s)]^2 + [w(\psi_s, \phi_s)]^2 \)) equal to the unity in pressure, for all \( \psi_s \) and \( \phi_s \).

The acoustic vector-sensor concept is versatile for direction-finding, due to these properties:

i) A single acoustic vector-sensor intrinsically possesses a two-dimensional azimuth-elevation directivity, because all three Cartesian components of the acoustic velocity-vector-field are simultaneously measured.

ii) Multiple incident sources’ azimuth-angles and the elevation-angles may be estimated and automatically matched with only one acoustic vector-sensor.

Please refer to [8] for an extended literature survey on acoustic vector-sensor based direction-finding algorithms, target-tracking algorithms, beam-pattern analysis, hardware implementations, sea trials, atmospheric trials, in-room trials, and suggested applications.

Though the above far-field measurement model in (1) was first introduced to the signal-processing literature by [3] and [5] over a decade ago, the corresponding near-field measurement-model has not been investigated. This overlooked issue is herein investigated. As Sections II-IV will show, the far-field measurement model’s independence of the signal frequency, the source/sensor distance, and the propagation-medium in (1) is invalid for the near-field case.

II. MATHEMATICAL DERIVATION OF THE MAIN RESULT

A. Review of the Pressure-Field Wave-Equation & the Particle-Velocity Wave-Equation

This Section II-A reviews the basic mathematics inter-relating the acoustic pressure-field wave-equation to the acoustic particle-velocity field wave-equation.1 This Section II-A will adopt the spherical coordinates customary in the acoustics literature, locating the emitter at the origin \( r_0 = (0, \theta, \phi) \) of the coordinates \( (R', \theta', \phi') \). These new coordinates differ from the \((R, \theta, \phi)\) coordinates of Sections I and II-B, customary to the direction-finding literature, with the sensor at the origin.

The aforementioned source emits into a quiescent isotropic homogeneous fluid, such as air or water. The resulting acoustic particle-velocity field-vector \( \mathbf{v}(r', t) \) is related by Euler’s equation2 to the corresponding pressure scalar field \( p(r', t) \) as follows:

\[ \rho_0 \frac{\partial \mathbf{v}(r', t)}{\partial t} = - \nabla p(r', t) \tag{2} \]

\[ \frac{1}{c^2} \frac{\partial^2 p(r', t)}{\partial t^2} = - \nabla \cdot \mathbf{v}(r', t) \tag{3} \]

where \( \rho_0 \) refers to the ambient fluid density, \( c \) symbolizes to the sound-wave propagation-speed, \( \nabla \) represents the gradient operator, and \( \nabla \cdot \) signifies the divergence operator.

To obtain the pressure-field wave-equation (namely[1, d’Alembert’s eq. (1.30)]): Take the partial derivative of (3) with respect to \( t \), and combine with (2), giving

\[ \frac{1}{c^2} \frac{\partial^2 p(r', t)}{\partial t^2} = \nabla^2 p(r', t). \tag{4} \]

1The authors would like to thank an anonymous reviewer for suggesting some of the materials in this Section II-A.

2I.e., a vector function at any spatial location \( r' = R' [\cos \phi' \sin \theta' \sin \phi + \sin \phi' \cos \phi \cos \theta] \) and any time \( t \).

3Please see [1, equations (1.58) to (1.59)], and [2, equation (1–3.7)].
To obtain the particle-velocity-field wave-equation ([1, eq. (1.33)]):
Take the gradient of (2), and combine with (3), giving
\[ \frac{1}{c^2} \frac{\partial^2 \mathbf{v}(t', t)}{\partial t'^2} = \nabla \left( \nabla \cdot \mathbf{v}(t', t) \right). \] (5)

To avoid distraction from the present focus on the array-manifold, a simple signal-model will be used: Let the emitted signal be a pure tone of angular frequency \( \omega \) and complex-amplitude \( A \). That is, \( p(t', t) = p(t') e^{j\omega t} \) and \( \mathbf{v}(t', t) = \mathbf{v}(t') e^{j\omega t} \). Then, the wave-equations become the Helmholz equations (i.e., [1, eq. (1.101), (1.102)])
\[ -\left( \frac{\omega}{c} \right)^2 p(t', t) = \nabla^2 p(t', t) \] (6)
\[ -\left( \frac{\omega}{c} \right)^2 \mathbf{v}(t', t) = \nabla \left( \nabla \cdot \mathbf{v}(t', t) \right). \] (7)

One solution to the above Helmholz equations is the spherical sinusoidal wave:
\[ p(t', t) = \frac{A}{R} \exp \left\{ j \left( \omega t - \frac{2\pi}{\lambda} R' \right) \right\} \] (8)
where \( \lambda \) signifies the signal wavelength.

B. To Derive the Acoustic Vector-Sensor Near-Field Array-Manifold

Fig. 1 shows the sensor-centric spherical coordinates \((R, \phi, \psi)\) customarily used in the direction-finding literature, with the acoustic vector-sensor at the origin \( r_0 = [0, \phi, \psi]^T \). Let \( R_s \) denote the distance between the acoustic vector-sensor and an emitting source located at \( r_s = (R_e, \phi_s, \psi_s)^T \). Hence, \( r_s = (0, \phi, \psi)' \) in the emitter-centric coordinates of \((R', \phi', \psi')\) from the acoustics literature in Section II-A) would correspond to \( r_s \) in the sensor-centric coordinates of \((R, \phi, \psi)\) in Section II-B here from the direction-finding literature.

The gradient of the pressure-field at the acoustic vector-sensor can be expressed as
\[ \nabla p(t, r) \bigg|_{r=r_0} = \frac{\partial p(t, r)}{\partial R} \hat{R} + \frac{1}{R} \sin \psi \frac{\partial p(t, r)}{\partial \phi} \hat{\phi} + \frac{1}{R} \cos \psi \frac{\partial p(t, r)}{\partial \psi} \hat{\psi} \] (9)
where \( \hat{R} \) and \( \hat{\psi} \), respectively, denote the unit-vectors along the azimuth-angle coordinate and the elevation-angle coordinate. Especially, in the spherical coordinate system \( r_0 = [0, \phi, \psi]^T \). Using the MATLAB Symbolic Math Toolbox, the following identities are found:
\[ \frac{\partial p(t, r)}{\partial R} \bigg|_{r=r_0} = -A \exp(j\omega t) \left( \frac{j2\pi R_e}{\lambda} + 1 \right) \exp(-j2\pi R_e) \] (10)
\[ \frac{\partial p(t, r)}{\partial \phi} \bigg|_{r=r_0} = 0 \] (11)
\[ \frac{\partial p(t, r)}{\partial \psi} \bigg|_{r=r_0} = 0. \] (12)
Substituting these three identities into (9), the gradient of the pressure-field at \( r_0 \) is obtained as
\[ \nabla p(t, r) \bigg|_{r=r_0} = -A \exp(j\omega t) \left( \frac{j2\pi R_e}{\lambda} + 1 \right) \exp(-j2\pi R_e) \hat{R} \] (13)

Likewise, the particle-velocity vector \( \mathbf{v}(r_0, t) \) may be represented as
\[ \mathbf{v}(r_0, t) = \frac{\nabla p(r_0, t)}{j\omega R_0} \] (14)
\[ = \frac{1}{-\rho_0 c} \left( \frac{j2\pi R_e}{\lambda} + 1 \right) \frac{A}{R_e} \exp \left\{ j \left( \omega t - \frac{2\pi}{\lambda} R_e \right) \right\} \hat{R} \] (15)

Using the definition of the source’s direction-vector \( \hat{R} \), (14) becomes
\[ \mathbf{v}(r_0, t) = p(r_0, t) \left[ \frac{\cos \phi_s \sin \psi_s}{\sin \phi_s \sin \psi_s} \right] \left[ \frac{1 + \left( \frac{\lambda}{2\pi R_s} \right)^2}{-\rho_0 c} \right] \exp(-j\arctan \frac{\lambda}{2\pi R_s}) \] (16)

From (15), the acoustic vector-sensor near-field array-manifold equals\(^5\):
\[ a_{near} = \left[ \frac{\cos \phi_s \sin \psi_s}{\sin \phi_s \sin \psi_s} \frac{\cos \psi_s}{\sqrt{1 + \left( \frac{\lambda}{2\pi R_s} \right)^2}} \exp(j\arctan \frac{\lambda}{2\pi R_s}) \right] \] (17)

A complex-phase difference thus exists between the velocity-sensor triad measurements and the pressure-sensor measurement in the near-field measurement-model in (16). This phase-difference depends on the wavelength-normalized source/sensor distance \( R_s/\lambda \) and the propagation-medium’s \( \rho_0 c \), but does not depend on the azimuth-elevation arrival-angles.

\(^5\)Concerning the “—” sign before the above \( \rho_0 c \), it arises from how the direction is defined above for the unit-vector \( \hat{R} \). This “—” sign may not appear in the acoustics literature.

\(^6\)In the far-field case, where \( R_s \gg \lambda \), it holds that \( \lambda/2\pi R_s \to 0 \). Thus, the pressure scalar field would relate to the particle-velocity vector-field as in (2), which presumes a planar wavefront upon the acoustic vector-sensor.
As the wavelength-normalized distance \( R_s/\lambda \to \infty \), the near-field array-manifold converges to
\[
\begin{bmatrix}
\cos \phi_s \sin \psi_s \\
\sin \phi_s \sin \psi_s \\
\cos \psi_s \\
-\rho_0 c
\end{bmatrix}
\tag{17}
\]

The above is consistent with the far-field array-manifold [3, eq. (2.5)] which normalizes the pressure-sensor gain from the above \(-\rho_0 c\) to become unity.

III. CRAMÉR-RAO BOUND ANALYSIS OF THE NEAR-FIELD MEASUREMENT MODEL

A. Defining the Statistical Data Model

To further characterize the acoustic vector-sensor’s array-manifold, this section will derive the Cramér-Rao bound for near-field (three-dimensional) source-localization by an acoustic vector-sensor. To avoid unnecessary distraction from focusing on the near-field array-manifold, a very simple signal statistical model will be used: Here the emitted signal \( s(t) = e^{j(\omega t + \theta_0)} \) is a pure tone at angular frequency \( \omega \) as before, with an initial phase of \( \theta_0 \). Both \( \omega \) and \( \theta_0 \) are deterministic unknown constants. At the \( m \)-th time-sample \( t = mT_s \), a \( 4 \times 1 \) data-vector \( \mathbf{z}(mT_s) \) is collected by the four-component acoustic vector-sensor
\[
\mathbf{z}(mT_s) = \mathbf{a}_{\text{near}} \mathbf{s}(mT_s) + \mathbf{n}(mT_s)
\tag{18}
\]
where \( T_s \) refers to the time-sampling period, and \( \mathbf{n}(t) \) denotes a \( 4 \times 1 \) vector of additive zero-mean spatiotemporally uncorrelated Gaussian noise, with an unknown deterministic covariance-matrix \( \mathbf{R}_0 = \text{diag} \left( \sigma_\phi^2, \sigma_\psi^2, \sigma_\phi^2, \sigma_\psi^2 \right) \). That is, \( \sigma_\phi^2 \) represents the noise-variance at each velocity-sensor, and \( \sigma_\psi^2 \) symbolizes the noise-variance at the pressure-sensor. The velocity-sensor and the pressure-sensor likely have different noise-variances, because of their different hardware implementations and the distinct physical wave-pressure measured [4]–[6]. With \( N \) number of time-samples, the collected \( 4 \times M \) data-set equals
\[
\mathbf{z} = \left[ (\mathbf{z}(T_s))^T, \ldots, (\mathbf{z}(M T_s))^T \right]^T = \mathbf{s} \otimes \mathbf{a}_{\text{near}} + \left[ (\mathbf{n}(T_s))^T, \ldots, (\mathbf{n}(M T_s))^T \right]^T
\tag{19}
\]
where \( \mathbf{z} = \mathbf{e}^T \left[ \mathbf{e}^{jT_\phi \mathbf{w}}, \mathbf{e}^{jT_\psi \mathbf{w}}, \ldots, \mathbf{e}^{jM T_\mathbf{w}} \right]^T \), \( \otimes \) symbolizes the Kronecker product, \( \mathbf{w} \) represents a \( 4 \times 1 \) noise vector with a spatio-temporal covariance matrix of \( \mathbf{R} = 1_4 \otimes \mathbf{R}_0 \), and \( 1_4 \) denotes an \( 4 \times 4 \) identity matrix. Therefore, \( \mathbf{z} \sim N(\mathbf{0}, \mathbf{R}) \).

The near-field source-localization problem is to estimate the azimuth-elevation arrival-angles \( \phi_s \) and \( \psi_s \), plus the radial distance \( R_s \), based on the \( 4 \times 1 \) collected data \( \mathbf{z} \).

B. Deriving the Cramér-Rao Bound for Near-Field Source-Localization by an Acoustic Vector-Sensor

In the statistical data model in Section III-A, there exist seven deterministic unknown entities, which are here collected into a \( 7 \times 1 \) vector,

\[
\boldsymbol{\theta} = \left[ \phi_s, \psi_s, R_s, \omega, \epsilon, \sigma_\phi^2, \sigma_\psi^2 \right]^T
\]

The resulting \( 7 \times 7 \) Fisher Information Matrix, \( \mathbf{J} \), would have the \((i, j)\)th entry

\[
J_{i,j} = 2 \Re \left\{ \frac{\partial \mathbf{J}}{\partial \theta_i} \frac{\partial \mathbf{J}}{\partial \theta_j}^H \right\} + \text{Tr} \left\{ \Gamma^{-1} \frac{\partial \Gamma}{\partial \theta_i} \frac{\partial \Gamma}{\partial \theta_j} \right\}
\tag{20}
\]

where \( \Re \{ \cdot \} \) signifies the real-value part of the entity inside the curly brackets, \( \text{Tr} \{ \cdot \} \) denotes the trace operation, and \( [\cdot] \) symbolizes the \( i \)th element of the vector inside the square brackets.

Straightforward calculus manipulations can express the Fisher information matrix entries in terms of the measurement-model parameters and statistical data-model parameters, as follows:

\[
\begin{align*}
J_{\phi_s, \phi_s} &= J_{1,1} = \frac{2M}{\sigma_\phi^2} \sin^2 \psi_s, \\
J_{\psi_s, \psi_s} &= J_{2,2} = \frac{2M}{\sigma_\psi^2}, \\
J_{R_s, R_s} &= J_{3,3} = \frac{M \rho_0 c}{\sigma_\phi^2 R_s k_1}, \\
J_{\omega, \omega} &= J_{4,4} = \frac{2M}{\sigma_\psi^2} \left[ k_1^2 + \left( \frac{\epsilon \omega R_s}{\sigma_\psi^2} - k_2 \right)^2 \right] + \frac{2M(M+1)}{\sigma_\psi^2 f_s^2} + \frac{\rho_0 c}{\sigma_\psi^2} + \frac{2M}{\sigma_\psi^2} \left( \frac{\epsilon \omega R_s}{\sigma_\psi^2} \right)^2 k_2 + \frac{M(M+1)(2M+1)}{f_s^2}, \\
J_{\epsilon, \epsilon} &= J_{5,5} = \frac{2M \rho_0 c}{\sigma_\phi^2}, \\
J_{\epsilon, \phi_s} &= J_{6,6} = \frac{3M}{\sigma_\phi^2}, \\
J_{\epsilon, \psi_s} &= J_{7,7} = \frac{3M}{\sigma_\psi^2}, \\
J_{R_s, \omega} &= J_{3,4} = J_{4,3} = \frac{M}{\sigma_\phi^2} \times \frac{\epsilon \omega(R_s)}{R_s k_1} \left[ 1 + \frac{\omega^2(M+1)}{2 f_s^2} \right], \\
J_{R_s, R_s} &= J_{5,5} = J_{5,4} = \frac{M}{\sigma_\phi^2} \epsilon \omega \rho_0 k_1, \\
J_{\omega, \epsilon} &= J_{4,5} = J_{5,4} = \frac{M}{\sigma_\psi^2} \epsilon \omega \rho_0 k_1 + \frac{M(M+1)}{f_s^2} k_3
\end{align*}
\tag{21–30}
\]

where \( k_1 = \frac{1}{\rho_0} \left( \frac{2 \rho_0 R_s \epsilon}{\epsilon \omega R_s} \right)^2 / \left[ 1 + (\epsilon \omega R_s)^2 \right]^2 \), \( k_2 = \frac{\rho_0 R_s (\epsilon \omega R_s)^2 / \left( 1 + (\epsilon \omega R_s)^2 \right)^2}{\rho_0^2 \epsilon^2 / \left[ 1 + (\epsilon \omega R_s)^2 \right]^2 \sigma_\phi^2 \sigma_\psi^2} \).

All other entries are zero in the Fisher information matrix. As a consequence

\[
\begin{bmatrix}
J_{1,1} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & J_{2,2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & J_{3,3} & J_{3,4} & J_{3,5} & 0 & 0 \\
0 & 0 & J_{4,4} & J_{4,5} & J_{4,6} & 0 & 0 \\
0 & 0 & J_{5,5} & J_{5,6} & J_{5,7} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & J_{6,6} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & J_{7,7}
\end{bmatrix}
\]

is block-diagonal.

\(^7\)Please see [7, eq. (8.34)].
Hence, the Cramér-Rao bounds of the emitter’s azimuth-elevation arrival-angles are

\[
\text{CRB}(\phi_*) = J_{1,1}^{-1} = \frac{\sigma_\phi^2}{2M \sin^2 \psi_*}
\]

(31)

\[
\text{CRB}(\psi_*) = J_{2,2}^{-1} = \frac{\sigma_\psi^2}{2M}
\]

(32)

As for the radial distance Cramér-Rao bound, see (33) at the bottom of the page, where \(\begin{bmatrix} 1 \end{bmatrix}^{T} \) represents the \((i, j)\)th entry of the matrix inside the square brackets. Substitution of (21) to (30) in (33) gives

\[
\text{CRB}(R_*) = \frac{\sigma_*^2}{(2M \lambda^2)} \left( \frac{\omega^2 R_*^2 + c^2}{\omega^2 c^2 \rho_0^2} \right)^{3/2}
\]

\[
\times \left[ \frac{\omega^2 R_*^2 + c^2}{\omega^2 c^2 \rho_0^2} \right] \frac{\sigma_*^2 + (\omega^2 R_*^2 + c^2) \sigma^2}{2} + O(M^{-1})
\]

(34)

By overlooking the \(O(M^{-1})\) terms, which contain multiples of \(M^{-1}, M^{-2}, \ldots\), the following approximation may be obtained:

\[
\text{(34a) CRB}(R_*) \text{ becomes largely constant with respect to } R_*/\lambda, \text{ for values of } R_*/\lambda \text{ that are typical for air-acoustics applications.}
\]

Table I shows how CRB \(R_*/\lambda\) may be approximated, under various degenerate cases in the data model. The following qualitative trends may be observed.

<table>
<thead>
<tr>
<th>Special Conditions in the Data Model</th>
<th>((\frac{2\pi}{\lambda})^2 \frac{2M}{\sigma_<em>^2} \text{ CRB}(R_</em>) \approx )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_a \gg \frac{1}{\lambda} )</td>
<td>((\frac{1}{\rho_0 c})^2 \left( \frac{2\pi R_a}{\lambda} \right)^6 \left( \frac{2\pi R_a}{\lambda} \right)^2 \left( \frac{2\pi R_a}{\lambda} \right)^{4+1} )</td>
</tr>
<tr>
<td>(R_a \ll \frac{1}{2\pi} )</td>
<td>((\frac{1}{\rho_0 c})^2 \left( \frac{2\pi R_a}{\lambda} \right)^4 \left( \frac{2\pi R_a}{\lambda} \right)^2 \left( \frac{2\pi R_a}{\lambda} \right)^{4+1} )</td>
</tr>
</tbody>
</table>

IV. QUALITATIVE OBSERVATIONS

The following qualitative trends may be observed of CRB \((\phi_*)\) from (31) and CRB \((\psi_*)\) from (32).

1. Both CRB \((\phi_*)\) and CRB \((\psi_*)\) are independent of the signal frequency \(\omega\), the source-sensor distance \(R_s\), the propagation-medium’s \(\rho_0 c\), the source’s azimuth-angle \(\phi_\ast\), and the pressure-sensor’s noise-variance \(\sigma_\rho^2\) (as the pressure-sensor offers no arrival-angle information).

2. CRB \((\psi_*)\) is unaffected also by the source’s elevation-angle \(\psi_\ast\).

8For an \(M\) as little as 100, simulations show that the approximate CRB \((R_*)\) in (35) can be graphically indistinguishable from the exact expression in (34).

\[
\text{CRB}(R_*) = \left[ J_{1,1}^{-1} \right]_{3,3} = \frac{J_{4,4} J_{5,5} - J_{4,5}^2}{J_{3,3} J_{4,4} J_{5,5} + 2J_{3,4} J_{4,5} J_{5,5} - J_{3,3} J_{4,5}^2 - J_{5,5} J_{4,4}^2 - J_{4,4} J_{5,5}^2}
\]

(33)

\[
\left( \frac{2\pi}{\lambda} \right)^2 \frac{2M}{\sigma_*^2} \text{ CRB}(R_*) \approx \left( \frac{1}{\rho_0 c} \right)^2 \left( \frac{2\pi R_a}{\lambda} \right)^2 \left( \frac{2\pi R_a}{\lambda} \right)^2 \left( \frac{2\pi R_a}{\lambda} \right)^{4+1} \left\{ \left( \frac{2\pi R_a}{\lambda} \right)^2 \left( \frac{2\pi R_a}{\lambda} \right)^2 \left( \frac{2\pi R_a}{\lambda} \right)^{4+1} \right\}
\]

(35)
relevant range of $R_s/\lambda$. This graphic observation also concur
with the approximation in Table I’s first row, which applies for
$R_s/\lambda \gg 1$. There, the far right fraction would approximate to
unity at the underwater acoustics’ very large $\rho_0 c$, thereby allowing
an independence from $H_c/\sigma_p$. For $R_s/\lambda < 1/2\pi$, applicable is
Table I’s second row, which is also independent of $H_c/\sigma_p$.

Fig. 3(a) and (b) reveals the dependence of $\text{CRB} (R_s)$ on the acoustic
medium’s $\rho_0 c$. These two figures plot $(2\pi/\lambda)^2 \text{CRB} (R_s)$ under the
degenerate condition of $H_c^2 = \sigma_p^2 = \sigma_c^2$ at 20 dB SNR at each com-
ponent-sensor, over ranges of $R_s/\lambda$ and $\rho_0 c$ relevant, respectively, to
air-acoustics and underwater acoustics applications. The following
qualitative trends may be observed.

(8) For the air-acoustic ranges of $R_s/\lambda$ and $\rho_0 c$, Fig. 3(a) shows
that $(2\pi/\lambda)^2 \text{CRB} (R_s)$ does not vary much with $\rho_0 c$, until the
near-field condition of $R_s/\lambda < 1/2\pi$ applies. There in the near
field, $(2\pi/\lambda)^2 \text{CRB} (R_s)$ decreases with increasing $\rho_0 c$.

(9) For the underwater acoustic Fig. 3(b), like the air-acoustic
Fig. 3(a), shows that $(2\pi/\lambda)^2 \text{CRB} (R_s)$ here decreases very
slightly with increasing $\rho_0 c$ whether inside of outside the near
field.

V. CONCLUSION

This correspondence is first to derive the near-field array-manifold
for an acoustic vector-sensor. Unlike the far-field array-manifold, a
complex-phase is found to exist between the pressure measurement
and the particle-velocity vector measurement. This phase-difference
depends on the source/sensor wavelength-normalized distance $R_s/\lambda$
and the propagation-medium’s $\sigma_p c$, but not on the azimuth-elevation ar-
riving angles. For three-dimensional source-localization, the azimuth-
elevation arrival-angle estimation accuracy could remain the same for
the near-field case as for the far-field case. However, the distance-es-
timation could have a wavelength-normalized accuracy that decreases
almost linearly with decreasing $R_s/\lambda$ outside the near field, but becomes largely flat inside the near field. Furthermore, this distance-estimation could also be independent of the source’s azimuth-elevation arrival direction.

REFERENCES