Cooperative Feedback for Multi-Antenna Cognitive Radio Networks
Kaibin Huang and Rui Zhang

Abstract

Cognitive beamforming (CB) is a promising technique for efficient spectrum sharing between primary users (PUs) and secondary users (SUs) in a cognitive radio network. With CB, the multi-antenna SU transmitter is able to suppress the interference to the PU receiver and maximize the SU link throughput. Existing designs on CB have assumed that the SU transmitter either has prior knowledge of the interference channel to the PU receiver or can acquire this knowledge by observing the PU transmission. Both assumptions may be impractical. In this paper, we propose a new and practical design paradigm for CB based on finite-rate cooperative feedback from the PU receiver to the SU transmitter. Specifically, for the case of multiple-input single-output (MISO) SU channel and single-input single-output (SISO) PU channel, the PU receiver collaboratively communicates to the SU transmitter the channel direction information (CDI), namely the quantized shape of the SU-to-PU MISO channel, and the interference power control (IPC) signal, which regulates the transmit power of the SU according to the tolerable interference margin at the PU receiver. We present two CB schemes for the SU transmitter based on quantized CDI and IPC feedback from the PU receiver. One is orthogonal beamforming where the SU transmit beamformer is restricted to be orthogonal to the feedback quantized SU-to-PU channel direction. The other is non-orthogonal beamforming for which the orthogonality constraint in the case of orthogonal beamforming is relaxed. Furthermore, cooperative feedforward of the SU CDI from the SU transmitter to the PU receiver is investigated to develop more efficient feedback schemes for the PU receiver. The resulting outage probabilities of the SU link with various CB designs and corresponding cooperative feedback/feedforward schemes are analyzed, from which the optimal bit allocation tradeoff between the CDI and IPC feedback is characterized.

Index Terms

Cognitive beamforming, cognitive radio, limited feedback, cooperative feedback, fading channel, multi-antenna system, outage probability.

K. Huang is with the School of Electrical and Electronic Engineering, Yonsei University, 262 Seongsanno, Seodaemun-gu, Seoul 120-749, Korea. R. Zhang is with Institute for Infocomm Research, 1 Fusionopolis Way, #21-01 Connexis, Singapore 138632. Email: huangkb@yonsei.ac.kr, rzhang@i2r.a-star.edu.sg.
I. INTRODUCTION

In a cognitive radio network, the secondary users (SUs) can access the spectrum originally allocated to a primary network, by transmitting over the frequency band of interest when the primary users (PUs) are not transmitting, or transmitting concurrently with the PUs provided that the SUs know how to control the resulting interference to the PUs within a tolerable margin. The former spectrum access model for the SUs is known as opportunistic spectrum access, while the latter is known as spectrum sharing [1]–[3]. In a cognitive radio network based on spectrum sharing, cognitive beamforming (CB) enables the SU transmitter with multiple antennas to suppress its interference to each PU receiver, and thereby transmit more frequently with larger power as compared with the case of single-antenna SU transmitter. CB requires the SU transmitter to have the channel state information (CSI) of its interference channel to each PU receiver. In [4], assuming perfect CSI for the SU-to-PU channels, the optimal CB design is proposed for maximizing the SU channel capacity subject to a given set of interference power constraints at the PU receivers. This assumption is relaxed in [5] and more practical CB designs are proposed where the SU transmitter estimates the required CSI by exploiting the channel reciprocity and periodically observing the PU transmissions. However, the channel estimation errors result in imperfect CB designs, which cause residual interference from the SU transmitter to PU receivers, and thus degrade the performance of primary links. Besides CB, the power of the SU transmitter can be adjusted opportunistically to further increase the SU throughput by exploiting the PU CSI as proposed in [6] and [7]. However, without a feedback channel from the PU receiver, the PU CSI is even more difficult for the SU to obtain than that of the SU-to-PU channel.

For the conventional multi-antenna or multiple-input multiple-output (MIMO) systems, CSI feedback from the receiver enables precoding at the transmitter, which not only enhances the MIMO channel transmission throughput and/or reliability, but also simplifies the transceiver design [8], [9]. However, CSI feedback can incur significant overhead due to the multiplicity of MIMO channel coefficients. This motivates active research on designing efficient feedback quantization algorithms for MIMO channels, called limited feedback [10]. Different designs of MIMO CSI quantizer have been proposed such as line packing [11], [12], subspace interpolation [13], and Lloyd’s algorithm [14]. Furthermore, different types of point-to-point limited feedback systems have been investigated, including beamforming [11], [12], precoded orthogonal space-time block codes [15], and precoded spatial multiplexing [16]. More recent research on limited feedback has been focusing on multiuser MIMO systems [17]–[21]. A comprehensive survey of latest research on limited feedback is given in [22]. In view of prior works, limited feedback for coexisting networks remains an open area. In particular, there are few results on limited feedback for
cognitive radio networks.

Motivated by the above discussions, we propose in this paper a new and practical design paradigm for CB, where the PU receiver collaboratively sends quantized side information to the SU transmitter for facilitating CB. This approach is justified by the fact that the PU receiver can more efficiently obtain the CSI of both the PU and the SU-to-PU channels than the SU transmitter. For example, the PU receiver can estimate the above CSI via the received training signals from the PU and SU transmitters. Specifically, this paper considers a simplified scenario with a multiple-input single-output (MISO) SU channel and a single-input single-output (SISO) PU channel. Under this setup, the PU receiver communicates to the SU transmitter the channel direction information (CDI), referring to the quantized shape of the SU-to-PU MISO channel, and the interference power control (IPC) signal, which regulates the transmit power of the SU according to the tolerable instantaneous interference power margin at the PU receiver. Based on the feedback CDI and IPC, the SU transmitter adapts its transmission by integrating opportunistic SU power control and CB. The main contributions of this paper are summarized as follows.

- We present two CB schemes for the SU transmitter based on the finite-rate CDI and IPC feedback from the PU receiver. One is orthogonal cognitive beamforming (OCB) where the SU transmit beamformer is restricted to be orthogonal to the quantized SU-to-PU channel shape in the CDI feedback, while the SU transmit power is limited by the IPC feedback. The other is non-orthogonal cognitive beamforming (NOCB), for which the orthogonality constraint on OCB is relaxed such that the SU transmit beamforming vector is allowed to spread over the feedback SU-to-PU channel direction. Furthermore, the maximum transmit power allocated in this direction is specified by the matching IPC feedback. We show that when the number of CDI feedback bits goes to infinity (i.e., perfect SU-to-PU MISO CSI is available at the SU transmitter), the proposed OCB and NOCB designs converge to the “zero-forcing” and “optimal” CB schemes proposed in [4], respectively.

- In addition to cooperative feedback from the PU receiver to the SU transmitter, we propose cooperative feedforward of the CSI for the SU MISO channel from the SU transmitter to the PU receiver. We show that the cooperative feedforward enables more efficient IPC feedback from the PU receiver and thereby permits larger transmit power for the SU CB.

- We analyze the performance of the SU link in terms of the signal-to-noise ratio (SNR) outage probability for OCB with various cooperative feedback/feedforward schemes. We show that the SU outage probability is lower-bounded in the high SNR regime due to feedback quantization. The lower bound is proved to decrease exponentially with the number of CDI feedback bits. Moreover, we derive the optimal bit allocation for CDI and IPC feedback under a sum feedback constraint,
which minimizes an upper bound on the SU outage probability.

The rest of this paper is organized as follows. Section II introduces the system model for the coexisting PU and SU links with cooperative feedback/feedforward between them. Section III presents the OCB and NOCB designs for the SU transmitter based on the cooperative feedback from the PU receiver, and the matching methods for the PU receiver to generate IPC feedback, with or without the SU CSI feedforward. Section IV and Section V present the analysis of the SU link outage probability and the optimal tradeoff between the CDI and IPC feedback, respectively, for the case of OCB. Section VI provides simulation results to corroborate the proposed designs in this paper. Finally, Section VII concludes the paper.

II. SYSTEM MODEL

We consider a primary link coexisting with a secondary link. The transmitter $T_p$ and the receiver $R_p$ of the primary link both have a single antenna, while the secondary link comprises a multi-antenna transmitter $T_s$ and a single-antenna receiver $R_s$. The multiple antennas at $T_s$ are employed for suppressing the interference from $T_s$ to $R_p$ as well as enhancing the reliability of the secondary link. The channel models for the primary, secondary, and interference (from $T_s$ to $R_p$) links comprise coefficients sampled from independent and identically distributed (i.i.d.) circular symmetric complex Gaussian random variables, each with zero-mean and unit-variance, denoted by $\mathcal{CN}(0,1)$, modeling the fading effects. Consequently, the signal received at $R_p$ has the power of $P_p g_p$, where $P_p$ is the transmit power at $T_p$ and $g_p$ is exponentially distributed with unit-variance, denoted by $\exp(1)$. The MISO channels from $T_s$ to $R_p$ and from $T_s$ to $R_s$ are represented by the $L \times 1$ vectors $h_i$ consisting of i.i.d. $\mathcal{CN}(0,1)$ elements and $h_s$ consisting of i.i.d. $\mathcal{CN}(0,\lambda)$ elements, respectively, where $0 < \lambda < 1$ is to account for the longer distance between $T_s$ and $R_p$ than that between $T_s$ and $R_s$ or that between $T_p$ and $R_p$. To facilitate the analysis, $h_i$ is decomposed into channel gain $g_i = ||h_i||^2/\lambda$ and channel shape $s_i = h_i/||h_i||$, and hence $h_i = \sqrt{\lambda} g_i s_i$; similarly, let $h_s = \sqrt{g_s} s_s$. The random variables $g_i$ and $g_s$ follow independent chi-square distributions with $L$ complex degrees of freedom. Last, all channels are assumed to follow independent blocking fading.

The primary receiver $R_p$ cooperates with the secondary transmitter $T_s$ to maximize the data rate of the secondary link under the constraint that the outage probability of the primary link is unaffected by the transmission of $T_s$. For this purpose, $R_p$ communicates to $T_s$ the constraint on the maximum transmit power of $T_s$, called interference power control (IPC) feedback, and the side information on $s_i$, called channel direction information (CDI) feedback, for $T_s$ to compute the transmit beamformer $f$. Assume that $R_p$ perfectly learns the CSI $h_i$ and $g_p$, as well as the SU transmit power constraint $P_{\text{max}}$. Using the above information, $R_p$ generates the IPC and CDI feedback.
Consider a finite-rate feedback channel from $R_p$ to $T_s$. Thus, IPC and CDI feedback must be both quantized. Let $\hat{s}_i$ denote the outputs of quantizing $s_i$. Following [23], [24], we adopt the quantization model where $\hat{s}_i$ lies on a hyper sphere-cap centered at $s_i$ and its radius depends on the quantization resolution. Specifically, the quantization error $\epsilon := 1 - |\hat{s}_i^\dagger s_i|^2$ with $\dagger$ denoting the conjugate transpose has the following cumulative distribution function (CDF) [23]

$$
\Pr(\epsilon \leq \tau) = \begin{cases} 
2^{B\tau L - 1}, & 0 \leq \tau \leq 2^{-\frac{\theta}{\epsilon}} \\
1, & \text{otherwise}
\end{cases}
$$ (1)

where $B$ is the number of CDI feedback bits. The algorithm for computing and quantizing the IPC feedback is described in Section III.

Feedback of $s_s$ from $R_s$ to $T_s$ is also required for beamforming design at $T_s$. For simplicity, this feedback is assumed perfect. This is justified by the fact that the quantization error of $s_s$ causes a small loss on $g_s$ but has no effect on the interference from $T_s$ to $R_p$. The mentioned loss can be made small by quantization on the Grassmannian manifold and a few bits of feedback as shown in [11], [12]. We assume no feedback of $g_s$ from $R_s$ to $T_s$.

We also consider the scenario where $T_s$ communicates to $R_p$, the secondary MISO channel shape $s_s$, called feedforward, prior to the primary feedback. This information is used by $R_p$ to predict the beamformer at $T_s$ and thereby tolerate larger transmit power at $T_s$. Detailed discussions on secondary feedforward are given in Sections III. For simplicity, the secondary feedforward CSI is assumed perfect. This assumption allows us to focus on the effects of finite-rate primary feedback.

The performance of both primary and secondary links are measured by the metric of outage probability. Accordingly, the data rates for the primary and secondary links are fixed as $R_p = \log_2(1 + \theta_p)$ and $R_s = \log_2(1 + \theta_s)$, respectively, where $\theta_p$ and $\theta_s$ specify the minimum receive SINR/SNR for correctly
decoding data transmitted over corresponding links. The receive SNR and signal-to-interference-plus-noise ratio (SINR) at \( R_p \) are given by

\[
\text{SNR}_p = \gamma_p g_p \quad \text{and} \quad \text{SINR}_p = \frac{\gamma_p g_p}{1 + \frac{\lambda g_i}{\sigma^2} |f^\dagger s_i|^2}
\]

where \( \gamma_p \) is the PU transmit SNR given by \( \gamma_p := \frac{P_p}{\sigma^2} \), and the noises at both \( R_p \) and \( R_s \) being i.i.d. \( \mathcal{CN}(0, \sigma^2) \) random variables. For simplicity, interference from \( T_p \) to \( R_s \) is assumed negligible and the receive SNR at \( R_s \) is

\[
\text{SNR}_s = \frac{g_s}{\sigma^2} |f^\dagger s_s|^2.
\]

The data rates \( R_p \) and \( R_s \) are chosen to satisfy the following outage probability constraints

\[
\Pr(\text{SNR}_p \leq \theta_p) \leq \zeta \quad \text{and} \quad \Pr(\text{SNR}_s \leq \theta_s) \leq \xi
\]

where \( 0 < \zeta \ll 1 \) and \( 0 < \xi \ll 1 \) are the prescribed outage probabilities for the primary and secondary links, respectively.

### III. COGNITIVE BEAMFORMING AND COOPERATIVE FEEDBACK: ALGORITHMS

In this section, we discuss the algorithms for beamforming design at \( T_s \) to control the interference from \( T_s \) to \( R_p \) based on the collaborative feedback from \( R_p \). The algorithms are designed to minimize the secondary link outage probability under the following constraint

\[
\Pr(\text{SNR}_p \leq \theta_p) = \Pr(\text{SINR}_p \leq \theta_p).
\]

In other words, the transmission by \( T_s \) should not increase the outage probability of the primary link. The OCB and NOCB algorithms together with matching IPC feedback designs are discussed separately in the following sections.

#### A. Orthogonal Cognitive Beamforming

The OCB beamformer at \( T_s \), denoted as \( f_o \), suppresses interference to \( R_p \), and yet enhances \( \text{SNR}_s \) in (3). To this end, \( f_o \) is constrained to be orthogonal to the feedback CDI \( \hat{s}_i \), thus giving the name OCB. Despite the orthogonality constraint, there exists residual interference from \( T_s \) to \( R_p \) due to the quantization error in \( \hat{s}_i \). The interference power can be controlled to satisfy the constraint in (5) using IPC feedback from \( R_p \) to \( T_s \). Specifically, the transmit power of \( T_s \) satisfies the constraint \( ||f_o||^2 \leq \hat{\eta} \), where \( \hat{\eta} \) is the quantized IPC feedback signal to be designed in the following subsections. Note that \( \hat{\eta} \leq P_{\text{max}} \). The computation of \( f_o \) at \( T_s \) under the above constraints can be formulated as the following optimization problem.

\[
f_o = \arg \max_{\mathbf{v} \in \mathbb{C}^L} |\mathbf{v}^\dagger s_s|^2, \quad \text{s.t.} \quad \mathbf{v}^\dagger \hat{s}_i = 0 \text{ and } ||\mathbf{v}||^2 \leq \hat{\eta}.
\]
To solve the above optimization problem, we decompose \( s_s \) as \( s_s = a\hat{s}_i + b\hat{s}_\perp \), where \( \hat{s}_\perp \in \mathbb{O}_L \) (denoting the space of \( L \times 1 \) complex vector with unit-norm), \( \hat{s}_\perp \perp \hat{s}_i \) (i.e., \( \hat{s}_\perp^\dagger \hat{s}_i = 0 \)), and the complex coefficients \((a, b)\) satisfy \( |a|^2 + |b|^2 = 1 \). With this decomposition, the optimization problem in (6) can be rewritten as

\[
\mathbf{f}_o = \arg \max_{\mathbf{v} \in \mathbb{C}^L} |b\mathbf{v}^\dagger \hat{s}_\perp|^2, \quad \text{s.t. } \|\mathbf{v}\|^2 \leq \hat{\eta}.
\]  

It follows that \( \mathbf{f}_o \) should implement maximum-ratio transmission [25] and is thus given as

\[
\mathbf{f}_o = \sqrt{\hat{\eta}}\hat{s}_\perp.
\]  

1) Design of Primary IPC Feedback: In this subsection, the unquantized IPC feedback signal, denoted as \( \eta \), is designed such that the constraint \( \|\mathbf{v}\|^2 \leq \eta \) is sufficient for enforcing that in (5). The quantization of \( \eta \) will be discussed in the next subsection. The constraint in (5) can be translated into one on the residual interference power from \( T_s \) to \( R_p \), denoted as \( \mathbf{I}_o \), as follows. Based on the CDI quantization model in Section II, we can decompose \( s_i \) as

\[
s_i = e^{j\theta_1}\sqrt{1-\epsilon}\hat{s}_i + e^{j\theta_2}\sqrt{\delta}\hat{s}_\perp + e^{j\theta_3}\sqrt{\epsilon-\delta}\mathbf{q}
\]  

where \( 0 \leq \delta \leq \epsilon \), \( \mathbf{q} \in \mathbb{O}_L \) lies in the null space of \((\hat{s}_i, \hat{s}_\perp)\), and the angles \( (\theta_1, \theta_2, \theta_3) \) represent appropriate phase rotations. Using the above expression and that of \( \mathbf{f}_o \) given in (8), \( \mathbf{I}_o \) can be upper-bounded as

\[
\mathbf{I}_o := \lambda g_i|\mathbf{f}^\dagger s_i|^2
\]  

\[
= \lambda g_i|\sqrt{\hat{\eta}}\hat{s}_\perp(e^{j\theta_1}\sqrt{1-\epsilon}\hat{s}_i + e^{j\theta_2}\sqrt{\delta}\hat{s}_\perp + e^{j\theta_3}\sqrt{\epsilon-\delta}\mathbf{q})|^2
\]  

\[
= \lambda g_i\hat{\eta}\delta
\]  

\[
\overset{(a)}{\leq} \lambda g_i\hat{\eta}\epsilon
\]  

where \((a)\) holds since \( \delta \leq \epsilon \). Note that computing \( \delta \) at \( R_p \) requires \( \hat{s}_\perp \) and thus feedforward of \( s_s \) from \( T_s \). Therefore, \( \mathbf{I}_o \) can be obtained at \( R_p \) using (11) for the case of feedforward or otherwise estimated using (12). Based on the principle of opportunistic power control in [6], \( \mathbf{I}_o \) must satisfy the following constraint that is equivalent to (5)

\[
\mathbf{I}_o \leq \omega := \sigma^2 \left( \frac{\gamma_pg_{p}}{\theta_p} - 1 \right) , \quad \text{if } \omega \geq 0
\]  

(i.e., the receive SNR at \( R_p \) can accommodate additional interference from \( T_s \) and yet maintain non-outage); otherwise, \( \mathbf{I}_o \) can be arbitrarily large since \( R_p \) experiences outage even without the interference from \( T_s \). For the case without feedforward, the constraint on the transmit power of \( T_s \) is obtained by
combining (12) and (13) as

\[
\eta = \begin{cases} 
\frac{\omega}{\lambda g_i \epsilon}, & \omega \geq 0 \\
 P_{\text{max}}, & \text{otherwise.}
\end{cases}
\]  
(14)

The counterpart of \( \eta \) for the case of feedforward, denoted as \( \hat{\eta} \), follows from (11) and (13) as

\[
\hat{\eta} = \begin{cases} 
\frac{\omega}{\lambda g_i \delta}, & \omega \geq 0 \\
 P_{\text{max}}, & \text{otherwise.}
\end{cases}
\]  
(15)

Note that the constraint in (15) on \( P_s \) is looser than that in (14) in the case of \( w \geq 0 \) since \( \delta \leq \epsilon \).

2) Quantization of Primary IPC Feedback : This subsection focuses on the quantization of the IPC feedback signal \( \eta \) for the case without feedforward. Let \( \hat{\eta} \) denote the output of quantizing \( \eta \). Based on (14), the feedback of \( \hat{\eta} \) is designed to comprise \((A + 1)\) binary bits. The first bit indicates whether there is an outage event at \( R_p \); the following \( A \) bits represent \( \hat{\eta} \) if \( R_p \) is not in outage (i.e., \( \omega \geq 0 \)) or otherwise neglected by \( T_s \). Given \( \omega \geq 0 \) or equivalently \( \gamma_p g_p \geq \theta_p \), \( \hat{\eta} \) is constrained to take on values from a finite set of \( N := 2^A \) nonnegative scalars, denoted by \( \mathcal{P} = \{p_0, p_1, \cdots, p_{N-1}\} \) where \( p_0 < p_1 < \cdots < p_{N-1} \). Note that the optimal design of \( \mathcal{P} \) for minimizing the secondary link outage probability requires additional knowledge of the secondary transmit rate and channel distribution at \( R_p \), and changes when such information is varied. Thus, we consider the suboptimal design of \( \mathcal{P} \), in which the elements partition the space of \( \eta \) using the criterion of equal probability. Specifically, \( p_0 = 0 \) and

\[
\begin{align*}
\Pr(p_n < \eta \leq p_{n+1} | \gamma_p g_p \geq \theta_p) &= \frac{1}{N}, & 0 \leq n \leq N - 2 \\
\Pr(\eta > p_n | \gamma_p g_p \geq \theta_p) &= \frac{1}{N}, & n = N - 1.
\end{align*}
\]  
(16)

Note that the \( \mathcal{P} \) given above is independent of \( P_{\text{max}} \). Given \( \mathcal{P} \), define the operator \([\cdot]_\mathcal{P}\) on \( x \geq 0 \) as \( [x]_\mathcal{P} = \max_{p \in \mathcal{P}} p \) subject to \( p \leq x \). Thus, \( \hat{\eta} \) is quantized as

\[
\hat{\eta} = \begin{cases} 
[\eta]_\mathcal{P}, & \omega \geq 0 \text{ and } \eta < P_{\text{max}} \\
 P_{\text{max}}, & \text{otherwise.}
\end{cases}
\]  
(17)

Note that \( \hat{\eta} \leq \eta \) and thus the constraint \( \|\hat{\eta}\|^2 \leq \hat{\eta} \) is sufficient for maintaining the constraint in (12) or its equivalence in (5). Also note that the above result can be easily extended to the case with feedforward by replacing \( \hat{\eta} \) with \( \tilde{\eta} \), which is the quantized IPC feedback for \( \tilde{\eta} \).

\(^1\)The IPC quantizer can be improved by limiting the quantization range to \( P_{\text{max}} \) and optimizing the set \( \mathcal{P} \) using Lloyd’s algorithm [26]. However, the related analysis is complicated. Thus, we use the current design for simplicity and do not pursue the optimization of the IPC quantization in this work.
B. Non-Orthogonal Cognitive Beamforming

The NOCB beamformer at $T_s$ is designed by relaxing the orthogonality constraint on OCB. First, we formulate the design of NOCB beamformer as a convex optimization problem and derive its close-form solution. Next, the IPC feedback compatible with the derived NOCB beamformer is designed.

The NOCB beamformer, denoted as $f_n$, is modified from the OCB counterpart by replacing the constraint $f_n^\dagger \hat{s}_i = 0$ with $|f_n^\dagger \hat{s}_i| \leq \hat{\mu}_1$ where $0 \leq \hat{\mu}_1 \leq P_{\text{max}}$. In other words, NOCB controls transmit power in the direction of quantized CDI feedback rather than suppressing it. In addition, $f_n$ satisfies a power constraint $\|f_n\|^2 \leq \hat{\mu}_2$ with $0 \leq \hat{\mu}_2 \leq P_{\text{max}}$. The parameters $\hat{\mu}_1$ and $\hat{\mu}_2$ constitute the quantized IPC feedback signal designed in the sequel. Under the above constraints, designing $f_n$ to maximize the receive SNR at $R_s$ can be formulated as the following optimization problem

$$f_n = \max_{v \in C_L} |v^\dagger s|, \quad \text{s.t.} \quad |v^\dagger \hat{s}_i| \leq \hat{\mu}_1 \text{ and } \|v\|^2 \leq \hat{\mu}_2.$$  \hfill (18)

To simplify the solution of the above problem, we write $f_n = \alpha \hat{s}_i + \beta \hat{s}_\perp + \rho q$ where $|\alpha|^2 + |\beta|^2 + |\rho|^2 \leq \hat{\mu}_2$, and $\hat{s}_\perp$ and $q$ follow those defined in Section III-A. An optimization problem having the same form as that in (18) is solved in [4]. Using the results in [4, Theorem 2] and $s = a\hat{s}_i + b\hat{s}_\perp$, we obtain the following lemma.

**Lemma 1:** The NOCB beamformer is given by $f_n = \alpha \hat{s}_i + \beta \hat{s}_\perp$ where

- If $\hat{\mu}_1 \geq |a|^2 \hat{\mu}_2$
  $$\alpha = a \sqrt{\hat{\mu}_2}, \quad \beta = b \sqrt{\hat{\mu}_2}$$  \hfill (19)

- If $0 \leq \hat{\mu}_1 < |a|^2 \hat{\mu}_2$
  $$\alpha = a \sqrt{\hat{\mu}_1}, \quad \beta = b \sqrt{\hat{\mu}_2 - \hat{\mu}_1}.$$  \hfill (20)

Note that the beamformer for the first case implements maximum ratio transmission [25].

In the remainder of this section, the IPC feedback signal $\hat{\mu} = (\hat{\mu}_1, \hat{\mu}_2)$ is designed to enforce the constraint in (5). To reduce the feedback overhead, each feedback instant of $\hat{\mu}$ comprises only one variable (either $\mu_1$ or $\mu_2$) with the other fixed as a designed constant. Thereby, the IPC feedback is effectively a scalar feedback with one additional bit specifying the variable index.

The unquantized version of $\hat{\mu}$, denoted as $\mu = (\mu_1, \mu_2)$, is first designed as follows. Similar to (13), the constraint in (5) can be transformed into the following constraint on the residual interference power from $T_s$ to $R_p$

$$I_n := \lambda g_i |f_n^\dagger \hat{s}_i|^2 \leq \omega, \quad \omega \geq 0$$  \hfill (21)

or otherwise $\|f_n\|^2 = P_{\text{max}}$. To implement the above constraint using the IPC feedback, $I_n$ is upper-
bounded as follows.

\[
I_n = (a) \lambda g_i |(\alpha \hat{s}_i + \beta \hat{s}_\perp)^\dagger(e^{j\theta_1} \sqrt{1 - \epsilon} \hat{s}_i + e^{j\theta_2} \sqrt{\delta} \hat{s}_\perp + e^{j\theta_3} \sqrt{\epsilon - \delta} q)|^2
\]

\[
= \lambda g_i |\alpha e^{j\theta_1} \sqrt{1 - \epsilon} + \beta e^{j\theta_2} \sqrt{\delta}|^2
\]

\[
\leq \lambda g_i (|\alpha| \sqrt{1 - \epsilon} + |\beta| \sqrt{\delta})^2
\]

\[
\leq \lambda g_i (|\alpha| \sqrt{1 - \epsilon} + \sqrt{P_{\text{max}} \delta})^2
\]

\[
\leq \lambda g_i (|\alpha| \sqrt{1 - \epsilon} + \sqrt{P_{\text{max}} \epsilon})^2
\]

(22)

(23)

where (a) uses Lemma 1 and (9), (b) results from \( \|f_n\|^2 = |\alpha|^2 + |\beta|^2 \leq P_{\text{max}} \), and (c) follows from \( \delta \leq \epsilon \). Recall that computing \( \delta \) at \( R_p \) requires secondary feedforward. Therefore, for the case without feedforward, the bound on \( I_n \) in (23) that is independent of \( \delta \) should be used in designing the IPC feedback. Specifically, combining (23) and (21) gives the following constraint on \( \alpha \)

\[
|\alpha| \leq \nu := \frac{\sqrt{\omega/\lambda g_i} - \sqrt{P_{\text{max}} \epsilon}}{\sqrt{1 - \epsilon}}.
\]

(24)

For \( \nu \geq 0 \), it follows that \( \mu_1 = \nu^2 \). For \( \nu < 0 \), the above constraint is invalid and thus we set \( |\alpha|^2 = \mu_1 = 0 \); as a result, the NOCB optimization problem in (18) converges to the OCB counterpart in (6), leading to \( \mu_2 = \eta \). Furthermore, it can be observed from (22) that setting \( \mu_2 = P_{\text{max}} \) for the case of \( \mu_1 > 0 \) does not violate the interference constraint in (21). Combining the above results gives the IPC feedback design as follows.

\[
(\text{No feedforward}) \quad \nu = \begin{cases} 
(\nu^2, P_{\text{max}}), & \nu \geq 0 \text{ and } \omega \geq 0 \\
(0, \eta), & \nu < 0 \text{ and } \omega \geq 0 \\
(P_{\text{max}}, P_{\text{max}}), & \omega < 0.
\end{cases}
\]

(25)

It follows that the quantized IPC feedback, denoted as \( \hat{\nu} \), is given as

\[
(\text{No feedforward}) \quad \hat{\nu} = \begin{cases} 
(\hat{\nu}, P_{\text{max}}), & \hat{\nu} \geq 0 \text{ and } \omega \geq 0 \\
(0, \hat{\eta}), & \hat{\nu} < 0 \text{ and } \omega \geq 0 \\
(P_{\text{max}}, P_{\text{max}}), & \omega < 0
\end{cases}
\]

(26)

where \( \hat{\nu} := [\nu^2]_{\mathcal{P}'} \) with \( \mathcal{P}' \) being a scalar quantizer codebook. This codebook can follow the design of \( \mathcal{P} \) as discussed in Section III-A.2. The feedback of \( \hat{\nu} \) is observed from (26) to involve the transmission of a quantized scalar (either \( \hat{\nu} \) or \( \hat{\eta} \)) with one additional bit for separating the first two cases in (26) (Note that the third case can be represented by setting \( \hat{\nu} = P_{\text{max}} \)).
For the case with secondary feedforward, the collaborative feedback is designed by applying the constraint in (21) to the upper bound on $I_n$ in (22) and following the similar steps as for the case without feedforward. The resultant quantized IPC feedback, denoted as $\tilde{\mu}$, is

\[
(\text{Feedforward}) \quad \tilde{\mu} = \begin{cases} 
(\hat{\nu}, P_{\text{max}}), & \hat{\nu} \geq 0 \text{ and } \omega \geq 0 \\
(0, \tilde{\eta}), & \hat{\nu} < 0 \text{ and } \omega \geq 0 \\
(P_{\text{max}}, P_{\text{max}}), & \omega < 0 
\end{cases}
\]

(27)

where $\hat{\nu}$ is the unquantized IPC feedback signal

\[
\hat{\nu} := \sqrt{\frac{\omega}{\lambda g_i} - \sqrt{\delta P_{\text{max}}}} \sqrt{1 - \epsilon}
\]

(28)

and $\tilde{\nu} := [\hat{\nu}^2]_P$ with $\tilde{P}$ being a suitable quantizer codebook. Note that $\mu \geq \hat{\mu}$ since $\nu \geq \hat{\nu}$ and $\tilde{\eta} \geq \tilde{\eta}$.

C. Comparison between Orthogonal and Non-Orthogonal Cognitive Beamforming

In this section, we compare the performance of OCB and NOCB beamforming designs in the asymptotic regimes of large SU transmit power (i.e., $P_{\text{max}} \to \infty$) and high-resolution collaborative feedback (i.e., $A, B \to \infty$).

Consider first the regime of $A, B \to \infty$. For OCB, it follows from (12) that the residual interference power $I_o \to 0$ and hence OCB converges to the perfect zero-forcing beamforming. Moreover, from (14) and (17), the limit of SU transmit power $\tilde{\eta} \to P_{\text{max}}$ for the case without feedforward. The same result holds with feedforward. For NOCB, the beamforming constraints in (18) converge as $\tilde{\mu} \to (\nu_0^2, P_{\text{max}})$ and $\tilde{\mu} \to (\nu_0^2, P_{\text{max}})$ for the cases with and without feedforward, respectively, where $\nu_0$ is given by $\nu$ in (24) or (28) with $\epsilon = 0$ and thus $\delta = 0$. The corresponding NOCB is identical to the optimal CB derived in [4] assuming perfect CSI of the interference channel and under a prescribed interference power constraint $\nu_0^2$ at $R_p$, which maximizes the receive SNR at $R_s$. In other words, for high-resolution collaborative feedback, NOCB outperforms OCB. In the non-asymptotic regime, NOCB should also perform better than OCB since from (17) and (26) we see that NOCB always has a more relaxed transmit power constraint for the SU than OCB.

The performances of OCB and NOCB converge in the regime of $P_{\text{max}} \to \infty$ and finite-rate CDI feedback ($\epsilon > 0$). To state this result, let $P_{\text{out}}$ and $\tilde{P}_{\text{out}}$ denote the SU outage probabilities for OCB and NOCB, respectively.

**Proposition 1:** For large $P_{\text{max}}$, the SU outage probabilities for OCB and NOCB converge as

\[
\lim_{P_{\text{max}} \to \infty} \tilde{P}_{\text{out}} = \lim_{P_{\text{max}} \to \infty} P_{\text{out}}.
\]

(29)
This result holds for both the cases with and without feedforward.

Proof: See Appendix A.

The above discussions are confirmed by simulation results in Fig. 3.

IV. Outage Probability

CDI typically requires more feedback bits than IPC since the former is for a complex $L \times 1$ vector and the latter is for a real scalar. For this reason, assuming perfect IPC feedback, this section focuses on characterizing the effects of CDI quantization on the SU outage probability for OCB. Similar analysis can be carried out for NOCB. However, the results are complicated with little insight and hence omitted.

A. Orthogonal Cognitive Beamforming without Feedforward

The cognitive beamforming and collaborative feedback algorithms proposed in this paper all satisfy the constraint in (5). Therefore the PU outage probability for all designs is given as

$$P_{\text{out}} = \Pr(\gamma_p g_p \leq \theta_p) = 1 - e^{-\frac{\theta_p}{\gamma_p}}$$

where $a$ follows from the fact that the primary channel gain $g_p$ is distributed as $\exp(1)$.

In the remainder of this section, we focus on deriving the SU outage probability $P_{\text{out}}$ for OCB without feedforward. This probability depends on the distribution of the transmit power of the SU, $P_s := ||f_o||^2$, as well as the secondary channel gain $g_s$. Given the OCB algorithm in Section III-A, the distribution of $P_s$ is obtained from (14) as shown in the following lemma.

Lemma 2: In the case of OCB without feedforward, the distribution of $P_s$ is given as

$$\Pr(P_s = P_{\text{max}}) = 1 - e^{-\frac{\theta_p}{\gamma_p}} \left[ \frac{(L - 1)\theta_p \lambda \gamma_{\text{max}}}{\gamma_p} \right] 2^{-\frac{\theta_p}{\gamma_p}} + O \left( 2^{-\frac{2\theta_p}{\gamma_p}} \right)$$

$$\Pr(P_s < \tau) = e^{-\frac{\theta_p}{\gamma_p}} \left[ \frac{(L - 1)\theta_p \lambda \tau}{\gamma_p \sigma^2} \right] 2^{-\frac{\theta_p}{\gamma_p}} + O \left( 2^{-\frac{2\theta_p}{\gamma_p}} \right), \forall 0 \leq \tau \leq P_{\text{max}}$$

where $\gamma_{\text{max}} := \frac{P_{\text{max}}}{\sigma^2}$.

Proof: See Appendix B.

For a sanity check, from the above results,

$$\lim_{B \to \infty} \Pr(P_s = P_{\text{max}}) = 1 \quad \text{and} \quad \lim_{B \to \infty} \Pr(P_s < P_{\text{max}}) = 0.$$

These agree with the fact that OCB with perfect CDI feedback ($B \to \infty$) nulls the interference from $T_s$ to $R_p$ by zero-forcing beamforming and thereby allows $T_s$ to always transmit using the maximum power.
Next, define the effective channel gain of the secondary link as
\[ \tilde{g}_s := g_s |\hat{f}_o s_s|^2, \] with \( \hat{f}_o := f_o / \sqrt{P_s} \).
The following result is obtained in [27] on zero-forcing beamforming for mobile ad hoc networks.

**Lemma 3:** The effective channel gain \( \tilde{g}_s \) is a chi-square random variable with \((L - 1)\) complex degrees of freedom, whose PDF is given as
\[ f_{\tilde{g}_s}(\tau) := \frac{\tau^{L-2}}{\Gamma(L-1)} e^{-\tau} \] (33)
where \( \Gamma(\cdot) \) denotes the Gamma function.

Using Lemmas 2 and 3, the main result of this section is obtained as shown in the following theorem.

**Theorem 1:** The SU outage probability in the case of OCB without feedforward is
\[ P_{\text{out}} = 1 - \frac{\Gamma \left( L - 1, \frac{\theta_s \gamma_{\text{max}}}{\gamma_p} \right)}{\Gamma(L-1)} + \varphi 2^{-\frac{B}{L-1}} + O \left( 2^{-\frac{2B}{L-1}} \right) \] (34)
where
\[ \varphi = e^{-\frac{\theta_s}{\gamma_p} (L-1) \lambda \theta_p \theta_s \Gamma \left( L - 2, \frac{\theta_s \gamma_{\text{max}}}{\gamma_p} \right)} \frac{\lambda \theta_p \theta_s \Gamma \left( L - 2 \right)}{\gamma_p \Gamma(L-1)}. \] (35)

**Proof:** See Appendix C.

The last two terms in (34) represents the increase in the SU outage probability due to the CDI quantization. The asymptotic outage probabilities for high SNRs and high-resolution CDI feedback is given in the following two corollaries.

**Corollary 1:** For high SNR, the SU outage probability in Theorem 1 converges as
\[ \lim_{P_{\text{max}} \to \infty} P_{\text{out}} = e^{-\frac{\theta_s}{\gamma_p} (L-1) \lambda \theta_p \theta_s 2^{-\frac{B}{L-1}}} + O \left( 2^{-\frac{2B}{L-1}} \right) \] (36)

This result in (36) shows that at high SNR, \( P_{\text{out}} \) saturates above a minimum level that depends on the quality of CDI feedback. The reason is that transmission at \( T_s \) contributes residual interference to \( R_p \). As a result, \( P_s \) is upper-bounded even with \( P_{\text{max}} \to \infty \). The saturation level of \( P_{\text{out}} \) in (36) decreases exponentially with increasing \( B \) since the corresponding residual interference power reduces. More details can be found in Fig. 2 and the related discussions in Section VI. To facilitate subsequent discussions, we refer to the range of \( P_{\text{max}} \) where \( P_{\text{out}} \) is constant as the *interference limiting regime*.

From (37), it can be observed that in the interference limiting regime \( P_{\text{out}} \) increases with the number of antennas \( L \). The reason is that the CDI quantization error increases with \( L \) if \( B \) is fixed, thus increasing the residual interference from \( T_s \) to \( R_p \). To prevent \( P_{\text{out}} \) from growing with \( L \) in the interference limiting regime, \( B \) must increase at least linearly with \((L - 1)\). However, \( P_{\text{out}} \) decreases with \( L \) outside the interference limiting regime, as shown by simulations results in Fig. 4.
Corollary 2: For perfect CDI feedback, the SU outage probability in Theorem 1 is

$$\lim_{B \to \infty} P_{\text{out}} = 1 - \frac{\Gamma \left( L - 1, \frac{\theta_s}{\gamma_{\max}} \right)}{\Gamma(L - 1)}. \quad (38)$$

In this asymptotic regime, $P_{\text{out}}$ decreases continuously with $P_{\text{max}}$ since the interference from $T_s$ to $R_p$ is eliminated by OCB. This result is verified by simulation in Fig. 2.

B. Orthogonal Cognitive Beamforming with Feedforward

Consider OCB with feedforward. Effectively, feedforward changes the analysis in the preceding section by replacing $\epsilon$ with $\delta$ defined in Section III-A. The distribution of $\delta$ is given in the following lemma.

Lemma 4: The PDF of $\delta$ is given as

$$f_{\delta}(\tau) = (L - 1)2^{\frac{n}{\tau - 1}} \left( 1 - 2^{\frac{n}{\tau - 1}} \right)^{L-2}, \quad 0 \leq \tau \leq 2^{-\frac{n}{\tau - 1}}. \quad (39)$$

Proof: See Appendix D. \qed

Let $\hat{P}_s$ represent the transmit power of $T_s$ for the case of OCB with feedforward. Using (15) and Lemma 4, the distribution of $\hat{P}_s$ is obtained as shown in the following lemma.

Lemma 5: The distribution of $\hat{P}_s$ is given as

$$\Pr(\hat{P}_s = P_{\max}) = 1 - e^{-\frac{\theta_p}{\gamma_p}} \left( \frac{\theta_p \lambda \gamma_{\max}}{\gamma_p} \right) 2^{-\frac{n}{\tau - 1}} + O \left( 2^{-\frac{2n}{\tau - 1}} \right) \quad (40)$$

$$\Pr(\hat{P}_s < \tau) = e^{-\frac{\theta_p}{\gamma_p}} \left( \frac{\theta_p \lambda \tau}{\gamma_p \sigma^2} \right) 2^{-\frac{n}{\tau - 1}} + O \left( 2^{-\frac{2n}{\tau - 1}} \right), \forall 0 \leq \tau \leq P_{\max}. \quad (41)$$

Proof: See Appendix E. \qed

The following theorem is proved using Lemma 5 and following the same procedure as for Theorem 1.

Theorem 2: For the case of OCB with feedforward, the SU outage probability is

$$\hat{P}_{\text{out}} = 1 - \frac{\Gamma \left( L - 1, \frac{\theta_s}{\gamma_{\max}} \right)}{\Gamma(L - 1)} + \frac{\varphi}{L - 1} 2^{-\frac{n}{\tau - 1}} + O \left( 2^{-\frac{2n}{\tau - 1}} \right) \quad (42)$$

where $\varphi$ is given in Theorem 1.

By comparing Theorems 1 and 2, it can be observed that feedforward reduces the increment of the outage probability due to CDI feedback quantization by a factor of $(L - 1)$. Thus, the outage probability reduction with feedforward is more significant for larger $L$.

V. TRADEOFF BETWEEN IPC AND CDI FEEDBACK

In this section, we consider both quantized CDI and IPC feedback. Using results derived in the preceding section and under a sum feedback constraint, the optimal allocation of bits to the IPC and CDI feedback is derived for OCB without feedforward and proved to hold also for OCB with feedforward.
Consider OCB without feedforward. The main challenge for deriving the optimal tradeoff between the IPC and CDI feedback in this case is the derivation of the distribution function of the transmit power of $T_s$, denoted as $\hat{P}_s$. This distribution is complicated given both CDI and IPC feedback quantization. Thus, we derive a close approximation of the optimal tradeoff as follows.

The loss on $\hat{P}_s$ due to IPC feedback quantization is bounded by a function of the number of IPC feedback bits $A$. Define the index $1 \leq n_0 \leq (2^A - 1)$ such that $p_{n_0-1} \leq P_{\max} \leq p_{n_0}$ where $p_n \in \mathcal{P}$.

Using this definition, the IPC quantization loss can be bounded by $\Delta P$ defined as follows

$$|\hat{P}_s - P_s| \leq \Delta P := \max_{1 \leq n \leq n_0} (p_n - p_{n-1}), \quad p_n \in \mathcal{P} \tag{43}$$

where $P_s$ is the counterpart of $\hat{P}_s$ for the case of perfect IPC feedback.

**Lemma 6:** $\Delta P$ defined in (43) is given by

$$\Delta P = \frac{\gamma_p \sigma^2}{(L-1)\theta_p \lambda} 2^{\frac{\theta_s}{\gamma_p \sigma^2}} + O\left(2^{\frac{\theta_s}{\gamma_p \sigma^2}}\right). \tag{44}$$

**Proof:** See Appendix F. □

Next, the CDF of $\hat{P}_s$ is upper bounded as shown below.

**Lemma 7:** The distribution of $\hat{P}_s$ satisfies

$$\Pr(\hat{P}_s = P_{\max}) = \Pr(P_s = P_{\max}) \tag{45}$$

$$\Pr(\hat{P}_s < \tau) \leq e^{-\frac{\tau}{\gamma_p \sigma^2}} \left(\frac{(L-1)\theta_p \lambda (\tau + \Delta P)}{\gamma_p \sigma^2}\right) 2^{-\frac{\theta_s}{\gamma_p \sigma^2}} + O\left(2^{-\frac{\theta_s}{\gamma_p \sigma^2}}\right), \quad \forall \ 0 \leq \tau \leq P_{\max} \tag{46}$$

where $\Pr(P_s = P_{\max})$ and $\Delta P$ are given in Lemma 2 and Lemma 6, respectively.

**Proof:** See Appendix G. □

Using Lemma 7 and following the procedure for proving Theorem 1, the outage probability for OCB without feedforward is bounded as shown below.

**Proposition 2:** Given quantized CDI and IPC feedback, the SU outage probability for the case of OCB without feedforward satisfies

$$\hat{P}_{out} \leq 1 - \frac{\varphi 2^{-\frac{\theta_s}{\gamma_p \sigma^2}} + \alpha 2^{-A} + O\left(2^{-\frac{\theta_s}{\gamma_p \sigma^2}}\right)}{\Gamma(L-1)} \tag{47}$$

where $\varphi$ is given in Theorem 1 and

$$\alpha := e^{-\frac{\theta_s}{\gamma_p \sigma^2}} \frac{\Gamma\left(L-1, \frac{\theta_s}{\gamma_p \sigma^2}\right)}{\Gamma(L-1)}. \tag{48}$$

Comparing the above result with (34), the increment over $P_{\text{out}}$ due to IPC feedback quantization is upper-bounded by the term $\alpha 2^{-A}$. The asymptotic result parallel to that in Corollary 1 is given below.

**Corollary 3:** For high SNR, the upper bound on the SU outage probability $\hat{P}_{out}$ converges as

$$\lim_{P_{\max} \to \infty} \hat{P}_{\text{out}} \leq \varphi 2^{-\frac{\theta_s}{\gamma_p \sigma^2}} + \alpha 2^{-A} \tag{49}$$
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where \( \varphi' := e^{-\frac{\theta_p}{\gamma_p} (L-1) \lambda \theta_s \theta_r} \) and \( \alpha' := e^{-\frac{\theta_p}{\gamma_p}} \).

The two terms on the right-hand-side of (48) characterize the effects of CDI and IPC quantization, respectively. The exponent of the first term, namely \( -\frac{B}{L-1} \), is scaled by the factor \( \frac{1}{L-1} \), which does not appear in that of the second term. The reason is that CDI quantization partitions the space of \( L \)-dimensional unitary vectors, while the IPC quantization discretizes the nonnegative real axis.

Consider the sum feedback constraint \( A + B = F \). Note that \( (F + 1) \) represents the total number of feedback bits where the additional bit is used as an indicator of an outage event at \( R_p \) (cf. Section III-A.2). Under this constraint, the feedback bit allocation to CDI and IPC feedback can be optimized for minimizing the outage probability upper bound in (47) as follows. Assume \( B \gg 1 \) and the second-order term \( O \left( 2^{-2 \frac{B}{L-1}} \right) \) in (47) is negligible. Under the constraint \( A + B = F \), the optimal value of \( B \), denoted as \( B^* \), to minimize the upper bound on \( \hat{P}_{out} \) is obtained as

\[
B^* = \min_{0 \leq B \leq F} \left( \varphi 2^{-\frac{B}{L-1}} + \alpha 2^{-A} \right), \quad \text{s.t.} \quad A + B = F.
\]

(49)

The function \( J(B) \) can be shown to be convex. Thus, by relaxing the integer constraint, \( B^* \) can be computed using the following equation

\[
\frac{dJ}{dB}(B^*) = -\frac{\ln 2 \times \varphi}{L-1} 2^{-\frac{B^*}{L-1}} + \ln 2 \times \alpha 2^{-F} 2^{B^*} = 0.
\]

(50)

It follows that

\[
B^* = \min \left[ \frac{L-1}{L} \left( F - \log_2 \chi \right)^+, F \right]
\]

(51)

where

\[
\chi := \frac{\gamma_p \Gamma(L-1, \frac{\theta_s \gamma_{max}}{\theta_p})}{\lambda \theta_p \theta_s \Gamma(L-2, \frac{\theta_s \gamma_{max}}{\theta_p})}.
\]

(52)

The value of \( B^* \) as computed above can then be rounded to satisfy the integer constraint. The derivation of (51) uses the first-order approximation of the upper bound on \( \hat{P}_{out} \) in Proposition Prop:Pout:QuantPwr, which is accurate for relatively small value of \( \frac{\theta_p \theta_s}{\gamma_p} 2^{-\frac{B}{L-1}} \). In this range, the feedback allocation using (51) closely predicts the optimal feedback tradeoff as observed from simulation results in Fig. 5. However, for large values of \( \frac{\theta_p \theta_s}{\gamma_p} 2^{-\frac{B}{L-1}} \), it is necessary to derive the optimal feedback allocation based on the exact analysis of \( \hat{P}_{out} \), which is tedious and gives no closed-form solution.

Next, consider OCB with feedforward. The feedforward counterpart of Proposition 2 is obtained as the corollary given below.
Corollary 4: Given quantized CDI and IPC feedback, the SU outage probability for the case of OCB with feedforward satisfies

\[
P_{\text{out}} \leq 1 - \frac{\Gamma(L - 1, \frac{\theta_s}{\gamma_{\text{max}}})}{\Gamma(L - 1)} + \frac{\varphi + \alpha 2^{-\lambda} - 2^{-\frac{B}{L - 1}}}{L - 1} + O\left(2^{-\frac{2B}{L - 1}}\right)
\]

The result in (53) shows that feedforward reduces the increment on outage probability due to IPC quantization by a factor of \((L - 1)\). Since the solution of the optimization problem in (49) minimizes the upper bound on \(P_{\text{out}}\) in (53), the optimal number of CDI feedback bits in (51) is also applicable for the case with feedforward.

VI. Simulation Results

In this section, various simulation results are presented for corroborating the theoretic results derived earlier and providing additional insight into the performance of cognitive beamforming and collaborative feedback algorithms. The common simulation parameters are the SINR/SNR thresholds \(\theta_p = \theta_s = 3\), the interference channel average power \(\lambda = 0.1\), and the noise variance \(\sigma^2 = 1\).

First, consider OCB with quantized CDI and perfect IPC feedback. This assumption is made for simulation results in Figs. 2, 3, and 4. Fig. 2 displays the curves of SU outage probability \(P_{\text{out}}\) versus maximum SU transmit SNR \(\gamma_{\text{max}}\) for the number of CDI feedback bits \(B = \{8, 12, 16, 20\}\). For comparison, we also plot the first-order terms of the \(P_{\text{out}}\) asymptotic values as given in Corollary 1 and Corollary 2. As observed from Fig. 2, \(P_{\text{out}}\) for a given \(B\) converges from above to the corresponding limit as \(\gamma_{\text{max}}\) increases, validating the result in Corollary 1. The limit of \(P_{\text{out}}\) in the interference limiting regime is observed to decrease exponentially with \(B\) as a result of more accurate CDI feedback.

Fig. 3 compares the performances of OCB and NOCB in terms of \(P_{\text{out}}\) with increasing \(\gamma_{\text{max}}\). As observed from Fig. 3, the performances of both beamforming algorithms converge in the interference limiting regime, as predicted by Proposition 1. The convergence is slower for larger \(B\). However, outside this regime, NOCB yields significantly lower SU outage probability than OCB. In the asymptotic case of \(B \to \infty\), NOCB outperforms OCB regardless of \(\gamma_{\text{max}}\) [4].

Fig. 4 compares the cases with and without secondary feedforward in terms of the SU outage probability. The OCB and NOCB beamforming designs are considered in Fig. 4(a) and Fig. 4(b), respectively. In Fig. 4(a) corresponding to OCB, it is observed that the reduction of \(P_{\text{out}}\) due to feedforward is more significant in the interference limiting regime and for larger \(L\). Furthermore, increasing \(L\) is found to result in higher outage probability in the interference limiting regime. These observations confirm the remarks following Corollary 1. The above observations also apply to Fig. 4(b) corresponding to NOCB.

Last, consider both quantized CDI and IPC feedback. Fig. 5 shows the curves of \(\hat{P}_{\text{out}}\) in the case
Fig. 2. SU outage probability for OCB versus maximum SU transmit SNR for quantized CDI and perfect IPC feedback. The number of antennas at $T_s$ is $L = 4$ and the PU transmit SNR is $\gamma_p = 10$ dB.

of OCB without feedforward versus the number of bits $A$ for IPC feedback under the sum feedback constraint $(A + B) = 12$ bits. It is observed from Fig. 5 that for a given $\gamma_{\text{max}}$, there exists an optimal combination of $(A, B)$ that minimizes $\hat{P}_{\text{out}}$. The optimal values of $A$ are indicated by the marker “o” and those computed using the theoretic result in (51) by the marker “x”. Despite the approximation involved in the analysis of the optimal feedback tradeoff, the simulation and theoretic results are close and differ at most by one bit as shown in Fig. 5. In particular, these results match each other for $\gamma_{\text{max}} = 6$ dB.

VII. CONCLUDING REMARKS

In this paper, we introduce a new operation model for coexisting PU and SU links in a spectrum sharing network, where the PU receiver collaboratively sends quantized side information to the SU transmitter for facilitating its opportunistic transmission, such that the resulted PU link performance degradation is minimized. Particularly, we consider the scenario where the SU transmitter is equipped with multiple antennas, and thus CB can be deployed along with the PU cooperative feedback to further improve the SU link throughput. We propose two CB schemes for the SU transmitter based on the finite-rate CDI and IPC feedback from the PU receiver, namely orthogonal and non-orthogonal cognitive beamforming, and analyze their performances in terms of SU outage probability. We show that due to feedback quantization, as the SU transmit power constraint increases, the SU outage probability converges to certain value,
which decreases exponentially with the number of CDI feedback bits. However, for a given SU transmit power constraint and PU feedback rate constraint, there exists an optimal allocation for the feedback bits between CDI and IPC feedback to minimize the SU outage probability. Moreover, we show that if additional cooperative feedforward of the SU CSI from the SU transmitter to the PU receiver is available, the IPC feedback of the PU receiver can be more efficiently designed to enable larger transmit powers for the SU CB.

To the authors’ best knowledge, this paper is the first attempt in the literature to study the design of cooperative feedback from the PU to the SU in a cognitive radio network. There are many important issues related to this design problem unaddressed in this paper and thus worth further investigation. For example, for simplicity, we have adopted in this paper a suboptimal equal-probability quantizer for the IPC feedback, while it is important to investigate the optimal quantizer in this case to minimize the SU outage probability. In addition, this paper has assumed single antennas for both PU and SU receivers. It would be interesting to extend the CB and cooperate feedback schemes developed in this paper to the more general case with multi-antenna PU and SU receivers. Last, in this paper, we have assumed a single SU link coexisting with a single PU link, while it is pertinent to investigate the more general system model with multiple coexisting PU and SU links.
Fig. 4. SU outage probability versus the maximum SU transmit SNR for (a) OCB and (b) NOCB. For each type of beamforming, both the cases of feedforward and no feedforward are considered. The number of antennas at $T_s$ is $L = 4$ and the PU transmit SNR is $\gamma_p = 10$ dB.
Fig. 5. SU outage probability versus the number of quantized IPC feedback bits. The total number of bits for CDI and IPC feedback is $A + B = 12$, the number of antennas at $T_s$ is $L = 4$, and the PU transmit SNR is $\gamma_p = 13$ dB.

APPENDIX

A. Proof of Proposition 1

For the case without feedforward, using (4), we can expand $\tilde{P}_{\text{out}}$ and $P_{\text{out}}$ as

$$
\tilde{P}_{\text{out}} = \Pr(|f_i^T h_s|^2 < \theta_s \sigma^2)
= \left[ \Pr(|f_i^T h_s|^2 < \theta_s \sigma^2 \mid \hat{\mu}_1 \geq 0, \omega \geq 0) \Pr(\hat{\mu}_1 \geq 0 \mid \omega \geq 0) + \Pr(|f_i^T h_s|^2 < \theta_s \sigma^2 \mid \hat{\mu}_1 = 0, \omega \geq 0) \Pr(\hat{\mu}_1 = 0 \mid \omega \geq 0) \right] \Pr(\omega \geq 0) + \Pr(|f_i^T h_s|^2 < \theta_s \sigma^2 \mid \omega < 0) \Pr(\omega < 0) \tag{54}
$$

$$
P_{\text{out}} = \Pr(|f_i^T h_s|^2 < \theta_s \sigma^2)
= \Pr(|f_i^T h_s|^2 < \theta_s \sigma^2 \mid \omega \geq 0) \Pr(\omega \geq 0) + \Pr(|f_i^T h_s|^2 < \theta_s \sigma^2 \mid \omega < 0) \Pr(\omega < 0) \tag{55}
$$

where $\omega$ is defined in (13) and $\hat{\mu}_1$ is the IPC feedback parameter in (26). Using (24) and (26)

$$
\lim_{P_{\text{max}} \to \infty} \Pr(\hat{\mu}_1 = 0 \mid \omega \geq 0) = \lim_{P_{\text{max}} \to \infty} \Pr(\nu < 0 \mid \omega \geq 0) = 1. \tag{56}
$$

Moreover, from Lemma 1 and (26)

$$
\lim_{P_{\text{max}} \to \infty} \Pr(|f_i^T h_s|^2 < \theta_s \sigma^2 \mid \omega < 0) = \lim_{P_{\text{max}} \to \infty} \Pr \left( P_{\text{max}} < \frac{\theta_s \sigma^2}{\gamma_s} \right) = 0. \tag{57}
$$
By combining (54), (56), and (57)

\[
\lim_{P_{\text{max}} \to \infty} \tilde{P}_{\text{out}} = \lim_{P_{\text{max}} \to \infty} \Pr(|\vec{f}_n^\dagger \vec{h}_s|^2 < \theta_s \sigma^2 \mid \hat{\mu}_1 = 0, \omega \geq 0) \Pr(\omega \geq 0) \\
= \lim_{P_{\text{max}} \to \infty} \Pr(|\vec{f}_o^\dagger \vec{h}_s|^2 < \theta_s \sigma^2 \mid \omega \geq 0) \Pr(\omega \geq 0) \tag{58}
\]

where \((a)\) follows from the IPC design in (26) where \(f_o = f_n\) for \(\hat{\mu}_1 = 0\). Next, based on the OCB algorithm specified by (8) and (17)

\[
\lim_{P_{\text{max}} \to \infty} \Pr(|\vec{f}_o^\dagger \vec{h}_s|^2 < \theta_s \sigma^2 \mid \omega < 0) = \lim_{P_{\text{max}} \to \infty} \Pr\left(P_{\text{max}} < \theta_s \sigma^2 \left| g_s |b|\right|^2 \right) = 0. \tag{59}
\]

Combining (55), (58) and (59) gives the desired result for the case without feedforward. The proof for the case with feedforward is similar and omitted for brevity.

**B. Proof of Lemma 2**

Given perfect IPC feedback and the corresponding OCB design specified by (8) and (14)

\[
\Pr(P_s = P_{\text{max}}) = \tilde{P}_{\text{out}} + \Pr\left(\frac{\omega}{\lambda g_i \epsilon} \geq P_{\text{max}}\right) \tag{60}
\]

where \(\tilde{P}_{\text{out}}\) is given in (30). From the definitions of \(P_b\) in (60) and \(\omega\) in (13)

\[
P_b = \int_0^\infty \int_0^\infty \int_0^\infty f_{g_y}(\tau_1) f_{g_i}(\tau_2) f_\epsilon(\tau_3) d\tau_1 d\tau_2 d\tau_3 \\
= \int_0^\infty \int_0^\infty \int_0^\infty \frac{e^{-\frac{\theta_p}{\gamma_p}(1+\lambda \gamma_{\text{max}} \tau_2 \tau_3)}}{(1+\gamma_{\text{max}} \tau_2 \tau_3)} f_{g_y}(\tau_2) f_\epsilon(\tau_3) d\tau_2 d\tau_3 \\
= \frac{e^{-\frac{\theta_p}{\gamma_p}}}{\Gamma(L)} \int_0^\infty \int_0^\infty \tau_2^{L-1} e^{-\left(b \tau_3 + 1\right) \tau_2} d\tau_2 f_\epsilon(\tau_3) d\tau_3, \quad u := \frac{\theta_p \lambda \gamma_{\text{max}}}{\gamma_p} \\
= e^{-\frac{\theta_p}{\gamma_p}} \int_0^\infty \frac{f_\epsilon(\tau)}{(1+u \tau)^L} d\tau \\
= e^{-\frac{\theta_p}{\gamma_p}} (L-1) 2^B \int_0^\infty \frac{\tau^{L-2}}{(1+u \tau)^L} d\tau \\
= e^{-\frac{\theta_p}{\gamma_p}} (L-1) 2^B \int_0^\infty \tau^{L-2} \left[1 - Lu \tau + O(\tau^2)\right] d\tau \\
= e^{-\frac{\theta_p}{\gamma_p}} \left[1 - (L-1) u 2^{-\frac{\theta_p}{L-1}}\right] + O\left(2^{-\frac{2\theta_p}{L-1}}\right). \tag{61}
\]
By combining (30), (60), and (61), (31) follows. Next, from (8) and (14)

\[
\Pr(P_s < \tau) = \Pr\left(0 \leq \frac{\omega}{\lambda g_s \epsilon} \leq \tau\right) = \Pr(\omega \geq 0) - \Pr\left(\frac{\omega}{\lambda g_s \epsilon} \geq \tau\right).
\]

Using the above equation, (32) is obtained following similar steps as \(P_b\) in (61) and the fact that \(g_p\) follows the \(\exp(1)\) distribution.

C. Proof of Theorem 1

Given the receive SNR at \(R_s\), \(P_s g_s\), it follows that

\[
\begin{align*}
P_{\text{out}} &= \Pr(P_s g_s \leq \theta_s \sigma^2) \\
&= \int_0^{\theta_s \sigma^2} \Pr(P_s \leq P_{\text{max}}) f_{\hat{g}_s}(\tau) d\tau + \int_{\theta_s \sigma^2}^{\infty} \Pr\left(P_s \leq \frac{\theta_s \sigma^2}{\tau}\right) f_{\hat{g}_s}(\tau) d\tau \\
&\overset{(a)}{=} 1 - \frac{\Gamma(L - 1, \frac{\theta_s \sigma^2}{\tau})}{\Gamma(L - 1)} + \exp\left(-\frac{\gamma p}{\theta_p}\right) \left[(L - 1)\lambda \theta_p \theta_s\right]^{-\frac{1}{\gamma p}} \int_{\theta_s \sigma^2}^{\infty} \frac{\tau^{L-3}}{\Gamma(L-1)} e^{-\gamma \tau} d\tau + O\left(2^{-2\frac{L}{L-1}}\right)
\end{align*}
\]

where (a) uses both Lemmas 2 and 3. The desired result follows from the above equation.

D. Proof of Lemma 4

Define the random variable \(\kappa := |s_1^\dagger s_2|^2\) where \(s_1\) and \(s_2\) are unit-norm isotropic vectors in \(\mathbb{C}^{L-1}\). The distribution function of \(\kappa\) is given as [12]

\[
\Pr(\kappa > \tau) = (1 - \tau)^{L-2}, \quad 0 \leq \tau \leq 1.
\]

As shown in [17], [23], \(\delta\) follows the same distribution as \(\kappa \epsilon\). Using the above results, the distribution of \(\delta\) is readily obtained as follows.

\[
\begin{align*}
\Pr(\delta \leq t) &= \Pr(\kappa \epsilon \leq t) \\
&= \int_0^{2^{-\frac{\theta_s}{\gamma_p}}} \Pr\left(\kappa \leq \frac{t}{\tau}\right) f_\epsilon(\tau) d\tau \\
&= \int_0^{t} \Pr\left(\kappa \leq \frac{t}{\tau}\right) f_\epsilon(\tau) d\tau + \int_{t}^{2^{-\frac{\theta_s}{\gamma_p}}} \Pr\left(\kappa \leq \frac{t}{\tau}\right) f_\epsilon(\tau) d\tau \\
&= 1 - \int_{t}^{2^{-\frac{\theta_s}{\gamma_p}}} \Pr\left(\kappa > \frac{t}{\tau}\right) f_\epsilon(\tau) d\tau \\
&\overset{(a)}{=} 1 - 2B(L - 1) \int_{t}^{2^{-\frac{\theta_s}{\gamma_p}}} \left(1 - \frac{t}{\tau}\right)^{L-2} \tau^{L-2} d\tau
\end{align*}
\]

(63)
where \((a)\) is by substituting (62). Differentiating both sides of the above equation gives the desired result.

### E. Proof of Lemma 5

Following (60) in the proof for Theorem 1, we can write that \(\Pr(\hat{P}_s = P_{\text{max}}) = \hat{P}_{\text{out}} + \hat{P}_b\) where \(\hat{P}_{\text{out}}\) is given in (30) and \(\hat{P}_b\) is modified from \(P_b\) in (61) by replacing \(\epsilon\) with \(\delta\). Thus, by modifying (63) accordingly, we have

\[
\hat{P}_b = e^{-\frac{\bar{s}}{\bar{r}}} \int_0^{2^{-\frac{\bar{d}}{\bar{r}}}} \frac{f_{\delta}(\tau)}{(1 + b\tau)^L} d\tau
\]

\[
= e^{-\frac{\bar{s}}{\bar{r}}} (L - 1)2^{\frac{\bar{d}}{L-1}} \int_0^{2^{-\frac{\bar{d}}{\bar{r}}}} (1 - 2^{-\frac{\bar{d}}{L-1}} \tau)^{L-2} [1 - Lb\tau + O(\tau^2)] d\tau
\]

\[
= e^{-\frac{\bar{s}}{\bar{r}}} \left\{ 1 - (L - 1)2^{\frac{\bar{d}}{L-1}} \int_0^{2^{-\frac{\bar{d}}{\bar{r}}}} (1 - 2^{-\frac{\bar{d}}{L-1}} \tau)^{L-2} [Lb\tau + O(\tau^2)] d\tau \right\}
\]

\[
= e^{-\frac{\bar{s}}{\bar{r}}} \left[ 1 - L(L - 1)2^{-\frac{\bar{d}}{L-1}} b \int_0^1 (1 - \tau)^{L-2} \tau d\tau + O \left( 2^{-\frac{\bar{d}}{L-1}} \right) \right]
\]

\[
= e^{-\frac{\bar{s}}{\bar{r}}} \left[ 1 - L(L - 1)2^{-\frac{\bar{d}}{L-1}} b \int_0^1 (1 - \tau)^{L-2} \tau d\tau + O \left( 2^{-\frac{\bar{d}}{L-1}} \right) \right]
\]

\[
= e^{-\frac{\bar{s}}{\bar{r}}} \left[ 1 - L(L - 1)2^{-\frac{\bar{d}}{L-1}} R(2, L - 1) + O \left( 2^{-\frac{\bar{d}}{L-1}} \right) \right] \tag{64}
\]

where \(R(\cdot, \cdot)\) represents the Beta function and defined as \(R(x, y) = \int_0^1 \frac{t^{x-1}(1 - t)^{y-1}}{t^x} dt\) [28, 8.380]. By substituting \(R(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}\) [28, 8.384] into (64), we have

\[
\hat{P}_b = e^{-\frac{\bar{s}}{\bar{r}}} \left[ 1 - L(L - 1)2^{-\frac{\bar{d}}{L-1}} b \frac{\Gamma(2)\Gamma(L - 1)}{\Gamma(L + 1)} + O \left( 2^{-\frac{\bar{d}}{L-1}} \right) \right]
\]

\[
= e^{-\frac{\bar{s}}{\bar{r}}} \left[ 1 - 2^{-\frac{\bar{d}}{L-1}} b + O \left( 2^{-\frac{\bar{d}}{L-1}} \right) \right] \tag{65}
\]

where \((a)\) uses the formula \(\Gamma(L + 1) = L!\) [28, 8.339]. Substituting (65) and (30) into \(\Pr(\hat{P}_s = P_{\text{max}}) = \hat{P}_{\text{out}} + \hat{P}_b\) gives (40). The desired result in (41) can be obtained following similar steps as given above.

### F. Proof of Lemma 6

Based on the IPC feedback quantization algorithm in Section III-A.2 and for \(1 \leq n \leq n_0\)

\[
\Pr(p_{n-1} \leq \eta \leq p_n \mid \gamma p g p \geq \theta_p) = \frac{\Pr(p_{n-1} \leq P_s < p_n)}{\Pr(\gamma p g p \geq \theta_p)}
\]

\[
\overset{(a)}{=} \frac{(L - 1)\theta_p A 2^{-\frac{\bar{d}}{L-1}}}{\gamma p \sigma^2} (p_n - p_{n-1}) + O \left( 2^{-\frac{\bar{d}}{L-1}} \right) \tag{66}
\]
where \((a)\) uses Lemma 2. Combining (66), (16) and \(N = 2^A\) gives
\[
p_n - p_{n-1} = \frac{\gamma_p \sigma^2}{(L-1)\theta_p \lambda} 2^{-\frac{n}{L-1} - A} + O\left(2^{-\frac{n}{L-1}}\right), \quad 1 \leq n \leq n_0.
\] (67)
The desired result follows from (43) and (67).

G. Proof of Lemma 7

The equality in \((45)\) follows from the quantized IPC feedback algorithm in Section III-A.2. Based on this algorithm
\[
\Pr(\hat{P}_s < \tau) = \Pr\left(0 \leq \left\lfloor \frac{\omega}{\lambda g_l \epsilon} \right\rfloor P \leq \tau\right)
\leq (a) \Pr\left(0 \leq \frac{\omega}{\lambda g_l \epsilon} \leq \tau + \Delta P\right)
\leq \Pr(\omega \geq 0) - \Pr\left(\frac{\omega}{\lambda g_l \epsilon} \geq (\tau + \Delta P)\right)
\] (68)
where \((a)\) follows from (43). The desired result in \((46)\) is obtained using (68) and following the similar steps as for \(\Pr(P_s < \tau)\) in Lemma 2.

REFERENCES


