Dynamic Analysis of Assembly Process with Passive Compliance for Robot Manipulators

K.-L. Du
Dept. of Electrical and Computer Engineering
Concordia University
Montreal, H3G 1M8, Canada
www.ece.concordia.ca/~kldu/

Biao-Biao Zhang
School of Aerospace and Mechanical Engineering
Australian Defence Force Academy
Canberra 2600 ACT, Australia

Xinhan Huang and Jianyuan Hu
Dept. of Control Science and Engineering
Huazhong University of Science and Technology
Wuhan 430074, China

Abstract—Assembly automation has become a research highlight for years. Dynamics of the most fundamental peg-in-hole mating, which represents an important topic of future research, is, however, far from being resolved. To this end the overall part-mating dynamics has been developed, and simulations have been implemented. The sensitivity analysis of each parameter on the assembly process has been made. The dynamic properties of assembly with the RCC have been concluded, which helps to implement active control. By selecting proper parameters of the dynamic remote center compliance, one can optimize assembly proceeding.

1 Introduction
Assembly Automation is one highlight of robotic application [1]. The peg-in-hole mating is a canonical form of assembly, and it is therefore crucial to study its mechanism for the sake of assembly techniques. The case of the quasi-static assembly has witnessed a rather complete analytical and experimental study [2], and the development of the remote center compliance (RCC) and equivalent compliance devices. For dynamic assembly, the situation is different since an increase in insertion force accompanies the increase in insertion speed. If the traditional RCC is applied, wedging and jamming will frequently occur. Passive compliance is known as the best solution to high-speed assembly [3]. However, dynamic wedging and jamming are still hard to avoid.

In the literature, references [4]-[8] provide much insight into this field. Asada et al [4]-[5] implemented the peg-in-hole assembly using their DRCC at a speed of 2m/s. In their work, the model is treated as dynamic phenomenon only at the instant that the peg bounces from the chamfer, while the motion process of the peg is quasi-static during the process. Shaninpoor et al [6] derived the condition of wedging and jamming for unchamfered part-mating. Wedging and jamming can be avoided by means of force control strategies. The assumption that insertion depth is a known function with regard to time helps to readily determine inertia force, which, however, is not in compliance with actual assembly process since positioning errors generally are of stochastic function type, and motion trajectory of peg is strongly dependent upon initial error. Trong et al [7]-[8] developed an overall dynamic analysis for compliantly supported rigid peg-in-hole assembly, which considers various factors that hold the possibility of affecting dynamic assembly process. This model is of much help to the research of floating compliance, which can effectively reduce insertion force and avoid wedging and jamming for unchamfered assembly. Their simulation result was similar to that of the quasi-static case [2]. Since there are many errors and ambiguities in their derivation, and the simulation results for dynamic process is similar to that of the quasi-static process, the result in [8] is doubtful. Based on D’Alembert’s Principle, we will develop a detailed derivation by following the framework applied in [8]. The simulation results are totally different from that of [2] and [8]. This, however, is supported by [9].

2 RCC Model and Assumptions
The general model of the RCC is shown in Fig. 1. To derive the dynamic equations, we make such assumptions as rigid peg and rigid hole, non-chamfered peg and chamfered hole, the lateral deviation always within the range of chamfer, equivalent dynamic and static friction coefficients, vertical insertion direction, constant insertion velocity, and no impact during contact.

For this model, the loadings of the compliance device \( F_x \), \( F_z \), and \( M \), which are the lateral force, the insertion force, and the torque of the RCC compliance exerting on the peg, respectively, are given by:

\[
F_x = K_x (x - x_0), \tag{1}
\]

\[
F_z = K_z (v + z_c - z_0), \tag{2}
\]

\[
M = K_m (y + z_c - z_0), \tag{3}
\]

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where $x_c$ and $z_c$ are the coordinates of point C, center of compliance, $x_{co}$ and $z_{co}$ are the coordinates of point C at the initial instant of contact, $\theta$ is the tilted angle, $\theta_0$ is the tilted angle at the initial instant of contact, and $v_i$ is the insertion speed. $K_a$, $K_o$ and $K_\theta$ denote axial stiffness, lateral stiffness, and angular stiffness respectively.

Fig. 2 illustrates two possible sets of contact configurations. In general, the evolution of assembly process is the chamfer-crossing stage, followed by the one-point contact stage and the two-point contact stage. If insertion depth is large, the two-point contact stage may be interrupted by the second one-point contact stage. As a matter of fact, only at the two-point contact stage can dynamic wedging and jamming occur. Although the forces are not large in the chamfer-crossing and the one-point contact stages, these stages determine the initial conditions of the two-point contact stage. In the following, we focus on the state set for $\theta=0$, which represents most of the successful insertions.

3 Chamfer Crossing

Fig. 3 is the schematic for dynamic analysis of the chamfer-crossing stage. The global coordinate system $xoz$ originates at the lower edge of the chamfer. We have geometric constraints as

$$x_c=R+e-L_c\sin\theta,$$
$$z_c=L_c\cos\theta'\sin\theta+z_{co},$$
$$x_G=R+e-L_c\sin\theta,$$
$$z_G=L_c\cos\theta+e\sin\theta+z_{co},$$

where $L_c$ is the distance from the lower-end surface to the center of compliance, $L_c$ is the distance from the lower-end surface to the centroid $G$, the gravity center of the gripper with the peg, $R$ and $r$ are the radii of the hole and the peg respectively, $e$ is the lateral deviation along the horizontal direction, which is a horizontal component of the displacement from any point in the center line to the center of the lower end of the peg, and $z_{co}$ is the vertical coordinate of point A, and $\alpha$ is the angle of the chamfer. Apparentely,

$$x_{co}=R+e-L_c\sin\theta_0,$$
$$z_{co}=L_c\cos\theta'+\sin\theta_0+(\cos\theta_0-e\sin\theta),$$

where $\theta_0$ is the value of $\theta$ at the initial instant of contact.

Derivation by D’Alembert’s Principle yields

$$M=K_a(\theta-\theta_0),$$

where $x_c$ and $z_c$ are the coordinates of point C, center of compliance, $x_{co}$ and $z_{co}$ are the coordinates of point C at the initial instant of contact, $\theta$ is the tilted angle, $\theta_0$ is the tilted angle at the initial instant of contact, and $v_i$ is the insertion speed. $K_a$, $K_o$ and $K_\theta$ denote axial stiffness, lateral stiffness, and angular stiffness respectively.

$$K_a=\sin\alpha-\mu\cos\alpha,$$
$$K_o=\cos\alpha+\mu\sin\alpha,$$
$$K_\theta=(L_cK_a-rK_o)cosh\theta+(L_cK_a+rK_r)\sin\theta.$$  

The loadings of the compliance device are calculated by (1)-(3).

Eliminating $F_a$ and inserting (6) and (7) up to their respective second-order derivatives into (8)-(10) leads to

$$f_1(\theta)\dot{\theta} + \ddot{\theta} + f_2(\theta)\dot{\theta}^2 + f_3(\theta, t) = 0,$$
$$g_1(\theta)\dot{\theta} + \ddot{\theta} + g_2(\theta)\dot{\theta}^2 + g_3(\theta, t, \xi) = 0,$$

where

$$C_1=(K_aL_c+rK_o)cosh\theta,$$
$$C_2=K_o(L_c\cos\theta+rsin\theta),$$
$$f_1(\theta)=C_1\cos\theta+C_2\sin\theta,$$
$$f_2(\theta)=C_1\cos\theta+C_2\sin\theta,$$
$$g_1(\theta)=-IK_\theta/mK_3(\theta)-L_c\cos\theta,$$
$$g_3(\theta, \xi, t)=K_\theta/[mK_3(\theta)](F_x\cos\theta+F_y\sin\theta)(L_c-L_G)-M/F_c,$$

Since $\theta$ is a small quantity, (11) and (12) can be linearized as

$$a_{11}\ddot{\theta} + \dddot{\theta} + a_{12}\dot{\theta} + a_{13}\dot{\theta}^2 + a_{14}\dot{\theta} + a_{15}\theta + \xi_0 = 0,$$

with initial conditions $\theta(0)=\theta_0, \dot{\theta}(0)=\dot{\theta}_0, \ddot{\theta}(0)=\ddot{\theta}_0, \xi(0)=\xi_0$, where subscript 0 denotes the value at the initial instant of contact.

When $z_i=0$, it steps into the one-point contact stage, and the instant $t_1$ is determined.

4 One-Point Contact

Fig. 4 is the schematic for dynamic analysis of the one-point stage. The geometric constraints are given by

$$x_c=rcos\theta-(h-cos\theta)sin\theta, \quad (13)$$
$$z_c=(L_c-h)cos\theta+rsin\theta, \quad (14)$$
$$x_G=rcos\theta-(L_c-h)sin\theta, \quad (15)$$
$$z_G=(L_c-h)cos\theta+rsin\theta, \quad (16)$$

where $h$ is the depth of insertion, which is the distance from the point of contact to the lower-end of the peg along the axial direction of the peg. The following relations hold

$$\dot{e}=h\cos\theta+rsin\theta, \quad \dot{\theta}+h\sin\theta.$$

Accordingly, we have the dynamic equations

$$K_xF_x+m\ddot{x}_c=0, \quad (17)$$
$$K_F\dot{F}_x-F_x\ddot{z}_c=0, \quad (18)$$

where $F_x$ is the force of the chamfer on the peg, $m$ and $I$ are the mass and the inertial moment of the gripper with the peg, $g=9.81ms^2$ is the coefficient of gravity, and $\mu$ is the coefficient of friction, $x_0$ and $z_0$ are the coordinates of the centroid, and
The loadings are calculated by (1)-(3).

After manipulation, we ultimately obtain
\[ p(h) \dot{\theta} + \dot{h} + p_0(h) \theta^2 + 2 \mu \dot{h} \theta + p_0(h, \theta, t) = 0, \]
\[ q_1(\theta, h) \dot{\theta} + q_2(\theta) \dot{h} + q_3(\theta, h) \theta^2 + q_4(\theta) h \theta + q_5(\theta, h, t) = 0, \]
\[ \text{where} \\
p_1(h) = -r + \mu(h - L_0), \]
p_2(h) = \mu(c_0 - h), \]
p_3(\theta, h, t) = K_F s / m - K_{c_0} F_s / m - K_{g_0},
\[ q_1(\theta, h) = -\sin^2(\theta + (h - L_0) \cos \theta - K_1 / [m(L_0 - h)], \]
\[ q_2(\theta) = \sin \theta, \]
\[ q_3(\theta, h) = ((L_0 - h) \sin \theta - r \cos \theta), \]
\[ q_4(\theta, h, t) = \left\{ F_s(K_1 + F_s \cos \theta + F_0 \sin \theta) \right\} / (L_0 - h) / m. \]

The initial conditions become \( \theta_0 = \theta, \) \( \dot{\theta}_0 = 0, \) \( h_0 = 0, \) \( \dot{h}_0 = -\left( \frac{\sin \theta}{\cos \theta} \right) \) / \( \cos \theta, \) where subscripts + and - represent the cases for pre- and post-transitions, respectively.

To avoid the failure illustrated in Fig. 5, it should be guaranteed that \( Z_2 = 2 \sin \theta - \cos \theta < 0, \) i.e. \( \theta < \theta = \arcsin \left( \left[ 1 -(rR)^2 \right] \right). \)

When
\[ B = 2R - h \sin \theta - 2r \cos \theta = 0, \]
the process steps into the two-point contact stage, and the instant \( t_2 \) is determined.

5 Two-Point Contact

The schematic for dynamic analysis of the two-point contact stage is shown in Fig. 6. The geometric constraints are given by
\[ x_c = 2R - r \cos \theta - L_0 \sin \theta, \]
\[ x_c = (L_0 - h) \cos \theta + r \sin \theta, \]
\[ x_c = 2R - r \cos \theta - L_0 \sin \theta, \]
\[ x_c = (L_0 - h) \cos \theta + r \sin \theta. \]

And the relations \( h = 2(R - r \cos \theta) / \sin \theta, \) \( \dot{h} = [2(R - r \cos \theta) / \sin \theta] \)

\[ F_s = \frac{m}{\mu K_1 + K_s} \left[ - \frac{h}{\sin \theta} \left[ 1 + \cos^2 \theta + \mu \cos \theta \sin \theta \right] + \frac{2}{\sin^2 \theta} \right] \left( R K_5 - \mu \right) + (r \cos \theta - h \sin \theta) - (L_0 + 2 \mu r) \]
\[ - \left( \left[ \frac{h}{\sin \theta} - \frac{r \cos \theta}{r} \right] K_4 + \mu L_0 - r \right) f_3(\theta, t) + \left( K_4 \frac{F_s}{m} + g \right) K_5 f_3(\theta, t) - \left( \frac{h}{\sin \theta} - r \cos \theta \right) K_4 + \mu L_0 - r \right] f_3(\theta, t). \]
occurs for dynamic assembly. The contact points are points overlap, wedging will appear inevitably. This also eously with respect to the hole and wedging cannot occur. reasons arising 6om impact or the robot controller, reduce during assembly process. For the quasi-static assembly, the sliding velocity at the contact point to zero, wedging if slide downward simultaneously or slide upward simultan-

The velocity of point B, which is in the tangential, very small, the impacts are very small, and geometric constraints can be retained at the instant of transitions. In the analysis, the one-point contact stage, there is an abrupt change in the direction of acting force, but there is no impact. Making use of the assumption of rigid peg and elastic hole, we assume that the edge point on the hole to be arc-transitional (A→A'), with a large arc radius, as shown in Fig. 7. The edge point on the peg slides along the arc and its velocity is in the tangential direction of the arc. Due to the short transitional period, the magnitude of the velocity almost remains constant \( V' = V_A \), but its direction changes abruptly, while the position changes continuously. Assume that geometric constraints can be maintained during the transition. In the analysis, \((\dot{x}_A, \dot{y}_A, \dot{\theta}_A)\) and \((\dot{x}_B, \dot{y}_B, \dot{\theta}_B)\) are the velocity vectors of pre- and post- transition respectively. The magnitude of the velocity at the edge point on the peg is \( V_A = \sqrt{\dot{x}_A^2 + \dot{y}_A^2 + \dot{\theta}_A^2} \), thus \( V_A = V'_A \), and the geometric constraints of the one-point contact stage naturally satisfy. Another constraint is needed to solve the velocity, this constraint depends upon the deformation process, and is very complex to determine. From motion analysis, we can assume \( \hat{\theta}_A = 0 \), thus \((\dot{x}_A, \dot{y}_A, \dot{\theta}_A)\) can be obtained. During the transition from the one-point contact stage to the two-point contact stage, there is impact. Since the horizontal component of the velocities at contact points are very small, the impacts are very small, and geometric constraints can be retained at the instant of transitions change. The velocity of point B, which is in the tangential, remains the same during the transition. We have two geometric constraints, thus \((\dot{x}_B, \dot{y}_B, \dot{\theta}_B)\) can be solved.

Jamming and wedging are two hindrances for assembly, which must be avoided. They usually occur at the two-point contact stage [2]. For the quasi-static assembly, if \( \theta - \theta_H = (R - r) / (R_H) \), the friction cones at the two contact points overlap, wedging will appear inevitably. This also occurs for dynamic assembly. The contact points are always in motion, thus the two contact points generally slide downward simultaneously or slide upward simultan-eously with respect to the hole and wedging cannot occur. However, if excessively large sliding friction, or other reasons arising from impact or the robot controller, reduce the sliding velocity at the contact point to zero, wedging will occur. Therefore at the two-point contact stage, one need to monitor the velocity directions at the contact points, and the condition \( \theta > \theta_H \) is still required to prevent wedging. When the ratios of the parameters of an assembly system are so irrational that they result in improper supporting forces, which makes point A in the peg sliding downward whereas point B upward, jamming occurs. The study of jamming can be transformed into the analysis of sliding condition, and can be derived in a similar manner as that given in [2].

7 Simulation

An amount of numerical experiments have been imple-
mented based on the following basic parameters:

\[
\begin{align*}
    m &= 0.1 \text{kg}, \\
    I &= 0.002 \text{kg} \cdot \text{m}^2, \\
    D &= 20 \text{mm}, \\
    d &= 19.8 \text{mm}, \\
    L_1 &= 150 \text{mm}, \\
    L_2 &= 100 \text{mm}, \\
    V &= 0.3 \text{m/s}, \\
    \mu &= 0.1, \\
    \varepsilon &= 1.0 \text{mm}, \\
    \phi &= 0.01 \text{m/s}, \\
    \theta &= 45^\circ, \\
    \theta_H &= 0.573^\circ, \\
    \theta' &= 0.573^\circ, \\
    K_e &= 1.0 \text{N/mm}, \\
    K_p &= 2.0 \text{N/mm}, \\
    K_o &= 20000.0 \text{N/mm/rad}, \\
    \text{depth of insertion} &= 100 \text{mm}. \\
\end{align*}
\]

A simulation result for the basic parameters is shown in Fig. 8. We study the effect of each parameter on assembly process by adjusting only the parameter and keeping all other basic parameters constant. The general results are as follows. In comparison with the whole process, the time for the chamfer-crossing stage is very short, during which the deviation decreases continuously. During the one-point contact stage, the depth of insertion increases continuously, which is true to intuition. It is interesting to notice that the depth of insertion decreases and the tilted angle increases during the two-point contact stage. This obviously differs from the results from the quasi-static case [2]. This can be explained that during the two-point contact stage the RCC adjusts its position in the hole by means of its elasticity to accomplish the peg-in-hole assembly. In attempting to avoid the peg’s coming out of the hole at this stage, it is necessary to ensure \( \theta < \theta_H \) , thus a smaller \( \theta_H \) improves the probability of successful assembly. As insertion proceeds, oscillation of adjustment occurs. This phenomenon was first observed by Seyferth and Pfeffer [9] experimentally and was interpreted as that the forces develop with uncertainty. At the chamfer-crossing stage and during the period following the appearance of the two-point contact, the speed of the peg decreases drastically until the occurrence of jamming. Only when the robot torque is sufficiently large does the motion continue in the two-point contact form and/or ultimately in the one-point contact form. The oscillation of the force evolution continues until the termination of the trajectory, which demonstrates the phenomenon of transient constraints [9].

Simulation results are concluded as follows. As \( K_e \) increases, \( \theta_H \), \( F_p \), and \( M \) will increase. As \( K_e \) grows, \( F_p \) will increase, meanwhile the chamfer-crossing time will shorten. If \( K_o \) increases, \( \theta_H \), and \( F_p \) reduce while \( M \) increases. A growth of \( v_i \) yields an increase of \( F_p \), but has little effect on \( \theta_H \). When \( v_i \) decreases, the number of
The oscillation is maintained by the axial elasticity of the RCC. If the stiffness in the axial direction is excessively large, jamming will occur. An increase in \( m \) yields a growth in the chamfer-crossing time. An increase in \( I \) yields a smaller \( \theta_{\text{max}} \). In both cases, \( F_x \) increases and the period of oscillation becomes longer. When \( a \) is in the vicinity of 45°, \( \theta_{\text{max}} \) the chamfer-crossing time, together with loadings \( F_x, F_y, \) and \( M \), reach their respective minimal values. When the clearance ratio becomes smaller, \( \theta \) decreases, and \( F_x, F_y \) and \( M \) increase, whereas \( \theta_{\text{max}} \) changes little. When \( L_c \) increases, \( F_x \) grows. If \( L_c \) is very small and close to zero, \( \theta_{\text{max}}, F_x, F_y, \) and \( M \) reach their respective minimal values. That is, when the center of compliance is at the distal end of the peg, best result can be achieved, which is in agreement with the quasi-static case presented in [2]. The growth of \( L_c \) generally has little influence on the process, only incurring a slight increase of \( F_x \) and a slight decrease of \( \theta_{\text{max}} \). Unlike the concept of generalized centroid by Asada et al [5], merely shifting the centroid toward the distal end of the peg may result in larger \( \theta_{\text{max}} \). An increase of \( \mu \) increases the value of \( F_x \), and has little effect on other performances. As \( \theta_0 \) or \( \epsilon_0 \) increases, \( \theta_{\text{max}} \) and loadings \( F_x, F_y, \) and \( M \) grow, the amplitude of oscillation becomes marked, and the amplitude of the reversing adjustment of the depth of insertion grows. \( \theta_0 \) and \( \epsilon_0 \) only influence the initial stage of the process. When \( \theta_0 \) is in the vicinity of zero, \( \theta_{\text{max}} \) becomes smaller. \( \epsilon_0 \) has little influence on the process, only when \( \epsilon_0 \) is fairly large in the positive direction, the chamfer-crossing time decreases and \( \theta_{\text{max}} \) increases slightly.

The dynamic properties lend themselves to our implementation of active control of assembly process, in attempting to improve successful assembly and to minimize the damage to robot system. Hence we can select a group of optimized parameters to promote assembly, that is, to obtain smaller \( \theta_{\text{max}} \) and smaller loadings.

When determining the kinematics relations at the instant of constraint transition, we have made use of some assumptions, whose rationality is still to prove.

8 Conclusion
A detailed geometric and dynamic analysis has been given for the compliantly supported rigid peg-in-hole assembly, and an amount of numerical experiments have been simulated. For the design of a dynamic RCC, it is advantageous to select larger inertial quantities (mass and rotational inertia), smaller friction coefficient, smaller initial deviation, smaller bilateral stiffness and axial stiffness, larger rotational stiffness, the chamfer angle in the vicinity of 45°, and the center of compliance in the vicinity of the distal end of the peg. A proper selection of the RCC parameters and the initial assembly state helps to minimize the tilted angle and the RCC loadings, thus facilitating successful assembly.

References

Figure 1. General model of RCC

Figure 2. Peg-in-hole assembly process