Adaptive sequential experimentation technique for 33 factorial designs based on revised simplex search

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Abstract: This paper is an extension to the work carried out in the field of sequential experimentation strategy by Siddiqui and Yang (2009). The methodology presented in this paper deals with expensive industrial experiments under the constraint of limited testing budget. This research focuses on involving three factors, each being at three levels. Another constraint appropriately assumed for these experiments is that of inadequate prior knowledge of the system, i.e., the behaviour of the system is not very well known to the experimenter. The aim of this research is to explore high quality parameter space in a minimum number of experimental runs in such situations. The explained experimentation strategy uses adaptive one factor at a time method, simplex downhill method, and response surface method.

Keywords: sequential experiments; adaptive experiments; simplex downhill method; design of experiments; one factor at a time; OFAT; 33 designs.


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1 Motivation

Factorially designed experiments are widely used in properly designing, conducting and analysing the experiments. In modern day practices, the analysis of such designs requires a complete experimental matrix. In the case of expensive industrial experiments, however, this matrix may remain incomplete as a result of cost constraint. We had listed such scenarios in one of our previous papers (Siddiqui and Yang, 2009), and are reproduced below:

- Combustion tests for aircraft engine or turbines: the prototypes of these combustion chambers are very expensive and the behaviour of the combustion chambers under different designs is very unpredictable.
- Computer experiment for complex engineering systems: each computer simulation run takes long time to finish and the results are difficult to predict.

Another problem with such complex systems is that the feasible region cannot be easily determined. It becomes very difficult to pin-point the good levels and hence define the operating window. It was, therefore, required to develop a strategy that would allow the experimenters to explore more desirable space in a minimum number of trials. We had developed such a strategy, the details of which can be found in our previous paper (Siddiqui and Yang, 2009). That strategy serves the following purposes:

1 Effective exploration of design space: the strategy allows effective exploration of the design space by utilising simplex downhill method (Spendley et. al. 1962) and response surface models (RSMs) (Montgomery, 2005). It outlines a certain course that allows the experimenter to explore the high-response points first if the response has higher-the-better characteristic and low-response points first if the response has smaller-the-better characteristic, considering the fact that there is inadequate prior knowledge about the behaviour of the system.

2 Minimum number of steps to reach the optimum point: because of the limited testing budget, and inadequate prior knowledge of the system, the experimenter would like to reach the optimum region in as few steps as possible. The use of Simplex Downhill Method enables the experimenter to reach the optimum quickly.

The motivation of this paper is to adapt the proposed strategy outlined in our previous paper (Siddiqui and Yang, 2009) so that it can be used for three factors. We found out that that for a 3\(^3\) design, the reflection of the worst point through the centroid of the rest
Adaptive sequential experimentation technique for 33 factorial designs

Of the simplex points do not always result in one of the design points. The adaptive sequential strategy will be able to serve the following scenarios:

1. Since there is little prior knowledge about the behaviour of the system, the initial settings of the levels of the factors are not guaranteed feasible. However, the best decision about initial settings of the factor levels will be made and the experiment conducted. The information from each experimental run will be used in the subsequent experiments.

2. The levels of each experimental factor can be changed in increments. In this paper, we have employed a $3^3$ design, thus there will be three factors each bearing three levels.

3. The total numbers of experimental runs are limited by the testing budget.

4. The goal of our adaptive sequential experiment is to locate the neighbourhood of the best design space, in terms of producing the best responses, by using the minimum number of experimental runs.

5. After locating the neighbourhood of the best design space, if we still can do more experimental runs within the budget, we will populate this neighbourhood with more experimental runs to further lean and model the best design space.

2 Introduction to the related terminologies and literature review

2.1 One factor at a time technique (OFAT)

Friedman and Savage (1947) outlined a particular design, which consisted of following steps:

Step 1: Select some particular combination of independent variables as a starting point.
Step 2: Arrange the independent variables in some order.
Step 3: Make tests at original starting point and at a series of points differing only in the value assigned the first independent variable, the other variables being held constant. Estimate the value of the first independent variable that maximises response.
Step 4: Holding the first independent variable fixed at the maximising value found in step 3 and the other independent variables, except the second, at the values originally assigned to them, repeat step 3 with the second independent variable.
Step 5: After all the variables have been varied (call this a ‘round’ of experiments), the whole process can be repeated, except that the original starting point is replaced by the combination of values of the independent variables reached at the end of the first round.

OFAT experimentation is preferred by some users because it is easy to conduct and analyse the experiments compared to the designing and analysis techniques involved in factorial experimentation. Many authors including Antony (2003) and Montgomery (2005) have renowned contributions towards design and analysis of experiments. They
M. Siddiqui and K. Yang have used DOE to properly design and analyse the experiments using well-established statistical tools.

There have been some efforts in combining the factorial experiments and the simplex downhill method but the use has been restricted to finding out the significant factors using factorial experiments and then applying simplex downhill method to reach the optimum. Passamontes and Callao (2006) used fractional factorial design to perform screening test on factors and then simplex algorithm for optimisation for a chemical process. Simplex algorithm was started using the point extracted from screening test study. A similar kind of technique was used by Martinotti et al. (1994), Sultan et al. (1998), Choisnard et al. (2002), Dahlen and Eckardstein (2005), Camacho and Munson-McGee (2005), Cantu et al. (2006) and Siro et al. (2006). Andersson et al. (2006) used fractional factorial designs and the simplex method to find the optimum for multivariate methods. Segade and Tyson (2006) used Plackett-Burman designs for finding out the influential parameters and then simplex procedure to find the optimum.

The problem associated with expensive industrial experiments is that the total number of experimental runs is limited because of limited testing budget. Some existing sequential experimentation techniques including the method of steepest ascent (Box and Wilson, 1951) and Evolutionary Operation (EVOP) (Box, 1957 and Box and Draper, 1969) require a lot of experimental runs. For example, EVOP is a sequential testing technique in which experiments are conducted at factorial points along with the centre points and then the next location of testing points is decided based upon RSMs. Conducting experiments at factorial points and centre points result in an excessive number of total experiments and the total experimental cost may be very high in case of expensive experimental runs.

Spendley et al. (1962) demonstrated a search method based upon a simplex for empirical optimisation in which a sequence of experimental designs in the form of a regular or irregular simplex was used. A simplex is an \( n+1 \) dimensional figure in \( n \) dimensions. Nelder and Mead (1965) modified the simplex approach towards optimisation by allowing the simplex to expand in favourable directions and contract otherwise. The methodology presented in this paper uses reflection phenomenon, therefore, we will give a brief explanation about reflection procedure.

Let \( y_i \) be the response value at a point \( P_i \), where \( P_i \) are the vertices of the simplex and \( i = 0, 1, \ldots, n \). Let the maximum and minimum response values be represented by \( y_{\text{max}} \) and \( y_{\text{min}} \) respectively and the corresponding points be \( P_{\text{max}} \) and \( P_{\text{min}} \) accordingly. Thus

\[
\begin{align*}
y_{\text{max}} &= \max (y_i) \\
y_{\text{min}} &= \min (y_i)
\end{align*}
\]

Here the response with smaller-the-better characteristic will be discussed.

### 2.2 Reflection phenomenon

Replace the point \( P_{\text{min}} \), also called worst point, by the reflected point, \( P_r \), where \( P_r \) is given by:

\[
P_r = (1+\alpha)\overline{P} - \alpha P_{\text{min}}
\]

where \( \overline{P} \) is centroid of \( \{P_0, P_1, \ldots, P_n\} \) and \( \alpha \): reflection coefficient.
$P_r$ lies on the line joining $P_h$ and $\overline{P}$ with $[P_r, \overline{P}] = \alpha[P_h, \overline{P}]$ (Figure 1). If the response at $P_r$, represented as $y_r$, is between $y_h$ and $y_i$, then $P_h$ is replaced by $P_r$ and we start with a new simplex.

**Figure 1** Reflection (see online version for colours)

2.3 Adaptive one-factor-at-a-time (AOFAT) technique

Frey et al. (2003) introduced AOFAT experimentation method. The procedure is described as follows, and shown in Figure 2.

**Figure 2** AOFAT method

Suppose, there are three factors $A$, $B$ and $C$ with two levels each. Thus it is a $2^3$ design. We can start the experiment from any point, called a baseline point. Suppose we start from point ‘bc’, the coordinates of which in coded units are $A = -1$, $B = +1$, $C = +1$. Refer to Figure 2. Then a factor, say $A$, is varied resulting in the point ‘abc’. If this change yields an improvement in the response, change in the level of factor $A$ will be retained, otherwise that change will be discarded. Suppose that the value of response at point ‘abc’ is better as compared to the one at ‘bc’. Thus the change will be retained.
After that, another factor, say, factor $B$, is varied. Again, if the change brings an improvement, the change will be kept; otherwise, we will retain the original level of B. Suppose that the response at the new point 'ac' is worse as compared to the one at 'abc'. Then the level of the third factor $C$ will not be changed from the current point 'ac', rather it will be changed from the previous point 'abc'. The process will continue until all the factors are tried out. Frey et al. (2003) and Frey and Jugulum (2006) conducted simulations to prove that AOFAT technique produces better results than orthogonal arrays under certain conditions and investigated the mechanisms by which this technique led to improvement.

2.4 Response surface method

Response surface methods are used to examine the relationship between one or more response variables and a set of predictors or experimental variables or factors. If we represent the observations of the responses obtained at the design matrix points as vector $Y$, the expected response by $E(Y)$, and corresponding vector of expected values by $\eta$, then the response surface will be: \[ \eta = E(Y) \]
also,
\[ \eta = \frac{L}{i=1} \beta_i X_i \]
thus
\[ \eta = X\beta \]
where $\beta$ is the vector of unknown constants.

First order RSM
\[ y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k + \varepsilon \]

Second order RSM
\[ y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_i x_i^2 + \sum_{i=1}^{k} \sum_{j=1}^{k} \beta_{ij} x_i x_j + \varepsilon \]

3 Revised simplex search method

We strongly recommend our previous paper (Siddiqui and Yang, 2009) for better explanation of this method along with detailed examples. However, for the sake of convenience, we have outlined only the main steps of this procedure below:

Step 0: Assuming that we have selected $n$ experimental factors, an initial experimental point is chosen to start our experiment.
Step 1: Apply AOFAT approach to determine the first \( n + 2 \) experimental runs. After the first \( n + 2 \) runs are completed, we will have \( n + 2 \) experimental response values corresponding to the first \( n + 2 \) experimental runs.

Step 2: After the first \( n + 2 \) completed experimental trials, construct as many simplexes as possible, fitting a RSM based on all completed experimental runs. For each simplex, apply simplex reflection algorithm to find its simplex reflection point, if the reflection point is out of experimental boundary, discard the point; if the reflection point is inside the experimental boundary, keep the point.

Step 3: For the new reflection point generated from the previous step, use the RSM to predict the response for the new reflection point. We will conduct a new actual experimental run at the reflection point if the RSM model gives a good predicted response value. Every execution of step 3 will create a new response value. Go to step 4.

Step 4: Construct as many new simplexes as possible based on available experimental runs and data, also, fitting a RSM based on all completed experimental runs. For each simplex, apply simplex reflection algorithm to find its simplex reflection point, if the reflection point is out of experimental boundary, discard the point; if the reflection point is inside the experimental boundary, keep the point. If all the feasible experimental points are tried, or the allocated experimental budget is used up, stop, otherwise, go to step 3.

Now we will explain the working of the procedure for a \( 3^3 \) factorial design.

### 3.1 Three factors at three levels (\( 3^3 \) design)

A \( 3^3 \) design has three factors, each at three levels. Figure 3 shows a \( 3^3 \) design matrix. Levels are numbered 1, 2, and 3, corresponding to low, medium and high levels respectively. The design points are numbered 1 through 27, whereas the level combinations are shown in parenthesis. For example, at point 20, A is at medium level, B at low level, and C at high level.

A simplex, in this case, will consist of four points. The worst point will be reflected through the centroid of the remaining three points. In some cases, reflection of the worst point through the centroid may lead to a point that is not in the factorial design matrix. In order to use the methodology of reflecting the worst point through the centroid, we need to move the centroid (\( \overrightarrow{CP} \)) to a point, reflecting through which will yield a design matrix point. The new centroid (\( \overrightarrow{CP} \)) will be calculated by finding the distances between the old centroid and the vertices, and the old centroid and the midpoints of all the sides of the triangle, and picking up the point that is located closest to the old centroid. This will make the reflection of the worst point yield a factorial design point.
We can understand it through an example. Consider a simplex consisting of points A, B, C and D, as shown in Figure 4, the coordinates of which are:

\[ A(1,1,1); B(1,2,1); C(2,2,1); D(1,2,2) \]

**Figure 4** (see online version for colours)
Suppose that \( A \) is the worst point. Hence \( A \) will be reflected through the centroid of \( B, C \) and \( D \). The coordinates of centroid \( \bar{P} \) will be

\[
\left( \frac{1+2+1}{3}, \frac{2+2+2}{3}, \frac{1+1+2}{3} \right) = \left( \frac{16}{3}, \frac{12}{3}, \frac{6}{3} \right) = \left( \frac{4}{3}, 4, 2 \right) = (1.33, 2, 1.33)
\]

Reflection of point \( A \) will take place as per the following equation:

\[
2 \bar{P} - W = \left( \frac{8}{3}, \frac{8}{3}, \frac{8}{3} \right) - (1, 1, 1) = \left( \frac{5}{3}, \frac{5}{3}, \frac{5}{3} \right)
\]

It is obvious that this point is not one of the factorial design points. Refer to Figure 5 below.

**Figure 5** (see online version for colours)

Now, find the distance between:

1. the centroid (\( \bar{P} \)) and every vertex of the triangle
2. the centroid (\( \bar{P} \)) and the midpoints of the sides.

These distances are shown in Figure 6 and calculated below:
1  Distance between the old centroid ($P$) and the vertices

- Distance between ($P$) and point B. $\overline{P,121} = 0.471$
- Distance between ($P$) and point C. $\overline{P,221} = 0.746$
- Distance between ($P$) and point D. $\overline{P,121} = 0.746$

2  Distance between the old centroid ($P$) and the midpoints of the sides

- Distance between ($P$) and point midpoint of BC. $\overline{P,M_{121,221}} = 0.373$
- Distance between ($P$) and point midpoint of CD. $\overline{P,M_{221,122}} = 0.236$
- Distance between ($P$) and point midpoint of BD. $\overline{P,M_{121,122}} = 0.373$

Now pick the least of all these distances, 0.236, which is the distance between the old centroid ($P$) and the midpoint of the side CD. Hence the shifted centroid ($P$) will be the midpoint of the side CD, the coordinates of which are $(3/2,2,3/2)$. The reflection of the worst point $(1,1,1)$ will take place as follows:

$$W = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$$

Thus $\overline{2W} = (3,4,3) - (1,1,1) = (2,3,2)$

$\Rightarrow A : \text{Medium level}; B : \text{High level}; C : \text{Medium level}$

Figure 7 shows that the reflection of the worst point through the shifted centroid ($P$) will result in a design matrix point.
The simplex may not always be of the shape as we just considered in Figure 7 above. It may also be as the one consisting of points ABDF, the coordinates of which are \( A(1,1,1); \ B(1,2,1); \ D(1,2,2); \ F(2,1,1) \), as shown below in Figure 8. If point A needs to be reflected through the centroid of BDF, the reflected point will not be a design point as is shown here.

\[
\begin{align*}
\bar{P} &= \left( \frac{1+1+2}{3}, \frac{2+2+1}{3}, \frac{1+2+1}{3} \right) = \left( \frac{4}{3}, \frac{5}{3}, \frac{4}{3} \right) = (1.33, 1.67, 1.33) \\
2\bar{P} - W &= \left( \frac{8}{3}, \frac{10}{3}, \frac{8}{3} \right) - (1,1,1) = \left( \frac{5}{3}, \frac{7}{3}, \frac{5}{3} \right),
\end{align*}
\]

which is not a design matrix point.

In Figure 8 shown below, the red point is the centroid of BDF, the coordinates of which are \( (4/3, 5/3, 4/3) \). It is clear that the reflected point is not a design point. However, if we move the centroid using the technique described above, it will be shifted to the midpoint of the side DF, and then the reflected point will be a design point. Refer to Figure 9.
which is a design point as shown in Figure 3.21 as point 'G'.

![Figure 9](see online version for colours)

3.2 Example

Table 1 shows the values of roughness that were determined against different level combinations of three factors viz pulse duration time (μs), current (amp) and volume of SiC (%) (Karthikeyan et al., 1997). In the original paper, there were two replicates but they have been averaged here for use in the analysis.

<table>
<thead>
<tr>
<th>Pulse duration time</th>
<th>Current</th>
<th>Volume of SiC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>2,2</td>
<td>9,7</td>
</tr>
<tr>
<td>2</td>
<td>4,4</td>
<td>12,10</td>
</tr>
<tr>
<td>3</td>
<td>5,6</td>
<td>13,13</td>
</tr>
<tr>
<td>2</td>
<td>2,3</td>
<td>9,9</td>
</tr>
<tr>
<td>2</td>
<td>4,5</td>
<td>11,12</td>
</tr>
<tr>
<td>3</td>
<td>6,6</td>
<td>14,13</td>
</tr>
<tr>
<td>3</td>
<td>2,4</td>
<td>11,9</td>
</tr>
<tr>
<td>2</td>
<td>5,5</td>
<td>12,12</td>
</tr>
<tr>
<td>3</td>
<td>8,6</td>
<td>14,14</td>
</tr>
</tbody>
</table>
Figure 10 shows all the design points, numbered 1 through 27. Level combinations of three factors are also shown in parenthesis. Values of response are indicated at each of the 27 design points. For example, at design point 8, levels of time, current and SiC are 2, 3 and 1, respectively, and the value of response at this point is 6.

**Figure 10** (see online version for colours)

Initial points are 222, 322, 232, 223, and 212. Response surface equation using the initial points will be:

\[ Y = -6 + 0.5T + 3.25C + 5.5S - 0.25C^2 \]  \hspace{1cm} (5)
Consider the initial simplex consisting of points 223, 222, 212, and 322. Reflecting 223 through the centroid of 222, 212, and 322 will not yield a design matrix point as shown below:

\[
\overrightarrow{P} = \left( \frac{2+2+3}{3}, \frac{2+1+2}{3}, \frac{2+2+2}{3} \right) = \left( \frac{7}{3}, \frac{5}{3}, \frac{6}{3} \right) = \left( \frac{7}{3}, \frac{5}{3}, \frac{2}{3} \right)
\]

\[
2\overrightarrow{P} - \mathbf{w} = \left( \frac{14}{3}, \frac{10}{3}, 4 \right) - \left( 2, 2, 3 \right) = \left( \frac{8}{3}, \frac{4}{3}, 1 \right)
\]

Now, as indicated earlier, the distances between the centroid (\( \overrightarrow{P} \)) and every vertex of the triangle, and the centroid (\( \overrightarrow{P} \)) and the midpoints of the sides will be calculated. The distances are shown as follows:

\[
\overrightarrow{P},222 = 0.471
\]

\[
\overrightarrow{P},212 = 0.746
\]

\[
\overrightarrow{P},322 = 0.746
\]

\[
\overrightarrow{P},M_{222,212} = 0.373
\]

\[
\overrightarrow{P},M_{212,322} = 0.236
\]

\[
\overrightarrow{P},M_{222,322} = 0.373
\]

The minimum of these is 0.236. Hence the centroid will be shifted to the midpoint of vertices 212 and 322. The coordinates of the shifted centroid will be \((5/2,3/2,2)\). Reflecting the worst point 223 through the shifted centroid will yield a design point as shown below:

\[
2\overrightarrow{P} - \mathbf{w} = 2 \left( \frac{5}{2}, \frac{3}{2}, 2 \right) - \left( 2, 2, 3 \right) = \left( 3, 1, 1 \right)
\]

Using equation (5), \(y_{311}\) is 4 whereas the observed value of response at this point is 3.

The following Table 2 shows the points in the simplex, new points that were obtained as a result of the reflection of the worst point, its predicted value using response surface equation, and the updated response surface equation. For example, as shown in row 2, in the simplex consisting of points 322, 222, 212 and 311, the worst point, 322, will be reflected through the centroid of 222, 212 and 311. The reflection will yield point 211, estimate of which will be obtained from the most recent response surface equation which is in the first row, and after performing the experiment at this point, the response surface equation will be updated as shown in row 2. If the reflection leads us outside of the design points, as can be seen in rows 3 and 8, then a new simplex should be chosen.
### Table 2

**Simplex points**

<table>
<thead>
<tr>
<th>Worst point</th>
<th>Remaining points</th>
<th>New point</th>
<th>Actual value</th>
<th>RSM pred.</th>
<th>Updated RSM equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>223</td>
<td>222 212</td>
<td>311</td>
<td>3</td>
<td>4</td>
<td>( y = -6 + 0.5 T + 3.25 C + 5.5 S - 0.25 C^2 )</td>
</tr>
<tr>
<td>322</td>
<td>222 212</td>
<td>311</td>
<td>2.5</td>
<td>3.5</td>
<td>( y = -13 + 2.5 T + 5.25 C + 8 S - 0.25 C^2 - 0.5 S^2 - 1.0 TC )</td>
</tr>
<tr>
<td>222</td>
<td>212 311</td>
<td>outside</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>322</td>
<td>222 212</td>
<td>211</td>
<td>2</td>
<td>1</td>
<td>( y = -10 + 5.25 C + 8 S - 0.5 T - 0.25 C^2 - 0.5 S^2 - 1.0 TC )</td>
</tr>
<tr>
<td>223</td>
<td>232 222</td>
<td>331</td>
<td>7</td>
<td>6.5</td>
<td>( y = -10 + 4.75 C + 8.5 S + 0.5 T - 0.25 C^2 - 0.5 S^2 - 0.75 TC - 0.25 TS )</td>
</tr>
<tr>
<td>232</td>
<td>222 322</td>
<td>331</td>
<td>5</td>
<td>5.75</td>
<td>( y = -8.25 - 0.1875 T + 3.25 C + 7.6875 S + 0.4375 T^2 + 0.125 C^2 - 0.4375 S^2 - 0.75 TC + 0.125 TS )</td>
</tr>
<tr>
<td>322</td>
<td>222 331</td>
<td>321</td>
<td>6</td>
<td>7</td>
<td>( y = -7.75 - 0.1875 T + 1.25 C + 8.9375 S + 0.4375 T^2 + 0.125 C^2 - 0.6875 S^2 - 0.25 TC - 0.375 TS + 0.5 CS )</td>
</tr>
<tr>
<td>222</td>
<td>231 331</td>
<td>outside</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>322</td>
<td>222 212</td>
<td>231</td>
<td>4</td>
<td>3.875</td>
<td>( y = -7.8095 - 0.3995 T + 1.4405 C + 9 S - 0.4762 T^2 + 0.1071 C^2 - 0.6905 S^2 - 0.2857 TC - 0.381 TS + 0.4762 CS )</td>
</tr>
<tr>
<td>222</td>
<td>212 231</td>
<td>221</td>
<td>4.5</td>
<td>4.14</td>
<td>( y = -8.167 + 0.167 T + 1.75 C + 8.667 S + 0.333 T^2 - 0.667 S^2 - 0.25 TC - 0.333 TS + 0.5 CS )</td>
</tr>
</tbody>
</table>
Till now, thirteen points have been explored and there are no more simplexes that can be used to get any other points. The so-far-explored points are shown by green circles in Figure 11.

The next step will be to explore the points which lie in the close vicinity of green points. The following Table 3 shows the estimates of those points, and they are circled orange in Figure 12. These estimates are obtained using response surface equation in the last row of Table 2.
Table 3

<table>
<thead>
<tr>
<th>Design matrix points</th>
<th>Actual value</th>
<th>RSM prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>131</td>
<td>5.5</td>
<td>6</td>
</tr>
<tr>
<td>112</td>
<td>8</td>
<td>8.83</td>
</tr>
<tr>
<td>312</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>122</td>
<td>11</td>
<td>11.33</td>
</tr>
<tr>
<td>132</td>
<td>13</td>
<td>13.83</td>
</tr>
<tr>
<td>332</td>
<td>14</td>
<td>14</td>
</tr>
</tbody>
</table>

Figure 12 (see online version for colours)
After obtaining the response estimates as listed in Table 3, the experiment will be performed at points in ascending order of their responses (as it is a smaller-the-better case). The order is given below:

131, 112, 312, 122, 132, 332

Using this approach, we have explored 19 points which is more than two-thirds of the total population. As you can see in Figure 12, the remaining points (remaining black circles) are the ones with higher or less desirable values of response.

### 3.3 Comparison with AOFAT method

If you start making all the possible paths, you’ll find 49 different ones. These paths were obtained after gathering $n + 2$ initial points. Please refer to our previous paper for details. Some of them will lead to the optimum point, whereas the others won’t. There were total 512 steps along those 49 paths. Hence AOFAT took, on the average, 10.45 steps to reach the optimum point. The proposed method took 7 steps to reach the optimum.

### 4 Conclusions and future research

A methodology to deal with expensive industrial experiments was presented in this paper. The paper emphasised on $3^3$ factorial designs. We considered the scenario where each experimental run was very costly, and the testing budget was limited. We also assumed the testing region to be unknown to the experimenter. Thus experimenter did not have a good idea about the behaviour of the system.

The presented methodology provided the experimenters with a procedure that can be used to reach to the high quality parameter space in a fewer number of steps. It is composed of AOFAT method, simplex downhill method, and response surface method. AOFAT method is used to initiate the search process. After that, the proposed strategy uses adaptive sequential simplex method along with RSM to further accomplish the task. AOFAT is good in acquiring initial points. But since it depends only upon the two current observations, and does not make use of all the previous observations, it is wise to shift to another technique such as the one presented to carry out the rest of the experiments. It takes advantage of all the previously conducted experiments and helps in exploring the more feasible design space efficiently.

Other exploration techniques should be used in conjunction with RSM to make the methodology robust to the initial points. Methods for dealing with highly irregular system behaviour should be developed so that highly non-linear responses can be modelled.

### References


Adaptive sequential experimentation technique for 33 factorial designs


