A genetic evolving ant direction DE for OPF with non-smooth cost functions and statistical analysis

K. Vaisakha, a,*, L.R. Srinivas b

a Department of Electrical Engineering, AU College of Engineering, Andhra University, Visakhapatnam-530003, AP, India
b Department of Electrical and Electronics Engineering, S.R.K.R. Engineering College, Bhimavaram-534204, AP, India

Article info
Article history:
Received 3 August 2009
Received in revised form 27 March 2010
Accepted 30 March 2010
Available online 18 May 2010

Keywords:
Evolving ant direction differential evolution
Optimal power flow
Genetic algorithm
Non-smooth cost functions
Voltage stability index
Statistical analysis

Abstract
This paper proposes an evolving ant direction differential evolution (EADDE) algorithm for solving the optimal power flow problem with non-smooth and non-convex generator fuel cost characteristics. The EADDE employs ant colony search to find a suitable mutation operator for differential evolution (DE) whereas the ant colony parameters are evolved using genetic algorithm approach. The Newton–Raphson method solves the power flow problem. The feasibility of the proposed approach was tested on IEEE 30-bus system with three different cost characteristics. Several cases were investigated to test and validate the robustness of the proposed method in finding the optimal solution. Simulation results demonstrate that the EADDE provides superior results compared to a classical DE and other methods recently reported in the literature. An innovative statistical analysis based on central tendency measures and dispersion measures was carried out on the bus voltage profiles and voltage stability indices.

© 2010 Elsevier Ltd. All rights reserved.

1. Introduction
In the present day power systems, optimal power flow (OPF) is an important tool for power system operators both in planning and operating stages. The main purpose of an OPF is to determine the optimal operating state of a power system and the corresponding settings of control variables for economic operation, while at the same time satisfying various equality and inequality constraints. The OPF problem, in general, is a large-scale highly constrained nonlinear non-convex optimization problem. Many mathematical programming techniques [1] such as linear programming [LP] [2,3], nonlinear programming (NLP) [4], quadratic programming (QP) [5], Newton method [6], and interior point methods (IPM) [7] have been applied to solve the OPF problem successfully. Usually, these methods rely on the assumption that the fuel cost characteristic of a generating unit is a smooth, convex function. However, there are situations where it is not possible, or even appropriate, to represent the unit’s fuel cost characteristics as a convex function. This situation arises when valve-points, units’ prohibited operating zones, and piece-wise quadratic cost characteristics are present [8].

In recent years, many heuristic algorithms, such as genetic algorithms (GA) [9], evolutionary programming [10], simulated annealing [11], tabu search [12], and particle swarm optimization [13] have been proposed for solving the OPF problem, without any restrictions on the shape of the cost curves. The results reported were promising and encouraging for further research in this direction. Moreover, many hybrid algorithms have been introduced to enhance the search efficiency. For instance, a hybrid tabu search and simulated annealing (TS/TA) [14] was applied to solve the OPF problem with flexible alternating current transmission systems (FACTS) device; a hybrid evolutionary programming and tabu search or improved tabu search (ITS) [15] was used to solve the economic dispatch problem with non-smooth cost functions. Meanwhile, an improved evolutionary programming (IEP) [16] was successfully used to solve combinatorial optimization problems.

In the recent past, Storn and Price introduced a powerful evolutionary algorithm called differential evolution (DE) to solve the OPF problems [17]. DE is a numerical optimization approach that is simple, easy to implement, significantly faster than other algorithms, and robust. DE combines simple arithmetic operators with the classical operators of crossover, mutation and selection to evolve from
a randomly generated starting population to a final solution. The fittest of an offspring competes one-to-one with the corresponding parent, which is different from the other evolutionary algorithms. This one-to-one competition gives rise to a faster convergence rate.

The DE has been successfully applied to various power system optimization problems such as generation expansion planning [18], hydro thermal scheduling [19], Figueroa and Cedero [20] applied DE for power system state estimation. Coelho and Mariani [21] used this algorithm for economic dispatch with valve-point effect. M. Basu [22] applied DE for solving the OPF problem incorporating FACTS devices. The hybrid differential evolution (HDE) has been employed for hydrothermal coordination [24], hydrothermal optimal power flow [25], and network reconfiguration problem [26].

Cololini [27] proposed the concept of ant system (AS) and applied to the traveling salesman problem (TSP) [28]. The Ant algorithm has been inspired by the behavior of real ant colonies, in particular, by their foraging behavior. Recently, the ant algorithm has been applied to various optimization problems, such as the short-term generation scheduling problem [29], unit commitment [30], and hydro electric generation scheduling [31].

In this paper, an efficient evolving ant direction DE based approach is proposed to solve the OPF problem with non-smooth cost functions. Evolving ant direction mutation operator selection is suggested to the original DE algorithm. Though there are five mutation operations stated in this paper, the EADDE uses only one mutation operator during the solution process. The proposed EADDE method embedded with the ant colony search is able to constantly choose different but most appropriate mutation operators during the solution process to accelerate the search for the global optimum solution. The proposed approach has been examined and tested on IEEE 30-bus standard test system with three different types of generator cost curves. Simulation results demonstrate that the EADDE algorithm is superior to the original DE algorithm and provides significantly better results compared to those reported in the literature.

The remainder of the paper is organized as follows: Section 2 describes the formulation of an optimal power flow problem, while section 3 explains the standard DE approach. Section 4 then details the procedure of proposed evolving ant direction DE. Section 5 presents the statistical analysis and section 6 presents the results of the optimization and compares methods to solve the case studies of optimal power flow problems with IEEE 30—bus system. Lastly section 7 provides the conclusion.

2. Problem formulation

The main goal of the OPF is to optimize a certain objective subject to several equality and inequality constraints. The problem can be mathematically modeled as follows:

Min \( OF(x, u) \)

subject to

\( g(x, u) = 0 \)

\( h_{\text{min}} \leq h(x, u) \leq h_{\text{max}} \)

where vector \( x \) denotes the state variables of a power system network that contains the slack bus real power output \((P_{G1})\), voltage magnitudes and phase angles of the load buses \((V_i, \delta_i)\), and generator reactive power outputs \((Q_G)\). Vector \( u \) represents control variables that consist of real power generation levels \((P_{Gi})\) and generator voltages magnitudes \((|V_{Gi}|)\), transformer tap setting \((T_K)\), and reactive power injections \((Q_{CS})\) due to volt-amperes reactive (VAR) compensations; i.e.,

\[
\mathbf{u} = [P_{G2}, \ldots, P_{GN}, V_{G1}, \ldots, V_{Gn}, T_1, \ldots, T_{NT}, Q_{C1}, \ldots, Q_{CS}]
\]

where \( N \) = number of generator buses,

\( NT \) = number of tap changing transformers

\( CS \) = number of shunt reactive power injections.

The OPF problem has two categories of constraints:

2.1. Equality constraints

These are the sets of nonlinear power flow equations that govern the power system, i.e.,

\[
P_{Gi} - P_{Di} - \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) = 0
\]

\[
Q_{Gi} - Q_{Di} + \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) = 0
\]

where \( P_{Gi} \) and \( Q_{Gi} \) are the real and reactive power outputs injected at bus \( i \) respectively, the load demand at the same bus is represented by \( P_{Di} \) and \( Q_{Di} \), and elements of the bus admittance matrix are represented by \( |V_i| \) and \( \theta_{ij} \).

2.2. Inequality constraints

These are the set of constraints that represent the system operational and security limits like the bounds on the following:

1) generators real and reactive power outputs

\[
p_{\text{min}} \leq P_{Gi} \leq p_{\text{max}}, i = 1, \ldots, N
\]

\[
Q_{\text{min}} \leq Q_{Gi} \leq Q_{\text{max}}, i = 1, \ldots, N
\]

2) voltage magnitudes at each bus in the network

\[
V_{i}^{\text{min}} \leq V_{i} \leq V_{i}^{\text{max}}, i = 1, \ldots, NL
\]

where \( NL \) = number of load buses.

3) transformer tap settings

\[
T_{i}^{\text{min}} \leq T_{i} \leq T_{i}^{\text{max}}, i = 1, \ldots, NT
\]

4) reactive power injections due to capacitor banks

\[
Q_{\text{min}} \leq Q_{Gi} \leq Q_{\text{max}}, i = 1, \ldots, CS
\]

5) transmission lines loading

\[
S_{i} \leq S_{i}^{\text{max}}, i = 1, \ldots, nl
\]

where \( nl \) = number of transmission lines.
Since voltage instability occurs when the system attains low voltages at load buses, the voltage stability L-index [32] is incorporated as an inequality constraint.

6) voltage stability index

\[
L_{ij} \leq L_{ij}^{\max}, i = 1, \ldots, NL
\]  

(13)

2.3. Handling of constraints

There are varied ways to handle constraints in evolutionary computation optimization algorithms. In this paper, the constraints are incorporated into a fitness function by means of a penalty function method. In this method, a penalty factor multiplied with the square of the violated value of variable is added to the objective function and any infeasible solution obtained is rejected.

The extended objective function to handle the inequality constraints of state variables including load bus voltage magnitudes, output variables with real power generation output at slack bus, reactive power generation output, and line loading can be de

\[
OF = \sum_{i=1}^{N} F_i(P_{Gi}) + K_p h(P_{Gi}) + K_q \sum_{i=1}^{N} h(Q_{Gi}) + K_v \sum_{i=1}^{NL} h(|V_i|) + K_x \sum_{i=1}^{nl} h(|S_i|)
\]  

(14)

where \( K_p, K_q, K_v, \) and \( K_x \) are the penalty constants for the real power generation at slack bus, the reactive power generation of all generator buses or PV buses and slack bus, the voltage magnitude of all load buses or PQ buses, and line or transformer loading, respectively. \( h(P_{Gi}), h(Q_{Gi}), h(|V_i|), \) and \( h(|S_i|) \) are the penalty function of the real power generation at slack bus, the reactive power generation of all PV buses and slack bus, the voltage magnitudes of all PQ buses, and line or transformer loading, respectively. The penalty function can be defined as:

\[
h(x) = \begin{cases} 
(x - x_{\max})^2, & \text{if } x > x_{\max} \\
(x_{\min} - x)^2, & \text{if } x < x_{\min} \\
0, & \text{if } x_{\min} \leq x \leq x_{\max}
\end{cases}
\]  

(15)

where \( h(x) \) is the penalty function of variable \( x \), and \( x_{\max} \) and \( x_{\min} \) are the upper limit and lower limit of variable \( x \), respectively.

3. Overview of differential evolution

The DE algorithm is a population based algorithm like genetic algorithms using the similar operators; crossover, mutation and selection. The main difference in constructing better solutions is that genetic algorithms rely on crossover while DE relies on mutation operation. This main operation is based on the differences of randomly sampled pairs of solutions in the population. The algorithm uses mutation operation as a search mechanism and selection operations to direct the search toward the prospective regions in the search space. The DE algorithm also uses a non-uniform crossover that can take child vector parameter from one parent more often than it does from others. By using the components of the existing population members to construct trial vectors, the recombination (crossover) operator efficiently shuffles information about successful combinations, enabling the search for a better solution space [33].

An optimization task consisting of D parameters can be represented by a D-dimensional vector. In DE, a population of NP solution vectors is randomly created at the start. This population is successfully improved by applying mutation, crossover and selection operators.

The main DE algorithm is described as follows:

3.1. Initialization

The initial population of NP vectors is randomly selected based on uniform probability distribution for all variables to cover the entire search uniformly. Each individual \( X_i \) is a vector that contains as many parameters as the problem decision variables D. Random values are assigned to each decision parameter in every vector according to:

\[
x_{ij}^0 \sim U(x_{ij}^{\min}, x_{ij}^{\max})
\]  

(16)

where \( i = 1, \ldots, NP \) and \( j = 1, \ldots, D; x_{ij}^{\min} \) and \( x_{ij}^{\max} \) are the lower and upper bounds of the \( j \)th decision variable; \( U(x_{ij}^{\min}, x_{ij}^{\max}) \) denotes a uniform random variable ranging over \([x_{ij}^{\min}, x_{ij}^{\max}]\). The population size \( NP \) is the initial \( j \)th variable of \( j \)th population. All the vectors should satisfy the constraints.

3.2. Evaluation

Evaluate the fitness value of each individual (in this work, the goal is to minimize the cost function).

3.3. Mutation

For each target vector \( x_{ij} \), a mutant vector is produced by

\[
v_{ij,G+1} = x_{ij} + K \cdot (x_{ij,G} - x_{ij}) + F \cdot (x_{ij,G} - x_{ij,G})
\]  

(17)

where \( i, r_1, r_2, r_3 \in \{1, 2, \ldots, NP\} \) are randomly chosen and must be different from each other.

In Eq. (17), \( F \) is the scaling factor which has an effect on the difference vector \( x_{ij,G} - x_{ij,G} \), \( K \) is the combination factor.

3.4. Crossover

The parent vector is mixed with the mutated vector to produce a trial vector \( u_{ij,G+1} \)

\[
u_{ij,G+1} = \begin{cases} 
v_{ij,G+1} & \text{if } (r_d < CR) \text{ or } j = r_e \text{ or } r_d \\
u_{ij} & \text{if } (r_d > CR) \text{ or } j \neq r_e
\end{cases}
\]  

(18)

where \( j = 1, 2, \ldots, D; r_e \in [0, 1] \) is the random number; \( CR \) is crossover constant \( \epsilon [0,1] \) and \( r_e \in [1,2, \ldots,D] \) is the randomly chosen index.

3.5. Selection

All solutions in the population have the same chances of being selected as parents without dependence of their fitness value. The child produced after the mutation and crossover operations is evaluated. Then, the performance of the child vector and its parent is compared and the better one is selected. If the parent is still better, it is retained in the population.

Price and Storn gave the working principle of DE with simple strategy in [17]. Later on, they suggested ten different strategies of DE [34]. Strategy-7 (DE/rand/1/bin) is the most successful and widely used strategy. The key parameters of control in DE are population size \( NP \), scaling factor \( F \) and crossover constant \( CR \). However, the selection of mutation operator is another very important issue in the DE application. A proper mutation
operator can accelerate searching out the global solution [35–37].

In this paper, the promising optimization approach such as ACO is applied for the selection of best mutation operator with best set of ACO parameters evolved by a genetic algorithm [38].

4. Proposed algorithm for OPF by evolving ant direction differential evolution

The main idea of EADDE is to use the ant colony search system to find the proper mutation operator to accelerate the search for the global solution [39]. The optimal values of ant colony parameters are evolved by genetic algorithm [38]. The EADDE is discussed in the following:

Step 1: Initialization

The initial population is generated randomly and is given by the following equation.

\[ X_i^0 = X_{i_{\text{min}}} + \text{rand}() \cdot (X_{i_{\text{max}}} - X_{i_{\text{min}}}), \; i = 1, \ldots, N_p \]  

where \( \text{rand}() \) denotes a uniformly distributed random number within the range [0,1]. This produces \( N_p \) individuals of \( X_i^0 \) randomly. During the initialization, the control variables real power generations, generator voltages, transformer taps and shunt reactive power injections are randomly generated within the allowable ranges. The maximum and minimum values of generator voltages, transformer taps and shunt reactive power injections are given in Table 1.

The ant colony parameters such as, the relative importance of the pheromone trail \( \alpha \), relative importance of the visibility \( \beta \), evaporation factor \( \rho \), and scaling factor for the modification of the trail \( Q \) are randomly generated as binary strings to be subjected to GA search. These parameters are converted into the values within the limits given in Table 2 as mentioned in reference [38] and pheromone trail \( t \) is initialized heuristically to a small value.

Step 2: Run power flow and evaluate the fitness value of each individual.

Step 3: Evolving ant direction search

In every generation, each of \( N_p \) ants selects a mutation operator according to the heuristic information and pheromone information. In normal ant direction search, the \( \alpha, \beta, \rho \) and \( Q \) are fixed, whereas in this proposed method these parameters are evolved using genetic algorithm method so that ant direction search progresses optimally. Here the standard GA implementation is used for evolving which includes reproduction by Roulette-wheel selection, crossover and mutation procedures to find the optimal set of parameters for the ant colony search. The fluctuant pheromone quantity is constructed based on the difference between the objective function value in the next generation and the best objective function value in the present generation and is given by

\[ D_{t_i} = \begin{cases} \frac{Q}{K} & \text{if proper mutation operator is chosen, } i = 1, \ldots, N_p \\ 0 & \text{otherwise} \end{cases} \]  

In Eq. (20), the proper mutation operator is chosen when the objective value of the next generation is better than the best objective function value of the present generation, where \( Q \) is the scaling factor for the modification of the trail. The \( LK \) in Eq. (20) can be defined as the following:

\[ LK = \frac{object_{\text{new}}}{best_{\text{present}} - object_{\text{new}}} \]  

Table 1
Ranges of control variables.

<table>
<thead>
<tr>
<th>Variable (p.u.)</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generator voltage</td>
<td>0.9</td>
<td>1.1</td>
</tr>
<tr>
<td>Transformer tap</td>
<td>0.9</td>
<td>1.1</td>
</tr>
<tr>
<td>Shunt injection</td>
<td>0.0</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 2
Parameters evolved and their ranges.

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 0.0 \leq \alpha \leq 5.0 )</td>
</tr>
<tr>
<td>2</td>
<td>( 0.0 \leq \beta \leq 10.0 )</td>
</tr>
<tr>
<td>3</td>
<td>( 0.0 \leq \rho \leq 1.0 )</td>
</tr>
<tr>
<td>4</td>
<td>( 0.0 \leq Q \leq 100.0 )</td>
</tr>
</tbody>
</table>

Fig. 1. Flow chart of the proposed EADDE algorithm.
where \( \text{best}_{\text{present}} \) expresses the best objective function value of the present generation and \( \text{obj}_{\text{new}} \) expresses the objective function value of the next generation.

The pheromone updating employs the following rule:

\[
\tau_{\text{new}} = (1 - \rho)\tau_{\text{old}} + \rho \Delta \tau_i
\]

(22)

It is observed that the probability of selecting a mutation operator is proportional to the pheromone quantity \( \tau_i \) and the information \( \eta_i \). The information \( \eta_i \) is defined as follows:

\[
\eta_i = \left( \frac{n}{\sum_{j=1}^{n} \left( \frac{X^{G+1}_{ij} - X^G_{ij}}{X^G_i} \right)^2} \right)^{0.5}
\]

(23)

where \( X^{G+1}_{ij} \) denotes the \( j \)th gene of the \( i \)th individual in a population in the \((G+1)\)th generation, \( n \) is the number of units, and \( X^G_{ij} \) denotes the \( j \)th gene of the best individual in a population in the \( G \)th generation. Hence, the probability of choosing a mutation operator is defined as follows:

\[
P_i(t) = \frac{\tau_i(t) \eta_i^\beta}{\sum_{i=1}^{N_p} \tau_i(t) \eta_i^\beta} \quad \text{where} \quad \alpha \text{ and } \beta \text{ are parameters to regulate the influence of } \tau_i \text{ and } \eta_i \text{ respectively.}
\]

Step 4: Mutation operation

According to the probability of choosing a mutation operator, the procedure for selecting the mutation operators in every generation is given here. For the given five mutation operators, the probability from 0.0 to 1.0 is divided into five parts say (0.2 0.4 0.6 0.8 1.0). Five integers are generated randomly between 1 and 5 say (3 2 5 1 4). As per the probabilities calculated in step 3, if the probability of one ant is greater than 0.2 and less than or equal to 0.4, then operator 2 is selected and this process is continued for the remaining ants. The five mutation operators are taken from [33] and are given below:

\[
X^{G+1}_{i1} = X^G_{\text{best}} + F \left( X^G_{12} - X^G_{12} \right)
\]

(25)

\[
X^{G+1}_{i1} = X^G_{\text{best}} + F \left( X^G_{12} - X^G_{12} \right) + F \left( X^G_{13} - X^G_{14} \right)
\]

(26)

\[
X^{G+1}_{i1} = X^G_{\text{best}} + F \left( X^G_{12} - X^G_{12} \right) + F \left( X^G_{13} - X^G_{14} \right)
\]

(27)

\[
X^{G+1}_{i1} = X^G_{\text{best}} + F \left( X^G_{12} - X^G_{12} \right) + F \left( X^G_{13} - X^G_{14} \right)
\]

(28)

\[
X^{G+1}_{i1} = \left( X^G_{11} + X^G_{12} + X^G_{13} \right) / 3 + (p_2 - p_1) \left( X^G_{11} - X^G_{12} \right)
\]

\[
+ (p_3 - p_2) \left( X^G_{12} - X^G_{13} \right) + (p_1 - p_3) \left( X^G_{13} - X^G_{11} \right)
\]

(29)

where \( F \) is the mutation constant, \( r_1 \neq r_2 \neq r_3 \neq r_4 \neq r_5 \neq i \) are randomly chosen indices from the population, \( N_p \). Eq. (29) is the trigonometric mutation operator [37], and the values of \( p_a \) \( a = 1, 2, 3 \) are obtained by

---

Table 3

Generator cost coefficients in Case 1.

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>( P_c ) (MW)</th>
<th>Cost coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
</tr>
<tr>
<td>1</td>
<td>50</td>
<td>200</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>80</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>35</td>
</tr>
<tr>
<td>11</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>13</td>
<td>12</td>
<td>40</td>
</tr>
</tbody>
</table>

---

Table 4

Evolved optimal ACO parameters for Case 1 with \( F = 0.5 \) and \( CR = 0.8 \).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Population size</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>1.1487 0.1794 1.5138</td>
</tr>
<tr>
<td>( \beta )</td>
<td>7.1260 4.8800 0.5364</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.8508 0.6173 0.8499</td>
</tr>
<tr>
<td>( Q )</td>
<td>11.0140 12.8450 64.9821</td>
</tr>
</tbody>
</table>
where \( VD \) is the voltage stability index of all load buses, \( P \) is the loss, and \( \text{perturbed individual} \) is to extend the diversity of further individuals at the next generation, by generating child individuals out of existing individuals. In order to reinforce prior successes and progress the crossover operation to generate the offspring, the \( CR \) is chosen by a random number and a crossover constant (\( CR \)) to determine if the newly generated individual is to be recombined. Each parameter \( j \) of the \( i \)th individual is reproduced from the perturbed individual \( X_i^{G+1} \) and the present individual \( X_i^G \) as follows:

\[
P_1 = \frac{f(X_i^G)}{p}, \quad p_2 = \frac{f(X_j^G)}{p'}, \quad \text{and} \quad p_3 = \frac{f(X_k^G)}{p''},
\]

with \( p' = |f(X_i^G)| + |f(X_j^G)| + |f(X_k^G)| \), where \( f(X) \) is the function to be optimized.

**Step 5: Crossover operation**

Crossover or recombination is a complementary process for differential evolution. Crossover aims at reinforcing prior successes by generating child individuals out of existing individuals. In order to extend the diversity of further individuals at the next generation, the perturbed individual of \( X_i^{G+1} \) and the present individual of \( X_i^G \) are chosen by a random number and a crossover constant (\( CR \)) to progress the crossover operation to generate the offspring. The \( CR \) is used to determine if the newly generated individual is to be recombined. Each parameter \( j \) of the \( i \)th individual is reproduced from the perturbed individual \( X_i^{G+1} \) and the present individual \( X_i^G \) as follows:

\[
\begin{align*}
X_{ij}^{G+1} &= \begin{cases} 
X_{ij}^G, & \text{if } \text{rand}(0, 1) < CR \\
\frac{X_{ij}^G + X_{ij}^{G+1}}{2}, & \text{Otherwise}
\end{cases} \quad (30)
\end{align*}
\]

where \( i = 1, ..., N_p; \ j = 1, ..., n; \ n = \text{number of parameters}. \)

**Step 6: Estimation and selection**

The fitness of the offspring is in competition with its parent. The parent is replaced by its offspring if the fitness of the offspring is better than that of its parent. On the other hand, the parent is retained in the next generation if the the offspring is less fit than that of its parent. These two forms are presented as follows:

\[
\begin{align*}
X_{i}^{G+1} &= \arg \max \left\{ f(X_i^G), f(X_i^{G+1}) \right\} \quad (31) \\
X_{b}^{G+1} &= \arg \max \left\{ f(X_i^G) \right\} \quad (32)
\end{align*}
\]

**Fig. 4.** Convergence of DE and EADDE with population 50 for Case 1.

**Fig. 5.** Convergence of EADDE with population size of 50 for different values of \( F \) and CR for Case 1.

---

**Table 5**

Control parameters and fuel costs of EADDE for Case 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Quadratic cost under different population size ($/h)</th>
<th>F</th>
<th>CR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.1</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>803.1615</td>
<td>802.8291</td>
<td>800.7214</td>
</tr>
<tr>
<td></td>
<td>803.0296</td>
<td>802.4005</td>
<td>800.4681</td>
</tr>
<tr>
<td></td>
<td>800.9309</td>
<td>800.2885</td>
<td>800.2041</td>
</tr>
</tbody>
</table>

**Table 6**

Optimal settings of control variables for Case: 1 with \( F = 0.5, \ CR = 0.8 \).

<table>
<thead>
<tr>
<th>Variables (p.u.)</th>
<th>EADDE algorithm Population</th>
<th>DE algorithm Population</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>( P_{G1} )</td>
<td>1.7770</td>
<td>1.7670</td>
</tr>
<tr>
<td>( P_{G2} )</td>
<td>0.4857</td>
<td>0.4892</td>
</tr>
<tr>
<td>( P_{G3} )</td>
<td>0.2088</td>
<td>0.2116</td>
</tr>
<tr>
<td>( P_{G4} )</td>
<td>0.2136</td>
<td>0.2087</td>
</tr>
<tr>
<td>( P_{G5} )</td>
<td>0.1194</td>
<td>0.1179</td>
</tr>
<tr>
<td>( P_{G6} )</td>
<td>0.1208</td>
<td>0.1295</td>
</tr>
<tr>
<td>( V_1 )</td>
<td>1.0836</td>
<td>1.0867</td>
</tr>
<tr>
<td>( V_2 )</td>
<td>1.0623</td>
<td>1.0677</td>
</tr>
<tr>
<td>( V_3 )</td>
<td>1.0280</td>
<td>1.0362</td>
</tr>
<tr>
<td>( V_4 )</td>
<td>1.0251</td>
<td>1.0414</td>
</tr>
<tr>
<td>( V_5 )</td>
<td>1.0911</td>
<td>1.0407</td>
</tr>
<tr>
<td>( V_6 )</td>
<td>1.0655</td>
<td>1.0543</td>
</tr>
<tr>
<td>( T_1 )</td>
<td>1.0079</td>
<td>0.9646</td>
</tr>
<tr>
<td>( T_2 )</td>
<td>1.0253</td>
<td>1.0517</td>
</tr>
<tr>
<td>( T_3 )</td>
<td>1.0157</td>
<td>0.9946</td>
</tr>
<tr>
<td>( T_{16} )</td>
<td>0.9732</td>
<td>0.9828</td>
</tr>
<tr>
<td>( Q_{10} )</td>
<td>0.0537</td>
<td>0.0602</td>
</tr>
<tr>
<td>( Q_{12} )</td>
<td>0.0558</td>
<td>0.0325</td>
</tr>
<tr>
<td>( Q_{15} )</td>
<td>0.0859</td>
<td>0.0953</td>
</tr>
<tr>
<td>( Q_{17} )</td>
<td>0.0672</td>
<td>0.0747</td>
</tr>
<tr>
<td>( Q_{20} )</td>
<td>0.0001</td>
<td>0.0986</td>
</tr>
<tr>
<td>( Q_{21} )</td>
<td>0.0947</td>
<td>0.0160</td>
</tr>
<tr>
<td>( Q_{23} )</td>
<td>0.0082</td>
<td>0.0007</td>
</tr>
<tr>
<td>( Q_{24} )</td>
<td>0.0839</td>
<td>0.0967</td>
</tr>
<tr>
<td>( Q_{29} )</td>
<td>0.0270</td>
<td>0.0290</td>
</tr>
<tr>
<td>Fuel cost ($/h)</td>
<td>800.7214</td>
<td>800.4681</td>
</tr>
<tr>
<td>( VD ) (p.u.)</td>
<td>0.8532</td>
<td>0.9133</td>
</tr>
<tr>
<td>( L_j ) max</td>
<td>0.1254</td>
<td>0.1252</td>
</tr>
<tr>
<td>( P ) loss (p.u.)</td>
<td>0.0913</td>
<td>0.0890</td>
</tr>
</tbody>
</table>

where \( VD \) – Sum of absolute voltage deviation for load buses, \( L_j \) max – Maximum voltage stability index of all load buses, \( P \) loss – Transmission loss.
where arg max means the argument of the maximum. Here arg max is used because the fitness function, \( f = 1 / O F \), where OF is the extended objective function that is to be minimized.

Step 7: Repeat steps 2 to 6 until maximum generation quantity is reached.

The flow chart of the proposed algorithm is shown in Fig. 1. The power flow algorithm is applied for each candidate solution to evaluate its fitness and determine the state variables. The optimization procedure stops whenever a predetermined number of generations is reached.

5. Statistical analysis

In the statistical analysis, when observations, discrete or continuous, are available on a single characteristic of a large number of individuals, sometimes it will be useful to represent the entire data as far as possible by a single number without losing any information of interest. The statistical constants [40] comprehend in a single effort the significance of the whole. They give us an idea about concentration of the values in the central part of the distribution. The following are the five measures of central tendency that are in common use:

5.1. Arithmetic mean (MN)

Arithmetic mean of a set of observations is their sum divided by the number of observations, e.g., the arithmetic mean \( x \) of \( n \) observations \( x_1, x_2, \ldots, x_n \) is given by

\[
x = \frac{1}{n} (x_1 + x_2 + \ldots + x_n) = \frac{1}{n} \sum_{i=1}^{n} x_i \tag{33}
\]

In case of the frequency distribution \( x_i f_i, i = 1, 2, \ldots, n \) where \( f_i \) is the frequency of the variable \( x_i \),

\[
\bar{x} = \frac{f_1 x_1 + f_2 x_2 + \ldots + f_n x_n}{f_1 + f_2 + \ldots + f_n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \frac{1}{N} \sum_{i=1}^{n} f_i x_i, \quad \sum_{i=1}^{n} f_i = N \tag{34}
\]

In case of grouped or continuous frequency distribution, \( x \) is taken as the mid value of the corresponding class.

5.2. Median (MD)

Median of a distribution is the value of the variable, which divides it into two equal parts. It is the value, which exceeds and is exceeded by the same number of observations, i.e., it is the value such that the number of observations above it is equal to the number of observations below it. The median is thus a positional average.

In case of ungrouped data, if the number of observations is odd then median is the middle value after the values have been arranged in ascending or descending order of magnitude. In case of even number of observations, there are two middle terms and median is obtained by taking the arithmetic mean of the middle terms.

The steps for calculating median are given below.

### Table 7

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>( P_c ) (MW)</th>
<th>Cost coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
</tr>
<tr>
<td>1</td>
<td>50</td>
<td>140</td>
</tr>
<tr>
<td></td>
<td>140</td>
<td>200</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>55</td>
<td>80</td>
</tr>
</tbody>
</table>

### Table 8

<table>
<thead>
<tr>
<th>Parameters</th>
<th>For population size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30</td>
</tr>
<tr>
<td>( a )</td>
<td>2.9125</td>
</tr>
<tr>
<td>( \beta )</td>
<td>5.0428</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.8724</td>
</tr>
<tr>
<td>Q</td>
<td>81.6907</td>
</tr>
</tbody>
</table>
find \(N_i/2\), where \(N = \sum f_i\).

- See the cumulative frequency just greater than \(N_i/2\).
- The corresponding value of \(x_i\) median.

In the case of continuous frequency distribution, the class corresponding to the c.f. just greater than \(1/2N\) is called the median class and the value of median is obtained by the following formula.

\[
\text{Median} = l + \left(\frac{h}{f} \right) \left(\frac{N}{2} - c\right)
\]

(35)

where \(l\) is the lower limit of the median class, \(f\) is the frequency of the median class, \(h\) is the magnitude of the median class, \(c\) is c.f. of the class preceding the median class, and \(N = \sum f\).

5.3. Mode (MO)

Mode is the value, which occurs most frequently in a set of observations and around which the other items of the set cluster densely. In other words, mode is the value of the variable, which is predominant in the series. Thus, in the case of discrete frequency distribution, mode is the value of \(x\) corresponding to maximum frequency.

In case of continuous frequency distribution, mode is given by the formula.

\[
\text{Mode} = l + h \left(\frac{f_0}{f_1 - f_0} - f_2 - f_1\right) = l + \frac{h(f_1 - f_0)}{f_1 - f_0 - f_2}.
\]

(36)

Where \(l\) is the lower limit, \(h\) the magnitude and \(f_1\) the frequency of the modal class \(f_0\) and \(f_2\) are frequencies of the classes preceding and succeeding the modal class respectively.

5.4. Geometric mean (GM)

Geometric mean of a set of \(n\) observations is the nth root of their product. Thus the geometric mean \(G\) of \(n\) observations \(x_i; \quad i = 1, 2, \ldots n\) is given by

\[
G = (x_1 x_2 \ldots x_n)^{\frac{1}{n}}
\]

(37)

The computation is facilitated by use of logarithms. Taking logarithm on both sides

\[
\log G = \frac{1}{n} (\log x_1 + \log x_2 + \ldots + \log x_n) = \frac{1}{n} \sum_{i=1}^{n} \log x_i
\]

(38)

In case of frequency distribution \(x_i f_i, (i = 1, 2, \ldots n)\) geometric mean, \(G\) is
Control parameters and fuel costs of EADDE for Case: 3.

Table 13: Parameters for population size

<table>
<thead>
<tr>
<th>Parameters</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>2.8254</td>
<td>1.9839</td>
<td>4.4108</td>
</tr>
<tr>
<td>( \beta )</td>
<td>9.1154</td>
<td>6.4279</td>
<td>1.4095</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.3425</td>
<td>0.2722</td>
<td>0.9295</td>
</tr>
<tr>
<td>( Q )</td>
<td>39.3652</td>
<td>94.2504</td>
<td>63.7079</td>
</tr>
</tbody>
</table>

\[ G = \left( x_1 \cdot x_2 \cdots x_n \right)^{1/n}, \text{ where } N = \sum_{i=1}^{n} f_i \]  \hspace{1cm} \text{(39)}

Taking logarithms of both sides, we get

\[ \log G = \frac{1}{N} \left( f_1 \log x_1 + f_2 \log x_2 + \cdots + f_n \log x_n \right) = \frac{1}{N} \sum_{i=1}^{n} f_i \log x_i \]  \hspace{1cm} \text{(40)}

Thus we find that logarithm of geometric mean is the arithmetic mean of the logarithms of the given values. From (40), we get

\[ G = \text{Antilog} \left( \frac{1}{N} \sum_{i=1}^{n} f_i \log x_i \right) \]  \hspace{1cm} \text{(41)}

In the case of grouped or continuous frequency distribution, \( x \) is taken to be the value corresponding to the mid-point of the class intervals.

5.5. Harmonic mean (HM)

Harmonic mean of a number of observations, none of which is zero, is the reciprocal of the arithmetic mean of the reciprocal of the given values. Thus, harmonic mean (\( H \)) of \( n \) observations \( x_i, i = 1, 2, \ldots, n \) is given by

\[ H = \frac{1}{\frac{1}{n} \sum_{i=1}^{n} \left( \frac{1}{x_i} \right)} \]  \hspace{1cm} \text{(42)}

In case of frequency distribution \( x_i/f_i, (i=1,2,\ldots,n) \),

\[ H = \frac{1}{\frac{1}{N} \sum_{i=1}^{n} \left( \frac{1}{x_i} \right)} = \frac{N}{\sum_{i=1}^{n} f_i} \]  \hspace{1cm} \text{(43)}

If \( x_1, x_2, \ldots, x_n \) are \( n \) observations with weights \( w_1, w_2, \ldots, w_n \) respectively, their weighted harmonic mean is defined as

\[ H = \frac{\sum_{i=1}^{n} w_i}{\sum_{i=1}^{n} x_i} \]  \hspace{1cm} \text{(44)}

The measures of central tendency give us an idea of the concentration of the observations about the central part of the distribution. If we know the central tendency measure alone, we cannot form a complete idea about the distribution.

The measures of central tendency are inadequate to give us a complete idea of the distribution. They must be supported and supplemented by some other measures. One such measure is Dispersion. The literal meaning of dispersion is ‘scattered ness’. “The measure of scattered ness of the mass of figures in a series about an average is called the measure of variation or dispersion”.

Various measures of dispersion can be classified into two broad categories.

Table 14: Optimal settings of control variables for Case: 3.

<table>
<thead>
<tr>
<th>Variables (p.u.)</th>
<th>EADDE algorithm Population</th>
<th>DE algorithm Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{c1} )</td>
<td>1.9320 1.9652 1.9750</td>
<td>1.9437</td>
</tr>
<tr>
<td>( P_{c2} )</td>
<td>0.5206 0.5206 0.5205</td>
<td>0.5206</td>
</tr>
<tr>
<td>( P_{c5} )</td>
<td>0.1556 0.1500 0.1500</td>
<td>0.1567</td>
</tr>
<tr>
<td>( P_{c8} )</td>
<td>0.1020 0.1000 0.1000</td>
<td>0.1000</td>
</tr>
<tr>
<td>( P_{c11} )</td>
<td>0.1057 0.1000 0.1000</td>
<td>0.1001</td>
</tr>
<tr>
<td>( P_{c13} )</td>
<td>0.1221 0.1200 0.1200</td>
<td>0.1216</td>
</tr>
<tr>
<td>( V_{1} )</td>
<td>1.0778 1.0479 1.0460</td>
<td>1.1000</td>
</tr>
<tr>
<td>( V_{2} )</td>
<td>1.0659 1.0127 1.0301</td>
<td>1.0780</td>
</tr>
<tr>
<td>( V_{3} )</td>
<td>1.0551 0.9836 1.0155</td>
<td>1.0431</td>
</tr>
<tr>
<td>( V_{4} )</td>
<td>1.0424 0.9999 0.9614</td>
<td>1.0478</td>
</tr>
<tr>
<td>( V_{11} )</td>
<td>1.0440 1.0318 0.9502</td>
<td>1.1000</td>
</tr>
<tr>
<td>( V_{12} )</td>
<td>1.0401 1.0296 0.9775</td>
<td>0.9883</td>
</tr>
<tr>
<td>( T_{11} )</td>
<td>1.0003 1.0212 1.0274</td>
<td>1.1000</td>
</tr>
<tr>
<td>( T_{12} )</td>
<td>1.0558 0.9673 0.9590</td>
<td>1.0959</td>
</tr>
<tr>
<td>( T_{15} )</td>
<td>1.0445 0.9014 0.9278</td>
<td>0.9025</td>
</tr>
<tr>
<td>( T_{16} )</td>
<td>1.0372 1.0174 0.9947</td>
<td>0.9741</td>
</tr>
<tr>
<td>( Q_{0.1} )</td>
<td>0.0916 0.0722 0.0991</td>
<td>0.1000</td>
</tr>
<tr>
<td>( Q_{0.2} )</td>
<td>0.0080 0.0172 0.0431</td>
<td>0.1000</td>
</tr>
<tr>
<td>( Q_{0.5} )</td>
<td>0.0819 0.0307 0.0926</td>
<td>0.0963</td>
</tr>
<tr>
<td>( Q_{0.7} )</td>
<td>0.0644 0.0731 0.0927</td>
<td>0.0900</td>
</tr>
<tr>
<td>( Q_{0.9} )</td>
<td>0.0203 0.0017 0.1000</td>
<td>0.0998</td>
</tr>
<tr>
<td>( Q_{1.1} )</td>
<td>0.0940 0.0558 0.0633</td>
<td>0.1023</td>
</tr>
<tr>
<td>( Q_{1.2} )</td>
<td>0.0311 0.0281 0.0948</td>
<td>0.0997</td>
</tr>
<tr>
<td>( Q_{1.4} )</td>
<td>0.0377 0.0998 0.0107</td>
<td>0.0000</td>
</tr>
<tr>
<td>( Q_{0.07} )</td>
<td>0.0180 0.0999 0.0948</td>
<td>0.0000</td>
</tr>
<tr>
<td>( Q_{0.15} )</td>
<td>0.0916 0.0722 0.0991</td>
<td>0.1000</td>
</tr>
<tr>
<td>Objective($/h)</td>
<td>938.9885 931.2328 930.7459</td>
<td>934.6909</td>
</tr>
<tr>
<td>( V_{D} )</td>
<td>0.5323 0.2813 0.5927</td>
<td>0.9076</td>
</tr>
<tr>
<td>( Lj ) max</td>
<td>0.1367 0.1375 0.1420</td>
<td>0.1276</td>
</tr>
<tr>
<td>P loss(p.u)</td>
<td>0.1077 0.1218 0.1315</td>
<td>0.1086</td>
</tr>
</tbody>
</table>

Fig. 9. Convergence of DE and EADDE with population size of 50 for Case 3.
The measures, which express the spread of observations in terms of distance between the values of selected observations. These are also termed as distance measures.

The measures, which express the spread of observations in terms of the average of deviations of observations from some central value, e.g., mean deviation and standard deviation.

5.6. Quartile deviation (QD)

Quartile deviation or semi-interquartile range $Q$ is given by

$$Q = \frac{1}{2}(Q_3 - Q_1)$$

(45)

where $Q_1$ and $Q_2$ are the first and third quartiles of the distribution respectively.

5.7. Mean deviation (MEDEV)

If $x_if_i, i = 1, 2, ..., n$ is the frequency distribution, then mean deviation from the average $A$ (usually mean, median or mode) is given by

$$\text{Mean deviation from average } A = \frac{1}{N} \sum_{i=1}^{n} f_i |x_i - A|, \quad \sum_i f_i = N$$

(46)

Table 15
Comparison of fuel costs.

<table>
<thead>
<tr>
<th>Method</th>
<th>Fuel cost ($/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1</td>
</tr>
<tr>
<td>EP [43]</td>
<td>802.907</td>
</tr>
<tr>
<td>TS [43]</td>
<td>802.502</td>
</tr>
<tr>
<td>TS/SA[43]</td>
<td>804.5560</td>
</tr>
<tr>
<td>TS/SA[43]</td>
<td>802.7880</td>
</tr>
<tr>
<td>TS/SA[43]</td>
<td>800.5842</td>
</tr>
<tr>
<td>Alsac-Stott [41]</td>
<td>802.4000</td>
</tr>
<tr>
<td>OPF</td>
<td>802.4650</td>
</tr>
<tr>
<td>SDE-ALM [46]</td>
<td>802.4040</td>
</tr>
<tr>
<td>OPPPSO [13]</td>
<td>804.5830</td>
</tr>
<tr>
<td>OPF/OPF [8]</td>
<td>804.5830</td>
</tr>
<tr>
<td>GPM [47]</td>
<td>804.5830</td>
</tr>
<tr>
<td>Alsac-Stott [41]</td>
<td>802.4000</td>
</tr>
<tr>
<td>DE</td>
<td>800.2041</td>
</tr>
</tbody>
</table>

Table 16
Statistical measures evaluated using voltage stability indices of OPF global solution.

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measures of central tendency</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Arithmetic Mean (MN)</td>
<td>0.0675</td>
<td>0.0692</td>
<td>0.0692</td>
</tr>
<tr>
<td>Median (MD)</td>
<td>0.0743</td>
<td>0.0800</td>
<td>0.0800</td>
</tr>
<tr>
<td>Mode (MO)</td>
<td>0.0800</td>
<td>0.0873</td>
<td>0.0873</td>
</tr>
<tr>
<td>Geometric Mean (GM)</td>
<td>0.0297</td>
<td>0.0304</td>
<td>0.0304</td>
</tr>
<tr>
<td>Harmonic Mean (HM)</td>
<td>0.0348</td>
<td>0.0351</td>
<td>0.0351</td>
</tr>
<tr>
<td>Measures of dispersion</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quartile Deviation (QD)</td>
<td>0.0257</td>
<td>0.0267</td>
<td>0.0267</td>
</tr>
<tr>
<td>Mean Deviation (MEDEV)</td>
<td>0.0269</td>
<td>0.0278</td>
<td>0.0278</td>
</tr>
<tr>
<td>Standard Deviation (SD)</td>
<td>0.0370</td>
<td>0.0377</td>
<td>0.0377</td>
</tr>
<tr>
<td>Coefficient of Variation (CV)</td>
<td>54.8848</td>
<td>54.4653</td>
<td>54.4653</td>
</tr>
<tr>
<td>Coefficient of Skewness (SK)</td>
<td>-0.3374</td>
<td>-0.4806</td>
<td>-0.4806</td>
</tr>
<tr>
<td>Beta2 ($\mu_2$)</td>
<td>2.2727</td>
<td>2.2287</td>
<td>2.2287</td>
</tr>
<tr>
<td>Kurtosis (Type of curve)</td>
<td>Platykurtic</td>
<td>Platykurtic</td>
<td>Platykurtic</td>
</tr>
</tbody>
</table>

Where $|x_i - A|$ represents modulus or the absolute value of the deviation $(x_i - A)$, where the negative sign is ignored.

5.8. Standard deviation (SD)

Standard deviation, usually denoted by the Greek letter small sigma ($\sigma$), is the positive square root of the arithmetic mean of the squares of the deviations of the given values from their arithmetic mean. For the frequency distribution $x_if_i, i = 1, 2, ..., n$

$$\sigma = \sqrt{\frac{1}{N} \sum_i f_i (x_i - \bar{x})^2},$$

(47)

where $\bar{x}$ is arithmetic mean of the distribution and $\sum_i f_i = N$.

The step of squaring the deviations $(x_i - \bar{x})$ overcomes the drawback of ignoring the signs in mean deviation. Standard deviation is also suitable for further mathematical treatment. Moreover, of all the measures standard deviation is affected least by fluctuations of sampling.

5.9. Coefficient of dispersion (CD)

Whenever we want to compare the variability of the two series which differ widely in their averages or which are measured in different units, we do not mere calculate the measures of dispersion but we calculate the coefficients of dispersion which are pure numbers independent of the units of measurement. The coefficients of dispersion (CD) based on standard deviation is
where $M$ is the mean, $M_b$ is the mode and $\sigma$ is the standard deviation of the distribution.

### 5.11. Skewness (SK)

Literally skewness means lack of symmetry. We study skewness to have an idea about the shape of the curve which we can draw with the help of the given data. A distribution is said to be skewed if

- Mean, Median and Mode fall at different points, i.e., mean $\neq$ median $\neq$ mode.
- Quartiles are not equivalent from median and mode.
- The curve drawn with the help of the given data is not symmetrical but stretched more to one side than to the other.

#### 5.11.1. Measures of skewness

The coefficient of skewness is given by

$$S_k = \frac{(M - M_0)}{\sigma} \quad (50)$$

where $M$ is the mean, $M_0$ is the mode and $\sigma$ is the standard deviation of the distribution.

#### 5.12. Kurtosis

If we know the measures of central tendency, dispersion and skewness, we cannot form a complete idea about the distribution as will be clear from the following figure in which all three curves A, B and C are symmetrical about the mean and have the same range.

In addition to these measures, we should know one more measure which Prof. Karl Pearson calls as the ‘Convexity of Frequency Curve’ or Kurtosis. Kurtosis enables us to have an idea about the ‘flatness or peakedness’ of the frequency curve. It is measured by coefficient $\beta_2$ or its derivation $\gamma_2$ is given by:

$$\beta_2 = \frac{\mu_4}{\mu_2^2}, \quad \gamma_2 = \beta_2 - 3 \quad (51)$$

Curve of the type ‘A’ which is neither flat nor peaked is called the normal curve or mesokurtic curve and for such a curve $\beta_2 = 3$, i.e., $\gamma_2 = 0$. Curve of the type ‘B’ which is flatter than the normal curve is known as platykurtic and for such a curve $\beta_2 < 3$, i.e., $\gamma_2 < 0$. Curve of the type ‘C’ which is more peaked than the normal curve is called Leptokurtic and for such a curve $\beta_2 > 3$, i.e., $\gamma_2 > 0$. The $\mu_2$ and $\mu_4$ represents the Sheppard’s corrections. The frequency curves for these three cases are shown in Fig. 2.

It is often useful to represent a frequency distribution by means of a diagram which makes the unwieldy data intelligible and conveys to the eye the general run of the observations. Diagrammatic representation also facilitates the comparison of two or more frequency distributions. In this paper histograms are plotted using the load bus voltage profiles and voltage stability indices based on predefined intervals.

### 6. Simulation results and discussion

The proposed algorithm was implemented in MATLAB computing environment with Pentium-IV, 2.66 GHz computer with 512 MB RAM. The standard IEEE 30-bus test system was used to test effectiveness of the EADDE approach. The test system consists of six generating units interconnected with 41 branches of a transmission network with a total load of 283.4 MW and 126.2 Mvar as shown in Fig. 3. The bus data and the branch data are taken from the ref. [41]. The original system has two capacitor banks installed at bus 5 and 24 with ratings of 19 and 4 MVAR respectively. These capacitor banks are not considered in this work, rather the shunt injections are provided at buses 10, 12, 15, 17, 20, 21, 23, 24 and 29 as given in the ref. [13]. In this case study, bus 1 is considered as the swing bus.

The cost coefficients and maximum and minimum limits of real power generations with three different types of generator cost curves are given in Tables 3, 7 and 11 respectively. The maximum and minimum values for the generator voltage and tap changing transformer control variables are 1.1 and 0.9 in per unit respectively. The maximum and minimum voltages for the load buses are...
considered to be 1.05 and 0.95 in per unit. The line flow limits are taken from the ref. [41].

In the normal ant colony search procedure the parameters $\alpha$, $\beta$, $\rho$, $Q$ are fixed. Generally, it is not easy to find optimal set of parameters for ant colony search. This is to be done heuristically through trial and error procedure. In this paper, these parameters are not fixed, but are evolved using genetic algorithm [38] so that better solution of OPF can be found with the best mutation operator using the optimal parameters of an ant colony search.

In this simulation study, minimization of different objectives under different cases is considered to test the performance of the proposed algorithm. The objective functions taken into consideration are quadratic fuel cost, piece-wise quadratic fuel cost, and quadratic cost model with valve-point loading effects. All the objective functions are augmented with the equality, inequality, and voltage stability constraints.

In each case study, two sets of 20 test runs for solving the OPF problem, were performed; the first set (DE-OPF) is based on the classical differential evolution algorithm and the second one (EADDE-OPF) is based on the evolving ant direction differential evolution algorithm. Moreover, various values of $F$ and $CR$ are also considered for simulations with different population sizes. The values of $F$ and $CR$, population sizes, maximum number of generations and penalty factors for all the three cases are given Table A.1. Note also that all control and state variables remained within their permissible limits for all the three cases.

Currently, the interconnected power systems are being operated under stressed conditions which impose a threat to the voltage stability due to low voltages [42]. Hence, the voltage stability index [32] is incorporated as an inequality constraint in the OPF problem.

6.1. Case 1: the OPF with quadratic cost curve model

In this case the fuel cost characteristics for all generating units are modeled by quadratic functions given by

$$f_i = a_i + b_i P_{G,i} + c_i P_{G,i}^2$$  \hspace{1cm} (52)

where $a_i, b_i$ and $c_i$ are the cost coefficients of the $i$th generating unit.

The extended objective function incorporating the constraints is given by:

$$OF = \sum_{i=1}^{N} \left( a_i + b_i P_{G,i} + c_i P_{G,i}^2 \right) + K_p \left( P_{G1} - P_{G1}^{lim} \right)^2 + K_v \left( V_i - V_i^{lim} \right)^2 + K_q \sum_{i=1}^{N} \left( Q_{G,i} - Q_{G,i}^{lim} \right)^2 + K_s \sum_{i=1}^{NL} \left( S_i - S_i^{lim} \right)^2 + K_l \sum_{j=1}^{NL} \left( L_j - L_j^{lim} \right)^2$$  \hspace{1cm} (53)
where $k_p$, $K_v$, $K_q$, and $K_L$ are the penalty factors, $NL$ is the number of load buses, $nl$ is number of transmission lines and $x_{lim}$ is the limit value of the dependent variable $x$ given as

$$x_{\lim} = \begin{cases} x^{\text{max}}, & x > x^{\text{max}} \\ x^{\text{min}}, & x < x^{\text{min}} \end{cases}$$ (54)

The minimum and maximum limits of real power generations and their cost coefficients are taken from [43] and are given in Table 3.

Table 4 gives the optimal set of evolved ant colony parameters. OPF results with quadratic fuel cost curves (case 1) for the various EADDE control parameters and population sizes are given in Table 5. Table 6 illustrates the best control variables of OPF solution with $F = 0.5$ and $CR = 0.8$ for different population sizes is depicted in Fig. 6. It is quite evident that EADDE gives the better results compared to DE.

6.2. Case 2: the OPF with piecewise quadratic cost curve model

The fuel cost characteristics for the generating units connected at bus 1 and 2 are now represented by a piecewise quadratic function to model different fuels given by

$$f_i(P_i) = \begin{cases} a_{i1} + b_{i1}P_{Gi} + c_{i1}P_{Gi}^2, & \text{fuel 1, } P_{Gi}^{\text{min}} \leq P_{Gi} \leq P_{Gi}^{\text{max}} \\ a_{i2} + b_{i2}P_{Gi} + c_{i2}P_{Gi}^2, & \text{fuel 2, } P_{Gi}^{\text{min}} \leq P_{Gi} \leq P_{Gi}^{\text{max}} \end{cases}$$ (55)

where $a_{im}$, $b_{im}$ and $c_{im}$ are cost coefficients of $i$th generating unit with fuel type $m$.

The cost coefficients for these two generating units are taken from ref. [43] and are given in Table 7.
Table 8 gives the optimal set of evolved ant colony parameters for case 2. OPF results with piecewise quadratic fuel cost (case 2) for the various EADDE control parameters and population sizes are given in Table 9. The best control variables of the OPF solution (case 2) with $F = 0.5$ and $CR = 0.8$ for population size 50 for a classical DE simulation and for population sizes 30, 40 and 50 using the EADDE algorithm are given in Table 10. The optimal fuel cost obtained by EADDE for case 2 is 629.7223 $/h, whereas for classical differential evolution the fuel cost is 630.8928 $/h. Note also that all control and state variables remained within their permissible limits. Fig. 7 shows the comparison of convergence characteristics of fuel cost of both the classical DE and EADDE. Fig. 8 depicts the convergence characteristics of fuel cost of the EADDE for different control parameter settings for a population size of 50. It is quite clear that EADDE gives better results compared to DE in case 2 also.

6.3. Case 3: the OPF with quadratic cost curve with valve point loadings

In this case a sine component is added to the cost curves of the generating units at bus 1 and 2 to reflect the valve point loading effects given by [44,45]:

$$f_i = a_i + b_i P_{G,i} + c_i P_{G,i}^2 + d_i \sin(e_i (p_{G,i}^{\text{min}} - P_{G,i}))$$  \hspace{1cm} (56)

where $a_i, b_i, c_i, d_i, e_i$ are the cost coefficients of the $i$th generating unit.

The fuel cost function is augmented with the constraints and is given by:

$$OF = \sum_{i=1}^{N} \left( a_i + b_i P_{G,i} + c_i P_{G,i}^2 + d_i \sin(e_i (p_{G,i}^{\text{min}} - P_{G,i})) \right) + k_p \left( P_{G1} - P_{G1}^{\text{lim}} \right) + k_v \sum_{i=1}^{NL} \left( V_i - V_i^{\text{lim}} \right)^2 + k_q \sum_{i=1}^{NG} \left( Q_{G,i} - Q_{G,i}^{\text{lim}} \right)^2 + k_s \sum_{i=1}^{nl} \left( S_i - S_i^{\text{lim}} \right)^2 + k_l \sum_{i=1}^{NL} \left( L_j - L_j^{\text{lim}} \right)^2$$  \hspace{1cm} (57)
permissible limits. Fig. 9 shows the comparison of convergence also that all control and state variables remained within their classical differential evolution the fuel cost is 934.6909$/h. Note obtained by EADDE algorithm for case 2 is 930.7459$/h, whereas for Fig. 10 depicts the convergence characteristics of the fuel cost of both the classical DE and EADDE. characteristics of fuel cost of both the classical DE and EADDE.

\[ f = \frac{1}{OF} \]  

(58)

where \( OF \) is the augmented objective function given in the above three cases.

Table 12 gives the optimal set of the evolved ant colony parameters for case 3. OPF results with quadratic fuel cost with valve point loading (case 3) for the various EADDE control parameters and population sizes are given in Table 13. Table 14 illustrates the best control variables of OPF solution (case 3) with \( F = 0.5 \) and \( CR = 0.8 \) for population size 50 for a classical DE and for population sizes 30, 40 and 50 for the EADDE algorithm. The optimal fuel cost obtained by EADDE algorithm for case 2 is 930.7459$/h, whereas for classical differential evolution the fuel cost is 934.6909$/h. Note also that all control and state variables remained within their permissible limits. Fig. 9 shows the comparison of convergence characteristics of fuel cost of both the classical DE and EADDE. Fig. 10 depicts the convergence characteristics of the fuel cost of EADDE for \( F = 0.5 \) and \( CR = 0.8 \) for different population sizes. It is quite clear that EADDE gives the better results compared to DE in case 3 also.

The most important observation from these simulations is that the optimal settings for \( F \) and \( CR \) are found to be 0.5 and 0.8 respectively. Table 15 gives the comparison of the proposed EADDE approach with a classical DE and other methods reported in the literature. Results clearly show the superiority of the EADDE algorithm over other methods. Fig. 11 shows the statistical evaluation of the mutation operator selection in all the three cases.

6.4. Comparison based on statistical analysis

The statistical central tendency measures and dispersion measures are evaluated using load bus voltages and voltage stability indices of the global solution of OPF and are given in Tables 16 and 17. The intervals considered for evaluating the statistical measures for both load bus voltages and voltage stability indices are given in the histograms shown in Figs. 12 and 13. Figs. 14–25 show the variation of statistical measures of best solution of OPF in each generation using both load voltage profiles and voltage stability indices.

Note: Coefficient of variation (CV) for voltage stability index is divided by 2000 and for voltage profile it is divided by 200 for the convenience of plotting figure. Coefficient of skewness (SK) is divided with 10 for both voltage stability indices and voltage profiles.

The histograms shown in Figs. 12 and 13 are constructed for load bus voltage profiles and voltage stability indices (L-indices) of the global best solutions of EADDE and classical DE methods. The Figs. 14–25 show the variation of both central tendency measures and dispersion measures of the best solution of EADDE method in each generation for all the three cases. From the Figs. 14, 18, 22, it can be observed that the values of central tendency measures, which are evaluated using voltage stability indices, are decreasing with the progress of iterations indicating the improvement of voltage stability, even though there are oscillations at the beginning. From the Figs. 15 and 19, and 23, it can be seen that the dispersion of L-indices is small and is decreasing with the increase in the generation number. From the Figs. 16 and 20, and 24, it can be observed that the central tendency measures, which are evaluated using load bus voltage profiles, are increasing with the increase in the generation indicating the improvement of voltage profiles. Also they are confined to the permissible limits as shown in histograms.

From the histograms it can be observed that the number of load bus voltages at the boundary points i.e. 0.95 and 1.05 are very less in number and majority of the voltages are in between 1.0 and 1.045 for EADDE indicating that proposed approach gives better voltage profiles and voltage stability. The Figs. 17, 21, 25, representing dispersion measures evaluated using voltage profiles, shows the improvement in bus voltage profiles with the increase in the generation/iteration number.

From the Table 17, it is also observed that the central tendency measures that are evaluated using voltage profiles in case 3 are more sensitive to voltages when compared to case 1 and 2.

7. Conclusion

This paper presents a novel approach for solving the OPF problem with non-smooth and non-convex generator fuel cost curves with the different inequality constraints through an evolving ant direction differential evolution (EADDE) algorithm. The EADDE algorithm effectively solves the OPF problem in all the three cases namely quadratic fuel cost, piecewise quadratic cost...
and quadratic cost with valve point loading. The robustness of the EA/DDDE algorithm has been demonstrated for the different control parameter settings and for different population sizes. The results clearly indicate that the algorithm has been successfully implemented for the task of real-world problems. The comparison shows that EA/DDDE outperforms other heuristic techniques when its parameters are properly tuned. Simulation results show that the EA/DDDE is superior to the original DE algorithm with regard to the convergence to the global optimum.

The proposed approach has been successfully and effectively implemented to find the optimal settings of the control variables of the IEEE 30 –bus test system. The comparison of the results using the proposed approach to those reported in the literature confirms its effectiveness and superiority to find the significant global solutions without any restrictions on the type of fuel cost curves. Also an innovative statistical analysis based on central tendency measures and dispersion measures was carried out on the bus voltage profiles and voltage stability indices.

### Appendix

#### Table A.1. Control parameter settings of EA/DDDE algorithm for IEEE 30 Bus System

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Setting Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size (Np)</td>
<td>30, 40, 50</td>
</tr>
<tr>
<td>Mutation Constant (F)</td>
<td>0.1, 0.3, 0.5</td>
</tr>
<tr>
<td>Crossover constant (CR)</td>
<td>0.4, 0.6, 0.8</td>
</tr>
<tr>
<td>Maximum number of generations (Camax)</td>
<td>250</td>
</tr>
<tr>
<td>Penalty factor of slack bus real power (Pe)</td>
<td>1000</td>
</tr>
<tr>
<td>Penalty factor of reactive power (Kc)</td>
<td>1000</td>
</tr>
<tr>
<td>Penalty factor of voltage magnitudes (Kv)</td>
<td>10000</td>
</tr>
<tr>
<td>Penalty factor of transmission line loadings (Kl)</td>
<td>10000</td>
</tr>
<tr>
<td>Penalty factor of voltage stability index (Ks)</td>
<td>10000</td>
</tr>
</tbody>
</table>

### References


Dr. K. Vaisakh received the B.E. degree in electrical engineering from Osmania University, Hyderabad, India in 1994, M.Tech degree from JNT University, Hyderabad, India in 1999, and Ph.D. degree in electrical engineering from the Indian Institute of Science, Bangalore, India in the year 2005. Currently, he is working as a Professor in the department of electrical engineering, AU College of Engineering, Andhra University, Visakhapatnam, AP, India. His research interests include optimal operation of power system, voltage stability, FACTS, power electronic drives and power system dynamics.

L. R. Srinivas received the B.Tech degree in electrical and electronics engineering from JNT University, Hyderabad, India in 1992, M.Tech degree from JNT University, Hyderabad, India in 2005. He is pursuing Ph.D. in the department electrical engineering, AU College of engineering, Andhra University. He is currently working as a Associate Professor in the department of electrical and electronics engineering, SRKR engineering college, Bhamavaram, W.G. Dist, A.P, India. His research interests include power system operation and control, power system analysis, power system optimization, soft computing applications.