Depolarization of Propagating Signals by Narrowband Ricean Fading Channels

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Abstract— In many cases of practical interest, the angle of arrival distribution at the receiver is sufficiently narrow that one can use knowledge of the mean received signal levels, Ricean K-factors and the cross-correlation coefficient that characterize fading signals observed on orthogonally polarized diversity branches to predict the first-order statistics of polarization state dispersion. This allows one to use simple power-only measurements of narrowband polarization diversity to predict the performance of alternative polarization diversity schemes or polarization adaptive antennas in realistic environments. Moreover, our results offer a useful geometric interpretation of how decorrelation between polarization diversity branches arises: As the angular spread of the polarization state distribution on the Poincaré sphere broadens, the correlation between branches decreases. However, knowledge of the angular spread alone is not sufficient to predict the cross-correlation coefficient; the Ricean K-factors on the branches must be known as well. Otherwise, the analogy to a similar relationship between the angle of arrival distribution and correlation between branches in space diversity is striking.

Keywords— channel model, fading channel, polarization diversity.

I. INTRODUCTION

Polarization diversity is a particularly attractive way to realize decorrelated diversity receiving branches when space diversity is either unwieldy or ineffective due to the narrowness of the angle of arrival distribution, e.g., at cellular base stations [1][2][3] and on earth-satellite links [4][5]. To date, however, most researchers have used the fading statistics and correlation properties of the signals that appear on each branch as the principal metric for characterizing such polarization diversity channels, e.g., [6]. While such a description is both adequate and appropriate for many purposes, a more complete description would capture the manner in which the polarization state of the received signal varies over time. In particular, knowledge of the first-order statistics of polarization state dispersion would facilitate comparison of alternative polarization diversity antenna configurations or the design and assessment of the performance of polarization adaptive antennas in realistic environments, e.g., [7][8]. Such knowledge would also potentially offer insights into the physical nature of polarization diversity itself.

In optics and radio astronomy, randomly polarized signals are commonly represented as the combination of an unpolarized component comprising a uniform distribution of polarization states over the entire Poincaré sphere and a completely polarized component comprising a single polarization state, e.g., [9]-[11]. Such a representation forms the basis for the Stokes vector representation of partially polarized waves and is particularly well-suited to describing noise-like signals that have been averaged over time or location. At first glance, this description seems to capture the essence of a Ricean channel – a strong fixed component combined with a noise-like scattered component. However, as the results that we present here show, an actual Ricean channel behaves quite differently.

Consider a narrowband wireless communications link over a mobility, fixed wireless or Earth-space path where fading on the diversity receiving branches is well-described by Ricean statistics. In most practical cases, the receiving antennas are chosen to be both symmetrically polarized (equidistant from the polarization state of the transmitting antenna in order to ensure equal mean received signal levels on both branches) and orthogonally polarized (in order to minimize correlation between branches). This constrains the polarization states of the receiving antennas to opposite points on the great circle that separates the co-polar and cross-polar hemispheres of the Poincaré sphere.

In the absence of fading, the polarization state of the signal that is observed at the receiver will describe a single point on the Poincaré sphere. As the depth of fading increases and as the correlation between the signals observed on the diversity receiving branches decreases, the polarization states observed by the receiver will begin to disperse across the Poincaré sphere. In the limit as the fading distribution on the branches becomes uncorrelated Rayleigh, i.e., the Ricean K-factor drops to zero, and the cross-correlation coefficient $\rho$ drops to zero, the polarization states will become uniformly distributed across the sphere. However, the specific details of the manner in which polarization states disperse for intermediate values of $K$ and $\rho$ are not as apparent.

In this work, our objective is to determine the form of the distribution that best describes the manner in which polarization states disperse across the Poincaré sphere over time, i.e., the first-order statistics of dispersion, and how the
parameters of this distribution are related to the Ricean K-factors and the cross-correlation coefficient that describe the diversity channel. The notion that it may be useful to consider the manner in which physical processes cause polarization states to disperse across the Poincaré sphere has been considered in applications as diverse as wireless propagation [12] and radio astronomy [13]. To the best of our knowledge, however, we are the first to consider how the distribution of polarization states is related to the fading statistics and cross-correlation coefficient between orthogonally polarized diversity branches.

The remainder of this paper is organized as follows: In Section II, we describe the concept in greater detail and explain our methodology. In Section III, we show how dispersion of polarization states over time is related to the conventional narrowband channel parameters. In Section IV, we summarize our contributions and their implications.

II. CONCEPT AND METHODOLOGY

We have previously shown that the first-order statistics of the Ricean fading signals \( g_1(t) \) and \( g_2(t) \) observed on diversity receiving branches are completely described by the average path gains \( G \) and Ricean K-factors \( K \) observed on each branch and the cross-correlation coefficient \( \rho \) between the time varying components of the signals on each receiving branch [6]. In such cases,

\[
g_1(t) = \sqrt{\frac{G}{K_1 + 1}} \left[ \sqrt{K_1} + x_1(t) \right] \tag{1}
\]

and

\[
g_2(t) = \sqrt{\frac{G}{K_2 + 1}} \left[ \sqrt{K_2} + x_2(t) e^{i\Delta \Psi} \right] \tag{2}
\]

where \( x_1(t) \) and \( x_2(t) \) are zero-mean complex Gaussian processes with unit standard deviation and \( \Delta \Psi \) is the phase offset between \( x_1(t) \) and \( x_2(t) \).

We denote the correlation between the complex Gaussian variates \( x_1 \) and \( x_2 \) by

\[
\rho = \frac{\bar{x}_1 x_2^*}{\mu_1 \mu_2} = \cos \theta \tag{3}
\]

If the phase of the fixed component is assumed to be zero, the power correlation coefficient \( \rho_{pwr} \) between the envelopes of \( g_1 \) and \( g_2 \) is related to \( \rho \), the correlation between the complex Gaussian variates \( x_1 \) and \( x_2 \), by

\[
\rho_{pwr} = \frac{\rho^2 + 2 \mu_1 \sqrt{K_1 K_2}}{\sqrt{(1 + 2K_1)(1 + 2K_2)}} \tag{4}
\]

where

\[
\mu_1 = \rho \cos \theta \tag{5}
\]

as described in [14][15]. Cross-correlated Ricean fading signals of this sort can be readily simulated using techniques such as those described in [16]. In the remainder of this paper, we use \( \rho_{pwr} \) to indicate the degree of cross-correlation between diversity branches.

When the angle of arrival distribution is sufficiently narrow that the antenna pattern appears to be constant over the range, e.g., as one might observe at a base station in a macrocell environment or at the satellite end of an earth-space link, the polarization state of the signal observed at the receiver at a given instant can be determined from knowledge of the amplitude and phase of the signals received on orthogonally polarized receiving branches at that instant. The arctangent of the ratio of the signal amplitudes gives the polarization angle \( \gamma \) while the difference between the signal phases gives the polarimetric phase \( \delta \). Trigonometric relations that relate the polarimetric angle and phase to the ellipticity angle \( \varepsilon \) and tilt angle \( \tau \) that define the polarization ellipse can be derived using spherical trigonometry, as described in [9]-[11].

Because the amplitude and phase distributions of a Ricean distributed signal do not lend themselves to the derivation of closed form expressions for the polarization angle and polarimetric phase distributions, we have opted to use numerical simulations to determine the manner in which the polarization states of the received signal disperse across the Poincaré sphere over time.

As noted earlier, polarization diversity receiving antennas are usually chosen to be both symmetrically and orthogonally polarized. For example, when the transmitted signal is vertically polarized, we might use forward and back slanted diagonally polarized receiving antennas. The inherent symmetry between the two branches suggests that the fading statistics on the two channels might reasonably be assumed to be identical, i.e., identical path gains \( G \) and Ricean K-factors \( K \). We refer to this as an ideal Ricean diversity channel. It is convenient to assume such a channel because it reduces the number of independent first-order channel model parameters to just three: \( G \), \( K \), and \( \rho_{pwr} \) while still being physically plausible.

Because the average path gains \( G \) on the two branches are identical, they cancel out when we calculate the polarization ratio and do not affect the polarization state. For each instance of \( K \) and \( \rho_{pwr} \) that we considered, we generated almost ten thousand values of \( g_1 \) and \( g_2 \) using a cross-correlated Ricean channel simulator based upon the techniques described in [16]. From the ratio \( g_1/g_2 \) at a given instant, we determined the polarization angle \( \gamma \) and the polarimetric phase \( \delta \), or, alternatively, the ellipticity and tilt angles \( \varepsilon \) and \( \tau \), that define the polarization state of the received signal. Finally, we plotted the polarization states on a Poincaré sphere, as suggested by Fig. 1, where the longitude coordinate \( \phi' = 2\tau \) and the colatitude coordinate \( \theta' = 90 - 2\varepsilon \) in degrees. The polarization states corresponding to horizontal and vertical, right and left circular and \( \tau = +45^\circ \) (forward slant) and \( +135^\circ \) (back slant) are indicated.
III. RESULTS

Polarization state distributions for various $K$ and $\rho_{pwr}$ are presented in Fig. 2. For ease of visualization as the distribution broadens, we displayed the rotated Poincaré sphere using Lambert’s equal area azimuthal projection. Because this projection maps equal areas on the sphere onto equal areas on the plane, the density of polarization states is preserved. In order to facilitate assessment of the rotational symmetry of the distributions, we have rotated the spherical coordinate frame used in Fig. 1 so that the polarization state corresponding to the mean direction is coincident with one of the poles of the rotated coordinate frame. Thus, the centre of the plot in Fig. 2 corresponds to vertical polarization while the outer circle corresponds to horizontal polarization.

In the absence of fading, i.e., a static channel with $K = \infty$, the polarization state of the signal that is observed at the receiver will describe a single point on the Poincaré sphere. As fading increases and as the cross correlation decreases, the polarization states will become increasingly dispersed across the sphere. The results are strikingly similar to those that we have observed experimentally in fixed wireless environments using a polarimetric channel sounder [17].

Fitting a known distribution to the simulated results is useful because it allows one to compactly represent the simulated results by the parameters of that distribution. Because the polarization state distributions in Fig. 2 appear to be unimodal and rotationally symmetric about the mean polarization state, an obvious trial candidate is the Fisher or spherical normal distribution given by

$$f(\theta, \phi) = \frac{\kappa \sin \theta}{2\pi e^{\kappa \theta}} e^{\kappa \sin \theta \sin \alpha \cos (\phi - \beta) + \cos \theta \cos \alpha}$$

where $\theta$ and $\phi$ are the co-latitude and longitude (elevation and elevation angle), $\kappa$ is the concentration parameter, and $\alpha$ and $\beta$ are the co-latitude and longitude of the mean direction [18][19]. If the mean direction is located at a pole of the sphere (e.g., $\alpha = 0^\circ$, $\beta$ arbitrary) then the distributions in $\theta$ and $\phi$ are independent and separable. In that case, the marginal distribution in $\phi$ is the uniform distribution, while the marginal distribution in $\theta$ is given by

$$f(\theta) = \kappa \sin \theta \frac{e^{\kappa \cos \theta}}{e^{\kappa} - e^{-\kappa}}.$$  

The maximum likelihood estimate of the mean direction is given by the sample mean direction [18]

$$\hat{\theta} = \arccos (\hat{z}) \quad \hat{\phi} = \arctan \left( \frac{\hat{y}}{\hat{x}} \right)$$

where $\hat{x}, \hat{y}, \hat{z}$ are given by

$$\hat{x} = \frac{1}{R} \sum_{i=1}^{N} x_i \quad \hat{y} = \frac{1}{R} \sum_{i=1}^{N} y_i \quad \hat{z} = \frac{1}{R} \sum_{i=1}^{N} z_i$$

Figure 1 - Dispersion of polarization states on the Poincaré sphere when fading signals observed on forward and back-slant polarized receiving antennas RX1 and RX2 are characterized by $K = 10$ and $\rho_{pwr} = 0.8$. The transmitted signal was vertically polarized but has been depolarized by the environment.

Figure 2 – Dispersion of polarization states on a Lambert equal area azimuthal projection of the Poincaré sphere when the fading signals observed on forward and back slant polarized receiving antennas are characterized by the indicated values of $K$ and $\rho_{pwr}$. 

$$f(\theta, \phi) = \frac{\kappa \sin \theta}{2\pi e^{\kappa \theta}} e^{\kappa \sin \theta \sin \alpha \cos (\phi - \beta) + \cos \theta \cos \alpha}$$

$$f(\theta) = \kappa \sin \theta \frac{e^{\kappa \cos \theta}}{e^{\kappa} - e^{-\kappa}}.$$
and $N$ is the number of samples, $R$ is the resultant length given by

$$R^2 = \left( \sum_{i=1}^{N} x_i \right)^2 + \left( \sum_{i=1}^{N} y_i \right)^2 + \left( \sum_{i=1}^{N} z_i \right)^2$$  \hspace{1cm} (10)

and $x$, $y$ and $z$ are the directional cosines of the spherical data,

$$x_i = \sin \theta_i \cos \phi_i , \quad y_i = \sin \theta_i \sin \phi_i , \quad z_i = \cos \theta_i .$$  \hspace{1cm} (11)

Experimental data may require estimators for the concentration parameter $\kappa$ that are robust against outliers, or have been modified for small ($N < 16$) sample sizes [20]-[23]. Because neither are issues here, we simply used the maximum likelihood estimator

$$\coth \kappa - \frac{1}{\kappa} = \frac{R}{N}$$  \hspace{1cm} (12)

that is given in [18]. The Fisher distributions that best fit the simulated data sets corresponding to selected values of $K$ between 1 and 100 and $\rho_{\text{pwr}}$ between 0 and 0.9 are shown in Fig. 3. It is apparent that the Fisher distribution fits the simulated data well for most values of $K$ and $\rho_{\text{pwr}}$ but slightly over predicts the polarization state dispersion when the channel is close to Rayleigh and the branches are highly correlated.

![Figure 3](image_url)  \hspace{1cm}

**Figure 3** – The Fisher distributions that best fit the co-latitude component of the polarization state distribution for selected values of $K$ between 0 and 100 and $\rho_{\text{pwr}}$ between 0 and 0.9.

Although our original goal was simply to determine how the Fisher concentration parameter $\kappa$ that determines the extent of the polarization state distribution depends upon the values of $K$ and $\rho_{\text{pwr}}$ that are observed on the diagonally polarized diversity branches, it is apparent that one can recast the relationship such that $\rho_{\text{pwr}}$ is the dependent variable and $K$ and $\kappa$ are the independent variables. We have done so in Fig. 4. Because $\kappa$ covers a large dynamic range, we have plotted it on a log scale. It is apparent that correlation between branches depends upon both the angular extent of the polarization state cluster and the depth of fading on each diversity branch.

![Figure 4](image_url)  \hspace{1cm}

**Figure 4** – The cross-correlation coefficient, $\rho_{\text{pwr}}$, between orthogonally polarized diversity branches when polarization state dispersion on the Poincaré sphere as a function of the Fisher concentration parameter $\kappa$ and the fading on each diversity branch as characterized by the Ricean K-factor $K$.

### IV. DISCUSSION

Although one generally needs to measure the phase between diversity branches to determine the received polarization state at a given instant, our results allow one to estimate the distribution of polarization states from estimates of $K$ and $\rho_{\text{pwr}}$, both of which can be estimated from simple measurements of received power vs. time [6]. The result is a fully polarimetric first-order statistical channel model that, among other things, can assist in the prediction of the performance of alternative polarization diversity schemes or polarization adaptive antennas in realistic environments.

Interpreting the cross-correlation coefficient as a function of $K$ and $\kappa$ offers a novel geometric interpretation of how decorrelation between polarization diversity branches arises: As the angular spread of the polarization state distribution on the Poincaré sphere broadens, the correlation between branches decreases. However, we’ve also shown that knowledge of the angular spread alone is not sufficient to predict the cross-correlation coefficient; the Ricean K-factors on the branches must be known as well. In all other respects, the analogy to a similar relationship between the angle of arrival distribution and correlation between branches in space diversity is striking. It’s possible that this geometric interpretation of polarization diversity may hold the key to
directly relating the physical nature of the propagation environment to the performance achievable using polarization diversity and/or polarization adaptation methods.

REFERENCES