Period Coded Phase Shifting Strategy for Real-time 3-D Structured Light Illumination

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Abstract—Phase shifting structured light illumination for range sensing involves projecting a set of grating patterns where accuracy is determined, in part, by the number of stripes. However, high pattern frequencies introduce ambiguities during phase unwrapping. This paper proposes a process for embedding a period cue into the projected pattern set without reducing the signal-to-noise ratio. As a result, each period of the high frequency signal can be identified. The proposed method can unwrap high frequency phase and achieve high measurement precision without increasing the pattern number. Therefore, the proposed method can significantly benefit real-time applications. The method is verified by theoretical and experimental analysis using prototype system built to achieve 120 fps at \(640 \times 480\) resolution.

Index Terms—Phase shifting methods, phase unwrapping, real-time 3-D

I. INTRODUCTION

S TRUCTURED Light Illumination (SLI) is an active range sensing method that performs triangulation between a projector and camera pair through projecting and capturing a group of light patterns [1], [2]. For SLI phase shifting methods [3], 3-D reconstructions are derived from the observed phase distortions caused by the object geometry. Because phase shifting methods involve projecting and processing a series of time-multiplexed patterns, it is not commonly associated with scanning and reconstructing scenes in real-time, but recent advances in imaging devices and computing units now allow high speed 3-D surface reconstructions in real-time [4]–[9]. Among these, high frequency Phase Measuring Profilometry (PMP) [10] has been widely used for its high precision and simplicity.

The measurement accuracy of phase shifting methods can be improved by increasing the number of grating stripes, but in order to achieve a complete correspondence between projected patterns and captured images, high frequency phase data has to be unwrapped into a single period [8]. However, the depth ambiguities that this process entails represents a major obstacle [11], as illustrated in Fig. 1 where Fig. 1 (top) shows two isolated surfaces where no method of spatial unwrapping can uniquely determine the period numbers in the captured image. Figure 1 (bottom) shows a depth discontinuity, parallel to the stripes, resulting in a continuity artifact in phase such that two different period numbers appear as the same one in the captured image.

Various methods have been investigated to enhance the reliability of phase unwrapping through the inclusion of extraneous spatial information [11]–[14]. But according to Saldner and Huntley [15], these approaches alone cannot solve the difficulties presented in Fig. 1. On the other hand, temporal unwrapping approaches [15], [16] have been proposed to achieve reliable unwrapping, but the use of extra patterns to remove the depth ambiguities is, again, unappealing for real-time acquisition.

Within the current SLI technology, phase information is the only signal coded into the patterns. In a projector, pixels with the same phase but different period numbers are assigned with the same intensity values. In order to unwrap high frequency data, either algorithms are used to “count” the period number from one direction to the other through a spatial correlation approach, or extra patterns are projected to obtain a reference unit frequency signal (or period cue). Over 20 years, this problem remains a challenge for SLI systems [17]. Current
high speed SLI applications prefer spatial unwrapping methods because fewer patterns are used, which, however, results in high failure rates (> 86%) in some extreme cases [18].

In an effort to develop robust high-frequency pattern schemes with low computational complexity, this paper presents a novel method of Period Coded Phase Shifting (PCPS) that does not use extra patterns. Instead, we propose using spatial intensity efficiency to measure the utilization efficiency of intensity values for all kinds of phase shifting pattern sets (Sec. II). For a pattern set with an efficiency less than 100%, a reference unit frequency signal (or period cue) can be embedded such that projection pixels having the same phase but at different periods are no longer assigned with the same intensities (Sec. III). We also propose pattern entropy as a measure of the distribution of those pixels (Sec. II). As a result, the optimal amplitude of the period cue can be achieved when the pattern entropy is maximized.

Based on the proposed spatial efficiency and pattern entropy measures, we develop a hybrid unwrapping method to take advantage of both spatial and temporal unwrapping approaches (Sec. IV). A triangular phase waveform is proposed for phase shifting methods that provides doubled frequency, compared with the traditional sawtooth phase waveform. The proposed approach works for most phase shifting schemes. The advantages of the proposed approach have been verified both mathematically (Sec. V) and experimentally using a real-time prototype system achieving 120 fps with the resolution of 640 × 480 (Sec. VI).

II. PHASE SHITING STRUCTURED LIGHT ILLUMINATION

Generally, the patterns \( \{ I_n^b : n = 0, 1, ..., N - 1 \} \) are designed as

\[
I_n^b(\vec{x}) = A + B s(\vec{x}), \tag{1}
\]

where \( A \) is a temporal DC value, \( B \) is the amplitude (or projector modulation) value of a periodical signal function \( s(\vec{x}) \). The coordinate \( \vec{x} = (x^j, y^j) \) is the corresponding coordinate in the projector. The captured images, \( \{ I_n^c : n = 0, 1, ..., N - 1 \} \), are then denoted as

\[
I_n^c(\vec{x}) = \alpha(\vec{x}) [I_n^b(\vec{x}) + \beta(\vec{x})], \tag{2}
\]

where the superscript \( c \) indicates that \( I_n^c \) is now in the camera space and \( \vec{x}^c = (x^c, y^c) \) is the camera coordinate [19]. In Eq. (2), \( \alpha(\vec{x}) \) represents the albedo within [0, 1] where 0 is pure black and 1 is pure white. The term, \( \alpha(\vec{x}) \beta(\vec{x}) \), represents the albedo image from ambient illumination \( \beta(\vec{x}) \).

Through a decoding function and a phase unwrapping algorithm, the phase \( \Phi \) that represents the coordinate of \( x^j \) or \( y^j \), can be obtained from the “wrapped” (or coded) phase \( \phi^w \) which is expressed as

\[
\phi^w(\vec{x}) = g \left[ \frac{U(\vec{x})}{V(\vec{x})} \right], \tag{3}
\]

where

\[
U(\vec{x}) = \sum_{n=0}^{N-1} a_n I_n^c(\vec{x}) \quad \text{and} \quad V(\vec{x}) = \sum_{n=0}^{N-1} b_n I_n^c(\vec{x}).
\]

The terms \( a_n \) and \( b_n \) are two coefficients in summations such that the terms of \( \beta(\vec{x}) \) and the DC value \( A \) are canceled, while the division between the two summations cancels the terms of \( \alpha(\vec{x}) \). The term \( g(\cdot) \) is a function that estimates the phase values \( \phi^w(\vec{x}) \in [0, 2\pi) \) out of image intensity values.

The modulation \( M(\vec{x}) \) is given by

\[
M(\vec{x}) = \gamma \sqrt{U^2(\vec{x}) + V^2(\vec{x})},
\]

where \( \gamma \) is a coefficient related with the number of patterns. Noting that the value of \( M(\vec{x})/\alpha(\vec{x}) \) represents the strength of the high frequency signal at pixel \( \vec{x} \), we define the Signal to Noise Ratio (SNR) for phase shifting methods as

\[
SNR(\vec{x}) = \lambda M(\vec{x})/|\alpha(\vec{x})|\sigma,
\]

where \( \lambda \) is a function of the number of patterns, \( N \), and \( \sigma \) is the standard deviation of the system noise. Since the temporal DC value \( A \) is canceled by the summations in \( U \) and \( V \), the SNR is only related to the temporal AC values \( B \) in \( \{ I_n^b \} \). The specific decoding functions of \( \phi^w(\vec{x}) \) and \( M(\vec{x}) \) are designed for the particular pattern strategy such that, after unwrapping the \( \phi^w \) into the final phase \( \Phi \in [0, 2P\pi) \), where \( P \) is the number of periods, the correspondence problem can be solved. The 3-D coordinates are finally recovered from pre-calibrated triangulation [20], [21].

For the specific phase shifting method of PMP, the patterns \( \{ I_n^b : n = 0, 1, ..., N - 1 \} \) of Eq. (1) are given by [10]

\[
I_n^b(\vec{x}) = A + B \sin[\Phi(\vec{x}) - \frac{2n\pi}{N}]. \tag{4}
\]

The sinusoidal light signal covers the whole resolution range of a projector [0, 1], if both \( A \) and \( B \) are given the value of 0.5. After capturing the PMP images \( \{ I_n^c \} \) in a camera, the decoding function of Eq. (3) is given by

\[
\phi^w(\vec{x}) = \arctan \left[ \frac{\sum_{n=0}^{N-1} a_n I_n^c(\vec{x}) \cos \left( \frac{2\pi n}{N} \right)}{\sum_{n=0}^{N-1} b_n I_n^c(\vec{x}) \sin \left( \frac{2\pi n}{N} \right)} \right] = \arctan \left[ \frac{U(\vec{x})}{V(\vec{x})} \right], \tag{5}
\]

where

\[
U(\vec{x}) = \frac{N}{2} \alpha(\vec{x}) B \sin[\Phi(\vec{x})], \tag{6}
\]

and

\[
V(\vec{x}) = \frac{N}{2} \alpha(\vec{x}) B \cos[\Phi(\vec{x})]. \tag{7}
\]

The modulation \( M(\vec{x}) = \alpha(\vec{x}) B \) becomes

\[
M(\vec{x}) = \frac{2}{N} \sqrt{U^2(\vec{x}) + V^2(\vec{x})}. \tag{8}
\]

Thus, the specific SNR for PMP pattern strategy is [22]:

\[
SNR(\vec{x}) = \frac{\sqrt{N} M(\vec{x})}{2\alpha(\vec{x}) \sigma}, \tag{9}
\]

that is \( \lambda = \sqrt{N/2} \).

A. Non-ambiguous Phase Unwrapping

In order to unwrap the high frequency \( \phi^w \) to \( \Phi \in [0, 2P\pi) \) where \( P \) is the number of periods, a secondary unit frequency signal (period cue) can be added into the projected patterns. In this sub-section, we first propose a simple approach to add the second signal such that, in case of PMP, the projected patterns are given by

\[
I_n^b(\vec{x}) = A + B \sin[\Phi(\vec{x}) - \frac{2n\pi}{N}] + \frac{p(\vec{x})}{2^k - 1}, \tag{10}
\]
where \( \{ p(\vec{x}) : 0, 1, ..., P-1 \} \) is the period number at \( \vec{x} \) when a \( k \)-bits per pixel (bpp) projector is used. After capturing the images \( \{ I^b_n \} \) through Eq. (2) and ignoring the effects of system noise and ambient light, the \( \phi^w(\vec{x}) \) is obtained from Eq. (5), while the period information \( p(\vec{x}) \) can be obtained by

\[
p(\vec{x}) = \frac{1}{\pi} \sum_{n=0}^{N-1} \frac{I_n^b(\vec{x})}{M(\vec{x})} B - A. \tag{11}
\]

Thus, \( \Phi(\vec{x}) = \phi^w(\vec{x}) + 2\pi p(\vec{x}) \), i.e., the high frequency signal is unwrapped. Compared to the traditional PMP, this method temporally unwraps the high frequency phase without projecting extra patterns; however, in order to leave room within the projector’s resolution range for the unit frequency signal \( p \), the high frequency amplitude needs to be reduced from \( B = 0.5 \) to \( 0.5 - \frac{P}{2^{464}} \). Thus, the obtained modulation \( M \) and the SNR of the high frequency signal has to be reduced.

With the ability to uniquely decode the period number, it is expected that an increase in the period number of the original signal will compromise the effects of noise to a greater extent. But it would be ideal to embedding a low frequency period cue without reducing the SNR. To this end, we note that there are intelligent ways to design and add this second signal such that the combined signal still falls within the resolution range of a projector without reducing the high frequency signal’s amplitude. Unlike the pattern strategy in Eq. (10), these intelligent approaches are robust to system noise and ambient illumination.

For developing our discussion of intelligent period cue design, we first examine the case of \( N = 4 \) and \( P = 2 \) pattern PMP (Fig. 2) to illustrate how a second signal can be added without reducing the original signal’s amplitude. Here, Eq. (5) is reduced to

\[
\phi^w(\vec{x}) = \arctan \left[ \frac{I_0^b(\vec{x}) - I_2^b(\vec{x})}{I_1^b(\vec{x}) - I_3^b(\vec{x})} \right]. \tag{12}
\]

where high frequency signal \( \phi^w(\vec{x}) \) remains constant as long as the ratio between \( \{ I_0^b(\vec{x}) - I_2^b(\vec{x}) \} \) and \( \{ I_1^b(\vec{x}) - I_3^b(\vec{x}) \} \) remain the same even though the values of \( \{ I_0^b(\vec{x}) \}, \{ I_2^b(\vec{x}) \} \) may change. As such, we define the action of “shifting” as the process of changing the intensity values of \( I^b_n(\vec{x}) \) such that the ratios of the distances among \( \{ I^b_n(\vec{x}) \} \) are changed. We further define the action of “scaling” as the process of changing the intensity values of \( I^b_n(\vec{x}) \) such that the ratios of the distances among \( \{ I^b_n(\vec{x}) \} \) remain the same.

Then, we can classify the points on the pattern: (1) shiftable points, and (2) shiftable and scalable points. As shown in Fig. 2 point \( E_1, \phi^w_{E_1} = 0.5\pi \) such that \( \{ I^b_0 = 1, I^b_1 = 0.5, I^b_2 = 0, I^b_3 = 0.5 \} \). The projected intensities of \( I^b_1 \) and \( I^b_3 \) can be shifted within \([0, 1]\) under the constraint that \( I^b_1 = I^b_3 \), and the obtained \( \phi^w_{E_1} \) will not change. The point \( E_1 \) is a shiftable point; however, the intensities at \( E_1 \) cannot be scaled up since \( I^b_0 \) achieves the supremum bound of the projector, and \( I^b_2 \) achieves the infimum bound. At the same time as shown in Fig. 2 point \( F, \phi^w_F = 1.25\pi \) and \( \{ I^b_0 = 0.1464, I^b_1 = 0.8536, I^b_2 = 0.8536, I^b_3 = 0.1464 \} \) where, since none of the four projected intensities achieves 0 or 1, the intensities can be scaled up as long as the ratio between \( (I^b_0 - I^b_2) \) and \( (I^b_1 - I^b_3) \) stay the same. And, the intensities at \( F \) are also shiftable.

As stated above for the point \( E_1 \), under the constraint of \( I^b_1(\vec{x}) = I^b_3(\vec{x}) \), the shifting of \( I^b_1(\vec{x}) \) and \( I^b_3(\vec{x}) \) affects neither the \( B \) value nor the SNR of \( \phi^w(\vec{x}) \); however, intensity scaling will cause a change in \( B \) in Eq. (4). By increasing the difference between intensities, the \( B \) value will be increased such that the SNR is improved.

In other words, it is the operation of intensity shifting that provides the possibility of encoding a second signal. But, for different pattern strategies, the capabilities of shifting intensities are not the same. For a particular strategy, if the intensities cannot be shifted at all, there will be no way to embed the second signal without decreasing the SNR. In order to measure the capability of intensity shifting, the spatial intensity efficiency, \( \eta \), is proposed for the phase shifting strategies.

### B. Spatial Intensity Efficiency

For phase shifting patterns (in projector space), there are \( P \) points, one point for each period of the projected pattern, having the phase value \( \phi^w(\vec{x}) = \Phi(\vec{x}) - 2p(\vec{x}) \pi \) where \( \phi^w(\vec{x}) \) is the wrapped phase value within the range \([0, 2\pi]\). As shown in Fig. 2, the points \( E_1 \) and \( E_2 \) are two such periodical points having the same wrapped phase value \( \phi^w \). Correspondingly, there are \( NP \) intensity values, \( \{ I_n(\phi^w) : n = 0, 1, ..., NP-1 \} \), projected from these periodical points across the pattern set. For each intensity \( I_n(\phi^w) \), there are \( 2^k \) available intensities that \( I_n(\phi^w) \) can choose from inside a \( k \)-bpp projector. We denote \( \Delta_n(\phi^w) \) as the number of different intensity values that result in the same value of phase, i.e., the shiftable intensity values. In Fig. 2, the value of \( I^b_1 \) at point \( E_1 \) can be shifted within \([0, 2^k-1] \). The spatial intensity efficiency, \( \eta \), for this phase shifting method is then defined as

\[
\eta = \frac{P \sum_{\phi^w=0}^{2\pi} \left[ 1 - \frac{\sum_{n=0}^{NP-1} \Delta_n(\phi^w)}{NP2^k} \right] \times 100\% }{L}, \tag{13}
\]

where \( L \) is the pattern length or the number of pixels across which the pattern is defined. It should be noted that, for patterns with length \( L \) and periods \( P \), there are \( L/P \) different \( \phi^w \) values. The spatial intensity efficiency is an average over all the wrapped phase values.

In fact, the spatial intensity efficiency measures the strength of the original high frequency signal. For a certain number of patterns, the overall information capability of phase shifting patterns can be regarded as \( 100\% \). For a strategy with \( \Delta_n(\phi^w) = 0 \) and \( \eta = 100\% \), all the intensities cannot be changed at all. The capacity is fully used and the SNR of...
Comparing the spatial intensity efficiency and pattern entropy of different pattern strategies.

<table>
<thead>
<tr>
<th>Pattern Strategy</th>
<th>$\eta$</th>
<th>$H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>High frequency PMP [10], [22]</td>
<td>65.80%</td>
<td>14.00 bit</td>
</tr>
<tr>
<td>2 + 1 method [9], [23]</td>
<td>59.39%</td>
<td>13.81 bit</td>
</tr>
<tr>
<td>Three patterns trapezoidal [4]</td>
<td>100%</td>
<td>13.71 bit</td>
</tr>
<tr>
<td>Fast three-step phase-shifting [25]</td>
<td>63.80%</td>
<td>14.00 bit</td>
</tr>
</tbody>
</table>

So an optimization procedure should be performed to maximize the distinctions among periodical points.

In order to measure and maximize the differences, we propose the definition of pattern entropy for phase shifting methods. The pattern entropy is a summation of entropies of $\{I_n(\phi^w)\}$ over $\phi^w \in [0, 2\pi)$. In information theory, entropy is a measure of the uncertainty associated with a random variable [26]. In order to extend the entropy analysis to phase shifting methods, we first define the probability mass function (pmf) of the intensity values. We assume the points with different wrapped phase values are independent. For the periodical points, we first sort the projected intensity values such that $\{I_n(\phi^w) : 0 \le I_0(\phi^w) \le I_1(\phi^w) \ldots \le I_{NP-1}(\phi^w) \le 1\}$. By assuming a uniform distribution in $[0.5(I_{n-1}(\phi^w) + I_n(\phi^w)), 0.5(I_n(\phi^w) + I_{n+1}(\phi^w))]$, the possibility at intensity value, $0.5(I_{n-1}(\phi^w) + I_n(\phi^w)) \le x \le 0.5(I_n(\phi^w) + I_{n+1}(\phi^w))$, is given by

$$q(x) = \sum_{I_n(\phi^w)=x}^{2} \frac{2}{NL[I_{n+1}(\phi^w) - I_{n-1}(\phi^w)]},$$

(16)

The range of intensity values is changed from $[0, 1]$ to $[-(2NP-2)^{-1}, 1 + (2NP-2)^{-1}]$, and the uniform distribution in $[0, 1]$ can achieve the highest pattern entropy. If $-(2NP-2)^{-1} \le x \le 0.5(I_0(\phi^w) + I_1(\phi^w))$, then $q(x) = 2(NP-1)/\{NL[1 + (NP-1)(I_0(\phi^w) + I_1(\phi^w))]\}$. And if $0.5(I_{n-2}(\phi^w) + I_{n-1}(\phi^w)) \le x \le 1 + (2NP-2)^{-1}$, then $q(x) = 2(NP-1)/\{NL[2NP-1-(NP-1)(I_{n-2}(\phi^w) + I_{n-1}(\phi^w))])\}. So the pattern entropy, $H$, is defined as

$$H = -\sum_{\phi^w=0}^{2\pi} \sum_{x=-(2NP-2)^{-1}}^{1+(2NP-2)^{-1}} q(x) \log_2 q(x).$$

(17)

The pattern entropy is an integral over all the phase values. Among the strategies with the same number of patterns, the higher pattern entropy indicates that the intensities of the periodical points are closer to a uniform distribution. By maximizing the pattern entropy, the distinctions among pixels with the same phase value $\{I_n(\phi^w)\}$ would be maximized and, therefore, the content of the added information $\{C_n\}$ is optimized.

In Table I, several phase shifting methods are listed with the values of pattern entropy, assuming a pattern length of 1,280 pixels on an 8-bpp projector. Pattern entropy measures the information in the projected patterns. In Table I, high frequency PMP achieves the highest pattern entropy since 4 patterns are used in this strategy. Not surprisingly, the two-step triangular pattern strategy has the lowest pattern entropy. The information content measured by the pattern entropy not only includes the high frequency phase but also some other signals. For a certain strategy, increasing the pattern entropy results in having more information in the patterns. Thus, when extra, useful information is embedded into the patterns, the pattern entropy can be employed to maximize the information content. In this paper, we use the pattern entropy to optimize the design of period cue.
III. CODIFICATION OF PROJECTED PATTERNS

In this section, we consider how the signals are coded into patterns in the projector space. For PMP, the spatial intensity efficiency is 63.80%, leaving a 36.20% margin of the available intensity dynamic range for adding a period cue signal. The proposed pattern strategy consists of two codification stages: (1) phase codification which introduces a triangular phase waveform, instead of saw-tooth, and encodes the high frequency phase \( \phi^w \), and (2) period codification which implements Eq. (14) and encodes the period cue \( D \) of the high frequency phase.

Traditional high frequency phase obtained from Eq. (5) uses only temporal intensity information, and its waveform is saw-tooth [10], [22]. However, the small neighboring information is actually also reliable and available in the phase image. By employing such small neighboring spatial information, the phase frequency can be further improved. Taking advantage of the spatial information, we propose a triangular waveform for the high frequency phase such that the wrapped phase from Eq. (5) is first decoded into traditional saw-tooth waveform by utilizing a small portion of spatial information. Thus, the triangular phase provides a doubled frequency.

We employ a set of sinusoidal wave patterns, \( \{ I_n^b : n = 0, 1, ..., N-1 \} \), as the base patterns which are given by

\[
I_n^b(\vec{x}) = A + B \sin[(1)^n \Phi(y^i) - \frac{2\pi n}{N}],
\]

where \( A \) and \( B \) are set to 0.5. The term, \( \Phi(y^i) \), is given by \( \Phi(y^i) = 2\pi P y^i / \lambda \), where \( \lambda \) is the pattern length. Note the dependence of the phase term, \( \Phi(y^i) \) on \( y^i \), which is assumed to be positioned vertically above/below the projector. When \( L = 1280 \) and \( P = 4 \), the created triangular phase is shown as the solid lines in Fig 3.

Given the captured images, \( \{ I_n^c : n = 0, 1, ..., N-1 \} \), the received phase of the sinusoidal wave at the camera coordinate \( \vec{z} \) is calculated from Eq. (5). To solve Eq. (5), at least 3 patterns are needed. When the number of periods, \( P \), is greater than 1, ambiguities in phase will appear as the phase values \( \Phi_{\text{even}}(\vec{x}) = 2p(\vec{x}) \pi - \phi^w(\vec{x}) \), \( p(\vec{x}) = \{ 1, 3, 5, ... \} \), and \( \Phi_{\text{odd}}(\vec{x}) = 2p(\vec{x}) \pi + \phi^w(\vec{x}) \), \( p(\vec{x}) = \{ 0, 2, 4, ... \} \), in Eq. (18), all have the same value \( \phi^w(\vec{x}) \) obtained from Eq. (5). Figure 3 shows that the wrapped phase \( \phi^w \) can be unwrapped into initial decoded phase \( \phi^i \) by judging the position of \( \phi^w \).

After the initial decoding, \( \phi^i \) is scaled to \( [0, 2\pi) \) such that the error in phase is reduced during the scaling. By using a small portion of spatial information to determine whether the phase is increasing or decreasing, triangular phase encoding provides a doubled-frequency from the point of view of the imaging sensor. We denote \( P^i \) as the number of initial decoded periods, where \( P^i = 0.5P \). For \( P \geq 4 \), \( P^i \geq 2 \) which means that ambiguities still exist. The high frequency \( \phi^i \) needs to be further decoded into unit frequency signal \( \Phi \) by referring the period cue.

For phase shifting methods with the spatial intensity efficiency less than 100%, we intend to embed a period cue signal into patterns within the remaining dynamic range. We develop pattern codification as a means of removing ambiguities in initial decoded phase by encoding the period cue \( P^i = \{ 0, 1, ..., (P^i-1) \} \) into the SLI patterns such that each point in the projector plane, \( (\vec{x}) \), is given by

\[
I_n(\vec{x}) = I_n^b(\vec{x}) + C_n(y^i),
\]

where \( I_n^b(\vec{x}) \) is defined in Eq. (18) and \( \{ C_n : n = 0, 1, ..., (N-1) \} \) represents the coding functions. Due to these added signals, the projected patterns are no longer sinusoidal wave patterns.

In an \( N \) pattern strategy, the number of coding functions is also \( N \). We need to define a parameter, period cue \( D(y^i) \), which is derived from the temporal values \( \{ C_n(y^i) \} \) and can be employed to identify the period \( p^i(y^i) \). The decoding filter function, \( F_D(\cdot) \), can be implemented in various forms as long as it translates the \( N \) temporal values \( \{ C_n(y^i) \} \) into a single value \( D(y^i) \). For the phase values \( \Phi(y^i) = 2\pi p^i(y^i) + \phi^i(y^i) \), \( p^i(y^i) = \{ 0, 1, ..., (P^i-1) \} \), \( D(y^i) \) varies with respect to \( p^i(y^i) \) such that the period number can be obtained from the value of \( D(y^i) \).

In practice, for a pattern strategy with the spatial intensity efficiency less than 100%, the coding functions \( \{ C_n(y^i) \} \), in Eq. (19), should have the following basic properties:

Property 1: Absence in phase: the calculation of phase value, in the camera space, by means of Eq. (5) should not be affected by adding the coding functions \( \{ C_n(y^i) \} \), in order to achieve an accurate 3-D reconstruction based on phase.

Property 2: Dynamic range of coding: the illumination patterns, \( \{ I_n(\vec{x}) \} \), in the projector space (Eq. (19)), should stay within the dynamic range of the projector, \( [0, 1] \); otherwise, the patterns have to be scaled into \( [0, 1] \), which will result in a reduced SNR of the high frequency phase.

Property 3: Maximum index probability: the information of period cue \( D(y^i) \) should maximize the probability of correctly indexing the period number \( p^i(y^i) \) in the presence of system noise. That is, the \( D(y^i) \) values for
different periodical points (points with the same \( \phi(y^j) \) values) should be as much distinct as possible, to prevent false estimation of \( p^i(y^j) \) under noisy conditions.

Property 4: Maximum pattern entropy: the illumination patterns, \( \{I_n(x^j)\} \), should achieve maximum pattern entropy such that the signal strength of period cue \( D(y^j) \), realized by adding the coding functions \( \{C_n(y^j)\} \), is maximized.

In order to devise such a coding method that satisfies the above properties, we will develop, first, an \( N = 4 \) pattern strategy where, as specified by Prop. (1), the values of \( U \) and \( V \) in Eqs. (6) and (7) will remain unchanged by adding \( \{C_n(y^j)\} \) if

\[
\sum_{n=0}^{N-1} C_n(y^j) \cos\left(\frac{2\pi n}{N}\right) = 0, \tag{20}
\]

and

\[
\sum_{n=0}^{N-1} C_n(y^j) \sin\left(\frac{2\pi n}{N}\right) = 0. \tag{21}
\]

For \( N \geq 3 \), solutions to Eqs. (20) and (21) always exist because the number of unknown parameters is less than the number of equations. Particularly since \( N = 4 \), \( C_0(y^j) = C_2(y^j) \) and \( C_1(y^j) = C_3(y^j) \) will satisfy the property of absence in phase. Thus, we define the period cue, \( D(y^j) \), for \( N = 4 \) as

\[
D(y^j) = \frac{C_0(y^j) + C_2(y^j) - C_1(y^j) - C_3(y^j)}{2}, \tag{22}
\]

which is a linear function of \( \{C_n(y^j)\} \). We further denote the value range of \( D(y^j) \) as \( R(y^j) \).

For \( N = 4 \), Prop. (2) should be satisfied because: (1) the intensity values cannot exceed \([0, 1]\) and (2) the dynamic range of the signals for computing high frequency phase should not be reduced. That is, \( B \) remains unchanged. Thus for a pixel \( I_n(x^j) \) of illumination patterns, the coding functions \( \{C_n(y^j)\} \) must satisfy \( C_0(y^j) \geq 0 - 1/4 \) and \( C_2(y^j) \leq 1/4 \).

From Eq. (15), the bounds of \( \{C_n(y^j)\} \) will limit the value range of period cue \( D(y^j) \), which is denoted as \( R(y^j) \) with supremum bound \( \sup \{R(y^j)\} \) and infimum bound \( \inf \{R(y^j)\} \), respectively. Figure 4 shows the bounds of \( D(y^j) \) when \( N = 4 \) and \( P = 4 \). As shown for different \( \Phi(y^j) \), \( R(y^j) \) is not the same. To simplify the problem, we set our bounds to a constant value \( R \) inside the theoretical bounds \( R(y^j) \).

With regard to Prop. (3), the period cue pattern can be of any shape as long as the cue values can be uniquely obtained from the received patterns, and for the phase values \( \Phi(y^j) = 2p^i(y^j)\pi + \phi(y^j), p^i(y^j) = \{0, 1, ..., P^i - 1\} \), the period cue values must be unique. So for purposes of coding period, the cue patterns should be continuous to avoid estimation errors, especially, when the resolution of camera is higher than that of the projector. In addition, the values of \( D(y^j) \) should be evenly spread in \( R \). So, we choose a linear period cue for the coding period in the \( N = 4 \) pattern strategy such that

\[
D(y^j) = \left(\frac{\sup \{R\} - \inf \{R\}}{L}\right)y^j + \inf \{R\}. \tag{23}
\]
Finally, after obtaining the coding functions, SLI patterns are computed through Eq. (19). For a pattern strategy where $N = 4$ and $P = 4$, the PCPS patterns are shown in Fig. 6, where cross sections of the four patterns indicate that the patterns are no longer sinusoidal waves due to the added coding functions. The spatial intensity efficiency and pattern entropy is listed in Table III. Compared with $N = 4$ and $P = 4$ high frequency PMP, the spatial intensity efficiency is increased from 63.80% to 91.53%, and the pattern entropy is increased from 14.00 bits to 15.91 bits.

### IV. DE-CODIFICATION OF RECEIVED IMAGES

In this section, we consider how the captured images are decoded in the camera space. There are three types of information we need to obtain: (1) the modulation of scanned object, (2) the wrapped phase, and (3) the period cue. With regards to modulation, the parameter $M(\bar{x}) = \alpha(\bar{x})B$, representing the amplitude of the observed high frequency signal reflected off the target, can be derived according to Eq. (8). The modulation $M$ is used as the texture image for the 3-D reconstruction. The wrapped phase value, $\phi^w$, is calculated by Eq. (5).

In order to achieve real-time operation, there are two facts that should be noticed: (1) the computational cost of $\sin(\cdot)$, $\cos(\cdot)$, and $\arctan(\cdot)$ in Eqs. (5) and (8) is high and (2) the value range of received images inside the camera are fixed. So, it becomes possible for us to build look-up table (LUT) based algorithms for resolving both Eqs. (5) and (8) such that $M(\bar{x})$ and $\phi^w(\bar{x})$ are obtained with very low computational cost, and because $B$ in Eq. (18), is constants of the projector, the albedo $\alpha(\bar{x})$ is obtained by

$$\alpha(\bar{x}) = \frac{M(\bar{x})}{B}. \tag{25}$$

Furthermore, the $\phi^w$ is initially decoded into $\phi^i$ as shown in Fig. 3.

To find the period $p^i(\bar{x})$, we present two methods for this purpose: (1) a temporal method, which decodes the period number point by point, and (2) a hybrid method, which decodes the period number with the help of spatial unwraping approaches. The temporal period decoding attempts to identify the $p^i(\bar{x})$ based on the period cue value $D(\bar{x})$

### Table III

<table>
<thead>
<tr>
<th>Patterns</th>
<th>Periods</th>
<th>Strategy</th>
<th>PMP</th>
<th>PCS</th>
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</thead>
<tbody>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>63.80%</td>
<td>91.53%</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>14.00 bit</td>
<td>15.91 bit</td>
</tr>
</tbody>
</table>

of each point. It decodes the phase with no dependence on neighboring points. The $D(\bar{x})$ value is calculated from Eq. (15). Particularly in a 4 pattern strategy, $D(\bar{x})$ is computed as

$$D(\bar{x}) = \frac{I_0(\bar{x}) + I_2(\bar{x}) - I_1(\bar{x}) - I_3(\bar{x})}{2\alpha(\bar{x})}, \tag{26}$$

where $\alpha(\bar{x})$ is obtained from Eq. (25).

The parameter, $D(\bar{x})$, is only related to the coding functions $\{C_n(\bar{x})\}$. So once $\phi^i(\bar{x})$ and $D(\bar{x})$ are obtained as shown in Fig. 7, the period $p^i(\bar{x})$ is calculated by

$$p^i(\bar{x}) = \text{round}\left(\frac{1}{2\pi} \left[ 2P^i \pi - \frac{D(\bar{x})}{\sup R} - \inf R - \phi^i(\bar{x}) \right] - 0.5 \right), \tag{27}$$

where $\sup$, $\inf$, and $R$ are defined in Sec. III, $P^i$ is the constant number of periods after initial decoding and $\text{round}(\cdot)$ notes the rounding function of mapping the operand to the nearest integer value. The phase $\Phi(\bar{x})$ is obtained by

$$\Phi(\bar{x}) = \phi^i(\bar{x}) + 2\pi p^i(\bar{x}). \tag{28}$$

When $D(\bar{x})$ is scaled to $[0, 4\pi]$ as shown in Fig. 7, if the absolute error $e_D(\bar{x})$ is smaller than $\pi$, the period will be correctly decoded.

We note that, when the period number $P$ increases and since the $R$ in Eq. (27) is unchanged, the points with incorrect decoded period increase. So, instead of decoding the period temporally, the spatial unwrapping approaches [11], [12], [18] can be combined with temporal period cues for indexing the period with higher accuracy. As such, we propose hybrid decodification where the initial decoded phase image is further processed line by line. As illustrated in Fig. 8 (left), the processed phase line is first segmented according to the corresponding period cue line. Then, the segment is unwrapped into segmental unwrapped phase, $\phi^s$, as shown in Fig. 8 (right). Since the start period, in pixels 287 to 322, is unknown, there is a distance between the scaled period cue $D$ and $\phi^s$. So the start period $p^s$ is computed as

$$p^s = \text{round}\left(\frac{1}{2\pi} \left[ 2P^s \pi - \frac{D - \inf R}{\sup R - \inf R} - \phi^s \right] - 0.5 \right), \tag{29}$$

which is similar to Eq. (27) except that $2P^s - \frac{D - \inf R}{\sup R - \inf R} - \phi^s$ is the average distance between $D$ and $\phi^s$. This averaging reduces the degrading effect from noise and gives a correct period value $p^s$. Lastly, the final decoded phase, $\Phi$, of the segment is obtained by adding $\phi^s$ with $2P^s \pi$ as Eq. (28). The overall flowchart of the PCPS strategy is shown in Fig. 9.

---

Fig. 6. Cross sections of the PCPS patterns when $N = 4$ and $P = 4$. From top to bottom are $I_0$, $I_1$, $I_2$, and $I_3$.

Fig. 7. The initial unwrapped phase is finally unwrapped according to the period cue. The period cue is scaled into $[0, 2P^i \pi]$.
V. ACCURACY ANALYSIS

In practice, uncertainty in $I_n^c(\tilde{x}^c)$ is introduced from camera noise [27] and projector noise [19], as well as other sources [28]. Since these effects degrade the quality of 3-D reconstructions, many studies have been performed on noise analysis to improve the signal to noise ratio through pattern strategy optimization [29]. In doing so, several researchers [22], [29], [30] have proposed modeling the combination of noise sources as additive, white, Gaussian noise, $w_n^c(\tilde{x}^c) \sim N(0, \sigma^2)$. Thus, the captured images, $I_n^c(\tilde{x}^c)$, in Eq. (2) can be expressed as

$$\tilde{I}_n^c(\tilde{x}^c) = \alpha(\tilde{x}^c)[I_n^c(\tilde{x}^c) + \beta(\tilde{x}^c)] + w_n^c(\tilde{x}^c),$$

(30)

where $\alpha$ denotes the observed variable polluted by noise. With the same period $p$, the phase can be regarded as unit frequency, i.e., $P = 1$. The phase value with noise, $\Phi(\tilde{x}^c)$, is then calculated by

$$\tilde{\Phi}(\tilde{x}^c) = \arctan \left[ \frac{\sum_{n=0}^{N-1} \tilde{I}_n^c(\tilde{x}^c) \cos(\frac{2\pi n}{N})}{\sum_{n=0}^{N-1} \tilde{I}_n^c(\tilde{x}^c) \sin(\frac{2\pi n}{N})} \right]$$

(31)

$$= \arctan \left[ \frac{\frac{1}{2} M(\tilde{x}^c) \cos[\Phi(\tilde{x}^c)] + \sum_{n=0}^{N-1} w_n^c(\tilde{x}^c) \cos(\frac{2\pi n}{N})}{\frac{1}{2} M(\tilde{x}^c) \sin[\Phi(\tilde{x}^c)] + \sum_{n=0}^{N-1} w_n^c(\tilde{x}^c) \sin(\frac{2\pi n}{N})} \right].$$

Then, the phase error $\Phi_e(\tilde{x}^c)$ is obtained by

$$\Phi_e(\tilde{x}^c) = \Phi(\tilde{x}^c) - \tilde{\Phi}(\tilde{x}^c) = \arctan \left( \frac{T(\tilde{x}^c)}{S(\tilde{x}^c)} \right),$$

(32)

where

$$S(\tilde{x}^c) = \frac{2}{N} \sum_{n=0}^{N-1} w_n^c(\tilde{x}^c) \sin[\Phi(\tilde{x}^c) - \frac{2\pi n}{N}] + M(\tilde{x}^c),$$

(33)

and

$$T(\tilde{x}^c) = \frac{2}{N} \sum_{n=0}^{N-1} w_n^c(\tilde{x}^c) \cos[\Phi(\tilde{x}^c) - \frac{2\pi n}{N}].$$

(34)

Further, it can be shown that $S(\tilde{x}^c)$ and $T(\tilde{x}^c)$ are Gaussian, and their correlation can be estimated by $E[S(\tilde{x}^c)T(\tilde{x}^c)] = \sigma^2 \sum_{n=0}^{N-1} \sin[2\Phi(\tilde{x}^c) - 4\pi n] = 0$. Thus, $S(\tilde{x}^c)$ and $T(\tilde{x}^c)$ are independent.

The joint Probability Density Function (PDF) of $\Phi_e$ and $M$ can be derived as in [31]. Integrating the joint PDF over $M$ yields the marginal PDF for $\Phi_e$ given by

$$f_{\Phi_e}(\phi_e) = \frac{1}{2\pi} \exp \left[ - \frac{NM^2}{4\sigma^2} \left[ 1 + \kappa \sqrt{\pi} \exp(\kappa^2)(1 + \text{erf}(\kappa)) \right] \right],$$

(35)

where $\text{erf}(\cdot)$ is the error function and $\kappa = \sqrt{NM \cos(\phi_e)/(2\sigma)}$. With sufficiently large values of $\sqrt{NM/(\sqrt{2}\sigma)} (>3)$, the error function will be close to 0. The term, $\kappa \sqrt{\pi} \exp(\kappa^2)(1 + \text{erf}(\kappa))$, will, therefore, dominate the constant 1 such that Eq. (35) reduces to a zero-mean, Gaussian distribution. Thus, the variance of $\Phi_e$ is approximated by $\sigma^2_{\Phi_e}(\tilde{x}^c) \approx \sigma^2/(NM^2(\tilde{x}^c))$. When using high frequency patterns, $\Phi(\tilde{x}^c)$ is decoded and scaled into $[0, 2\pi)$ before 3-D reconstruction. The phase error, $\Phi_e(\tilde{x}^c)$, is divided by $P$,

$$\sigma^2_{\Phi_e}(\tilde{x}^c) \approx \frac{2\sigma^2}{NP^2M^2(\tilde{x}^c)},$$

(36)

which is the same as that of the two frequency PMP strategy derived by Li [22] with only half the number of patterns. Also from Eq. (36), it should be noted that when $P$ is less than $M(\tilde{x}^c)$, where $M(\tilde{x}^c)$ is generally larger than 64 on 8 bpp devices, increasing $P$ will more efficiently reduce $\sigma^2_{\Phi_e}(\tilde{x}^c)$ than increasing $M(\tilde{x}^c)$.

VI. RESULTS AND DISCUSSIONS

To demonstrate the proposed pattern strategy, we developed a prototype SLI system shown in Fig. 10, based on an 8 bpp, monochrome, Prosilica GC640M, gigabit ethernet camera with 640 x 480 pixel resolution. The projector is composed of a Texas Instrument’s Discovery 1100 board with ALP-1 controller and LED-OM with 225 ANSI lumens. The resolution of the 8 bpp, monochrome, projector is 1024 x 768 (W x L), with
a maximum frame rate of 150 fps. The camera and projector are synchronized by an external triggering circuit with a baseline distance between camera and projector of 120 mm. The scanned object was placed approximately 600 mm away. Gamma correction was performed on the received images [32], while a lookup table was created to correct optical distortion.

We programmed the experimental system using Microsoft Visual Studio 2005 with managed C++. As our processing unit, we used a Dell Optiplex 960 with an Intel Core 2 Duo Quad Q9650 processor running at 3.0 GHz. And in the first two experiments, stationary objects were scanned with a camera exposure time of 2.4 ms. Due to the low illumination of our projector, the standard deviation of system noise, $\sigma$, was 1.5413, higher than SLI systems using commercial projectors.

In the first experiment shown in Fig. 11, three separate white foam boards were carefully placed and scanned such that the left-most board was isolated from two boards otherwise positioned to create a phase ambiguity. In Fig. 11 (a) is the corresponding phase term, $\phi^w$, while Fig. 11 (f) shows $\phi^w$ for the 500th image column, which dissects that ambiguous foam boards in half. Visually, there is no apparent discontinuity in phase to suggest that more than two distinct objects appear in the scene in either (a) or (f).

After initial decoding in Fig. 11 (b) and (g), the phase values $\phi'$ of the right two boards still appear to form a single, continuous surface; however, the depth discontinuity does show up in the period cue image of Fig. 11 (c) and (h), that is the $D$ information. Thus, PCPS works to detect otherwise ambiguous phase, where it should be noted that because of the focus problem [24], the received information is low-pass filtered. As a result, the inflection points of phase in Fig. 11 (c) and (h) are not as sharp as our theory predicts. In practice, the phases of these points can be corrected after obtaining the final decoded phase [4], [24], [33].

The final 3-D reconstruction is shown in Fig. 11 (d), (e), (i) and (j), where because of the heavy noise in our system ($\sigma = 1.5413$), the temporal period decoding method may produce wrong period numbers for some heavily contaminated points, e.g. the impulse in Fig. 11 (i). But even in our noisy system with $P = 16$, 99.56% points were correctly decoded through the temporal approach. By implementing the hybrid method, however, all the points were correctly decoded (Fig. 11 (e) and (j)). After having obtained the final phase by the hybrid method, 3-D surfaces were reconstructed as shown in Fig. 12. The side and top views, of the reconstructed surfaces, demonstrate that the proposed PCPS works correctly in situations like Fig. 1.

In order to demonstrate the accuracy of the PCPS, we performed the second experiment by scanning a textured, flat,
The board was further scanned and reconstructed using Gray Code + Phase Shifting (GC+PS) [16], [34], [35] and two frequency PMP [22] strategies. For a fair comparison, both GC+PS and two frequency PMP were consisted of 4 phase shifting patterns with 16 periods. To unwrap the high frequency phase, 4 extra gray code patterns and 4 extra unit frequency PMP patterns were employed in GC+PS and two frequency PMP methods respectively. Thus, there were totally 8 patterns for either GC+PS or two frequency PMP method.

For comparing the PCPS with a method that has the same number of patterns, we developed a simple Step Pattern + Phase Shifting (SP+PS) method. The SP+PS is consisted of high frequency and unit frequency components. For the high frequency component, the phase shifting patterns (3 patterns with 16 periods) were used, whereas, for the unit frequency component, a step pattern was employed such that there are 16 different intensity values in the step pattern corresponding to the 16 periods in the phase shifting patterns. So the total number of patterns in the SP+PS is 4.

In this experiment, the textured board was scanned 2000 times, for each strategy of SP+PS, GC+PS, two-frequency PMP and PCPS. For all the methods, the numbers of reconstructed points were 267749, and the mean values of phase error, obtained from the averaging of the 2000 times’ scanning, were very small. The theoretical variance value was obtained from Eq. (36). As demonstrated, the experimental variances of phase error of GC+PS, two frequency PMP and PCPS are very close for all numbers of periods $P$ (see Fig. 14 (left)) and modulation values $M$ (see Fig. 14 (right)). However, it should be noted that 8 patterns were used in GC+PS and two frequency PMP strategies, whereas only 4 patterns were projected in PCPS. Thus, PCPS achieves the same accuracy as high frequency PMP by using only half the number of patterns.

For the SP+PS strategy, although the number of patterns was the same as PCPS, the variances for different $P$ and $M$ values are nearly 1.33 times of those of PCPS. Further, in practice, since only one pattern is used for unwrapping the high frequency, we noticed a dramatically decrease of $SNR$ in the low albedo regions if the intensity in the step pattern is also very low. The unwrapping in those regions is very unreliable. However, by using the PCPS, because multiple patterns with both high and low intensities are projected onto the low albedo regions, the unwrapping in those regions become much more reliable than by using the SP+PS.

In the third experiment, a stationary, textured, statue and a slowly moving hand were scanned as shown in Fig. 10, and the execution performance of PCPS was analyzed. In order to achieve a high speed of scanning, the exposure time of the camera was reduced to 0.8 ms, resulting in an increased variance of 2.3165. In this experiment, we projected the 4 pattern PCPS. After capturing the most recent 4 patterns, the 4 images were re-ordered and stored in a buffer. We then used the lookup table (LUT) 3-D reconstruction techniques to calculate the 3-D point cloud and modulation results. Every time when a new image was captured, only one image in the buffer was replaced. Because the speed of the camera/projector pair was 120 fps, the speed of final 3-D reconstruction also achieved 120 fps.

To observe the experimental time cost, we implemented the algorithm in our system by using only one CPU core. Our tests were based upon processing the full resolution of $640 \times 480$ pixels. The processing time each important procedure, averaged over 10,000 frames, is listed in Table IV. The period decoding is by means of the temporal method. The computational cost of hybrid decoding depends on the scene, since segmentation is involved. However, the one dimensional segmentation is done by determining the period cue values. Indeed, the hybrid method will not introduce much more computational workload. The computation frame rate can be further improved by ignoring the low modulation points (un-illuminated points) and using multiple cores or GPU programming. In our case, the computation speed is fast enough for the maximum frame rate of the camera/projector pair. For comparison, we also implement Li’s algorithm [22], while Zhang et al. reported a reconstruction frame rate of 25.56 fps with a resolution of $532 \times 500$ when employing quality-guided phase unwrapping and using GPU processing on nVidia Quadro FX 3450 [23]. The comparison result is listed in Table V.

The PCPS works well under different background illumination conditions as long as there is no saturation in the images. In those saturated regions, the phase information and period cue are distorted. As a matter of fact, the background illumination (i.e. the ambient illumination $\beta$ in Eq. (2)) can be canceled out during the computation, as indicated by Eqs. (5), (8) and (26). However, an increased background illumination, such as turning on the light in the room, will introduce more noise into the system. And in order to avoid the saturation of captured images, the aperture size or exposure time of the

### Table IV

<table>
<thead>
<tr>
<th>Function</th>
<th>Time cost</th>
<th>Remark</th>
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<tbody>
<tr>
<td>Wrapped phase and modulation</td>
<td>$1.72 \text{ ms}$</td>
<td>$\phi^w + M$</td>
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<tr>
<td>Period cue</td>
<td>$0.59 \text{ ms}$</td>
<td>$D$</td>
</tr>
<tr>
<td>Initial decoded phase</td>
<td>$0.96 \text{ ms}$</td>
<td>$\phi^i$</td>
</tr>
<tr>
<td>Final decoded phase</td>
<td>$1.24 \text{ ms}$</td>
<td>Temporal period decoding</td>
</tr>
<tr>
<td>Overall data processing</td>
<td>$7.87 \text{ ms}$</td>
<td>Include 3-D reconstruction</td>
</tr>
</tbody>
</table>
camera has to be reduced. As a result, the intensity dynamic ranges of the captured images are reduced, resulting in more phase errors.

When the scanned object moves away from the imaging system, the noise in the phase and period cue increases and the intensity dynamic range of the captured images decreases. Thus, the number of correctly unwrapped points decreases. To compensate the reduced SNR, the aperture size or exposure time should be increased. In practice, we employ the modulation information as a shadow detector to avoid the 3-D reconstruction of regions with low SNR.

In summary, the proposed PCPS strategy uses the remaining 36.20% intensity dynamic range to add the second reference signal. As shown in the first experiment, the PCPS works in situations where traditional phase shifting methods either fail or require more patterns, with high system noise standard deviation (1.5413). In the second experiment, we demonstrated Eq. (36), showing that the proposed PCPS method achieves the same accuracy as high frequency PMP with half the number of patterns. In the last experiment, we implemented the PCPS strategy in a real-time system that can achieve 120 fps for 3-D acquisition and reconstruction. The analysis of execution performance shows that the computational cost of PCPS is considerably low. Finally, it should be noted that for the SLI systems with lower noise level than our prototype system, the quality of both phase and period cue will be improved, leading to more accurate unwrapped phase values.

For simplicity, we have presented the PCPS strategy for $N = 4$. In fact, this new pattern strategy can be extended to any number of patterns as long as the pattern is obtained from Eq. (18) and satisfies the four coding properties in Sec. III. As a phase shifting pattern strategy, the theoretical minimum number of patterns of PCPS is 3. As the first example for $N = 3$ and $P = 4$, the base pattern is derived from Eq. (18). From Eqs. (20) and (21), the coding functions have $C_0(y') = C_1(y') = C_2(y')$. Thus, we define Eq. (15) as $D(y') = [C_0(y') + C_1(y') + C_2(y')]/3$.

The period cue is designed as shown in Fig. 16, which is no longer linear. The wrapped phase, $\phi^n(\vec{x})$, is first mapped to an initial decoded phase $\phi^*(\vec{x})$ where the points, with the same $\phi^*(\vec{x})$, are compared such that the $p^i(\vec{x})$ with higher $D(\vec{x})$ are assigned to 1 while the $p^i(\vec{x})$ with lower $D(\vec{x})$ are assigned to 0. Limited by the number of patterns, the $\beta(\vec{x})$ cannot be removed from $D(\vec{x})$. Thus, it becomes unreliable to decode patterns with $P > 4$. The 3-D reconstruction results of 3 pattern PCPS with $P = 4$ are shown in Fig. 17 (b).

As a second example, we present the pattern strategy when $N = 6$, where from Eqs. (20) and (21), the coding functions satisfy $C_1(y') = C_2(y') = C_3(y') = C_4(y')$ and $2C_0(y') + C_1(y') + C_5(y') = 2C_3(y') + C_2(y') + C_4(y')$, which provide even more coding freedom than 4 pattern strategy. Thus, it is possible to design more complicated period cues such that higher accuracy period decoding can be achieved. For simplicity, we define the period cue as $D(y') = [2(C_0(y') + C_3(y')) - (C_1(y') + C_2(y') + C_4(y') + C_5(y'))]/4$, which is a linear period cue. The decoding process is similar to the 4 pattern strategy, where because of the additional freedom, the range of $D(y')$ is increased. Comparison between $N = 4$...
two techniques, we have developed a novel period information embedded pattern strategy (PCPS) for fast, reliable 3-D data acquisition and reconstruction. This strategy includes codification of patterns, de-codification of images, real-time implementation, accuracy analysis, and performance analysis. A major advantage of the proposed PCPS strategy includes removing the depth ambiguity associated with traditional phase shifting patterns without reducing phase accuracy or increasing the number of projected patterns. The computational cost of PCPS is low and up to 120 fps can be achieved for 3-D acquisition and reconstruction and can be extended to other phase shifting methods. Future research will be focused on the performance improvement for the 3-pattern strategy as well as finding the optimal signal ratio between high frequency phase signal and the period cue.

VII. CONCLUSION

This paper presents two novel concepts of spatial intensity efficiency and pattern entropy for phase shifting methods. When the spatial intensity efficiency is less than 100%, a secondary signal can be embedded into the projection patterns. The pattern entropy can be used to optimize the magnitude of the embedded secondary signal. Based on these

Fig. 17. 3-D reconstructions for different number of patterns strategies. The standard deviation of system noise was 1.5413. No filter was applied. (a) Scanned Object. (b) The depth rendering result using 3 patterns 4 period PCPS. (c) The depth rendering result using 4 patterns 16 period PCPS. (d) The depth rendering result using 6 patterns 32 period PCPS.

and $N = 6$ shows that the denominator increases from 2 to 4 such that the period cue noise is reduced by half. The 3-D reconstruction results of 6 pattern PCPS with $P = 32$ is shown in Fig. 17 (d).

Applying the period coded method to other phase shifting methods is similar to the procedures of PMP. It should be noted that, in this paper, the SNR of the high frequency information is unchanged; however in practice, the projector amplitude value $B$, in Eq. (18), can be reduced to produce a reduction in SNR of the high frequency signal such that the $M$ value, in Eq. (36), is reduced, but a smaller $B$ value leaves more dynamic range for period codification. As a result, the $P$ value, in Eq. (36), will be increased, and, based on Eq. (36), when the $P$ value is less than $M$, the increasing of $P$ will more efficiently reduce the phase error than increasing $M$. By reducing the projector modulation, the period information can be embedded into phase shifting methods even with the spatial intensity efficiency of 100%. The tradeoff between the dynamic ranges of the high frequency phase and the period cue indeed depends on the distribution of system noise.

REFERENCES

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