Optimal Solution of Fuzzy Nonlinear Programming Problems
with Linear Constraints

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Abstract

The nonlinear relationships between the objective function and the decision variables, the constraints and the parameters introduce varieties of complexities in optimization models. Last century was an era of development in the field of Nonlinear modeling of real life problems. Imprecision in every aspect of nature and human life compelled to see these models through a system called the fuzzy systems. Here in this paper a new method is proposed to find optimal solution to fuzzy nonlinear programming problems. Several authors have proposed algorithms those include linear programming problems with linear constraints. The proposed method stands on the previous works available in the literature and is verified through numerical examples.

Keywords: Fuzzy non-linear programming problems, fuzzy optimal solution, Ranking function, Triangular fuzzy numbers.

1. Introduction

In an earlier work Osman [11] have introduced the notion of solvability set, stability set of first kind and second kind and analyzed these concepts for parametric convex nonlinear programming problems. Osman and EleBanna [11] have introduced the qualitative analysis of the stability set of the first kind for fuzzy parametric multi objective nonlinear programming problems. Loganathan and Sherali [5] have presented an interactive cutting plane algorithm for determining a best compromise solution to a multi-objective optimization problem [17] in situations with an implicitly defined utility function. Zimmerman [6] proposed the first formulation of fuzzy linear programming. Nasseri [14, 15] derived a method to solve linear programming problems and Amit kumar and Jagdeep Kaur [1] and B. Kheirfam [4] introduced the fuzzy optimal solution of fuzzy non linear programming problems(FNLPP) with inequality constraints. In their result they have taken all the coefficients and decision variables to be fuzzy numbers and all the equations to be linear. Here in our present paper we have assumed the objective function to be nonlinear and the constraints to be linear, both inequality and equality type. All the variables, coefficients are fuzzy triangular numbers [10]. We have used the Langrangian method after converting the fuzzy non linear programming problem (FNLP) for equality constraints into crisp form with the help of ranking function [18] and KKT Conditions in non linear programming problem with inequality constraints into crisp form with the help of ranking function [3, 12, 18]. The motivation behind our result is that nonlinear objectives are equally important to be discussed under linear constraints.

This paper is organized as follows: in section 2, some basic definitions and arithmetic operations of triangular fuzzy numbers are reviewed. In section 3 formulation of fuzzy nonlinear programming problems and application of ranking function for solving FNLP problems are discussed. In section 4 and 6, a new method is proposed for solving FNLP problem. To illustrate the proposed method, numerical examples are solved in section 5 and 7. The conclusion is in section 8.
2. Preliminary Definitions and Earlier Results

In this section some definitions and notations of fuzzy set theory are reviewed.

**Definition 2.1**: The membership function \( \mu_A \) of a crisp set \( A \subseteq X \) assigns a value either 0 or 1 to each member in \( X \). This function can be generalized to a function \( \mu_A \) such that the value assigned to the element of the universal set \( X \) falls within a specified range i.e. \( \mu_A : X \rightarrow [0,1] \). The assigned value indicates the membership grade \( [2] \) of the element in the set \( A \). The function \( \mu_A \) is called the membership function and the set \( \{ (x, \mu_A(x)) : x \in X \} \) defined by \( \mu_A \) for each \( x \in X \) is called a fuzzy set.

**Definition 2.2**: A fuzzy number \( \tilde{a} = (a, b, c) \) is said to be a triangular fuzzy number if its membership function is given by

\[
\mu_{\tilde{a}}(x) = \begin{cases} 
\frac{x-a}{b-a}, & a \leq x \leq b \\
\frac{x-c}{b-c}, & b \leq x \leq c \\
0, & \text{otherwise}
\end{cases}
\]

**Definition 2.3**: A triangular fuzzy number \( (a, b, c) \) is said to be non-negative iff \( a \geq 0 \).

**Definition 2.4**: A ranking function \([18]\) is a function \( R : F(R) \rightarrow \mathbb{R} \), where \( F(R) \) is a set of fuzzy numbers, defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists. Let \( \tilde{a} = (a, b, c) \) be a triangular fuzzy number. Amit Kumar and Jagdeep Kaur \([1]\) have defined the ranking function \( R(\tilde{a}) = (a + 2b + c)/4 \).

**Definition 2.5**: Let \( \tilde{a} = (a_1, b_1, c_1) \) and \( \tilde{b} = (a_2, b_2, c_2) \) be two triangular fuzzy numbers, then

(i) \( \tilde{a} \preceq \tilde{b} \) iff \( a_1 \leq a_2, b_1 \geq b_2, c_1 \leq c_2 \)

(ii) \( \tilde{a} \succeq \tilde{b} \) iff \( a_1 \geq a_2, b_1 \leq b_2, c_1 \geq c_2 \)

(iii) \( \tilde{a} \simeq \tilde{b} \) iff \( a_1 = a_2, b_1 = b_2, c_1 = c_2 \)

**Definition 2.6**: The fuzzy optimal solution of FNLP Problem is a fuzzy number if it satisfies the following characteristics:

(i) \( \tilde{X} \) is a non-negative fuzzy number

(ii) \( \tilde{A} \otimes \tilde{X} = \tilde{b} \)

(iii) If there exists a non-negative fuzzy number \( \tilde{X}' \) such that \( \tilde{A} \otimes \tilde{X}' = \tilde{b} \) then

\[
R(\tilde{C}^T \otimes \tilde{X}^n) > R(\tilde{C}^T \otimes \tilde{X}^n')
\]

for a maximization problem and

\[
R(\tilde{C}^T \otimes \tilde{X}^n) < R(\tilde{C}^T \otimes \tilde{X}^n')
\]

for a minimization problem.

**Arithmetic operations**:

Let \( \tilde{a} = (a,b,c) \) and \( \tilde{b} = (e,f,g) \) be two triangular fuzzy numbers defined on the set of real numbers \( R \). Then

(i) \( \tilde{a} \oplus \tilde{b} = (a+e,b+f,c+g) \)

(ii) \( -\tilde{a} = (-c,-b,-a) \)

(iii) \( \tilde{a} - \tilde{b} = (a-g,b-f,c-e) \)

(iv) \( \tilde{a} \otimes \tilde{b} = \begin{cases} 
(ae,bf,cg), & a \geq 0 \\
(af,be,cd), & a < 0, c \geq 0 \\
(af,be,ce), & c < 0 
\end{cases} \)
Theorem 1: The pareto optimal solutions [8] of fuzzy multi-objective linear programming problems

\( P_1 \): Maximize \( \tilde{Z}_r = \sum \tilde{C}_i x_j \), \( r = 1,...,q \)

subject to \( \sum \tilde{a}_{ij} x_j \leq \tilde{b}_i \), \( i = 1,2,....,m \)

and \( x_j \geq 0 \)

where, \( \tilde{a}_{ij} \) & \( \tilde{C}_i \) are trapezoidal fuzzy numbers

and

\( P_2 \): Maximize \( R(\tilde{Z}_r) = \sum R(\tilde{C}_i) x_j, \) \( r = 1, ........ q \)

subject to \( \sum R(\tilde{a}_{ij}) x_j \leq R(\tilde{b}_i), \) \( i = 1,........, m. \)

and \( x_j \geq 0 \),

are equivalent, where \( R \) is the linear ranking function [8].

By the use of ranking function the optimal solution of a fuzzy multi-objective linear programming problem becomes a pareto optimal solution.

Note: P. Senthil Kumar and G. Rajendran [13] derived a solution of fuzzy linear programming problem (FLP) by using \( \alpha \)-cut with \( \alpha = 0 \) and \( \alpha = 1 \).

For \( \alpha = 0 \) they obtained the solution set \( x_0^0, x_2^0, \ldots, x_n^0 \) and \( x_1^0, x_2^0, \ldots, x_n^0 \) and for \( \alpha = 1 \) the solution set is \( x_1^1, x_2^1, \ldots, x_n^1 \) and \( x_1^1, x_2^1, \ldots, x_n^1 \).

Using these two solution sets finally they have obtained the optimal solution to be \( x_j = (x_j^1 - x_j^0) \alpha + x_j^0 \) and \( x_j = (x_j^1 - x_j^0) \alpha + x_j^0 \), \( j = 1, 2, \ldots, n \)

Theorem 2.2: Karush, Kuhn-Tucker Conditions (KKT Conditions) [16]

Let \( S = \left\{ x \in \mathbb{R}^n \mid \tilde{A} \otimes x \geq_R \tilde{b}, x \geq 0 \right\} \) be nonempty. Then \( x^* \in S \) is an optimal solution to the fuzzy linear programming problem;

Minimize \( \tilde{C} \otimes x, \)

subject to \( \tilde{A} \otimes x \geq_R \tilde{b} \)

and \( x \geq 0 \),

where \( \tilde{A} \in F_l(\mathbb{R}^{m \times n}), \tilde{b} \in F_l(\mathbb{R}^m), \tilde{c} \in F_l(\mathbb{R}^n), \) \( x \in \mathbb{R}^n \) if and only if \( \left( x^*, \alpha, \tau \right) \in \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^n \) is a solution to the following system:

\( \tilde{A} \otimes x \geq_R \tilde{b}, x \geq 0, \)

\( \omega \otimes \tilde{A} \otimes v \approx_R \tilde{c}, \omega \geq 0, v \geq 0; \)

\( \omega \otimes (\tilde{A} \otimes x \otimes \tilde{b}) \approx_R \emptyset, \forall x = 0 \)

Lemma 2.1[16]: Let \( R \) be any linear ranking function. Then \( \tilde{a} \geq \tilde{b} \) if and only if \( \tilde{a} - \tilde{b} \geq \emptyset, \) which implies \( \tilde{b} \geq -\tilde{a} \).

Further \( \tilde{a} \geq \tilde{b} \) and \( \tilde{c} \geq \tilde{d} \) \( \Rightarrow \tilde{a} + \tilde{c} \geq \tilde{b} + \tilde{d} \).

\( \textbf{P}^3: \) Maximize (or minimize) \( z = \sum_{j=1}^{n} c_j x_j^n \),

\( n \geq 2 \) as the objective function is non linear

subject to \( \sum_{j=1}^{n} a_{ij} x_j^n \leq (\geq) \ b_i, \ i = 1, 2, \ldots, m \)

and \( x_j \geq 0 \)

We consider this Non linear programming problem under three different cases in fuzzy domain.

Case-1:
Here \( c_j \) and \( a_{ij} \) are fuzzy numbers for all \( i = 1, 2, \ldots, m \) and \( j = 1, 2, \ldots, n \) and the variables \( x_j \geq 0 \) are crisp.

\( \textbf{P}^3 \) takes the form:

\( \textbf{P}_4: \) maximize (or minimize) \( z = \sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j^n \)

subject to \( \sum_{j=1}^{n} \tilde{a}_{ij} \tilde{x}_j^n \leq (\geq) \ \tilde{b}_i, \ \ i = 1, 2, \ldots, m \)

and \( \tilde{x}_j \geq 0 \)

Case-2:
Here \( c_j \) and \( a_{ij} \) both are crisp for all \( i = 1, 2, \ldots, m \) and \( j = 1, 2, \ldots, n \) and the variables \( b_i \) and \( x_j \geq 0 \) are fuzzy numbers.

\( \textbf{P}^3 \) takes the form:

\( \textbf{P}_5: \) Maximize (or minimize) \( z = \sum_{j=1}^{n} c_j \tilde{x}_j^n \)

subject to \( \sum_{j=1}^{n} a_{ij} \tilde{x}_j^n \leq (\geq) \ \tilde{b}_i, \ \ i = 1, 2, \ldots, m \)

and \( \tilde{x}_j \geq 0 \)

Case-3:
Here all the parameters \( c_j, a_{ij}, b_i \) and \( \tilde{x}_j \geq 0 \) are fuzzy numbers.

\( \textbf{P}^3 \) takes the form:

\( \textbf{P}_6: \) Maximize (or minimize) \( z = \sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j^n \)

subject to \( \sum_{j=1}^{n} \tilde{a}_{ij} \tilde{x}_j^n \leq (\geq) \ \tilde{b}_i, \ \ i = 1, 2, \ldots, m \)

and \( \tilde{x}_j \geq 0 \)
We use the Ranking function on (P₃) to get

\[ P₇: \text{ Maximize (or minimize)} \quad z' = \sum_{j=1}^{n} R(\tilde{c}_j x_j^n) \]

subject to

\[ \sum_{j=1}^{n} R(\tilde{a}_{ij} x_j^n) \leq (\geq) R(\tilde{b}_i), \quad i = 1, 2, \ldots, m \]

and

\[ x_j \geq 0 \]

which is equals to

\[ P₈: \text{ Maximize (or minimize)} \quad z' = \sum_{j=1}^{n} R(\tilde{c}_j x_j^n) = \sum_{j=1}^{n} c'_j x_j^n \]

subject to

\[ \sum_{j=1}^{n} R(\tilde{a}_{ij} x_j^n) \leq (\geq) R(\tilde{b}_i') \quad i = 1, 2, \ldots, m \]

\[ \Rightarrow \sum_{j=1}^{n} a'_{ij} x_j^n \leq (\geq) b'_i \]

and

\[ x_j \geq 0 \]

**Lemma 3.1:** The solutions of P₄ and P₈ are equivalent.

**Proof:**
Let \( M_1 \) and \( M_2 \) be the set of feasible solutions of (P₄) and (P₈) respectively.

\[ x \in M_1 \text{ if and only if} \quad \sum_{j=1}^{n} \tilde{a}_{ij} x_j^n \leq (\geq) \tilde{b}_i, \quad i = 1, 2, \ldots, m \]

\[ \Rightarrow \sum_{j=1}^{n} R(\tilde{a}_{ij} x_j^n) \leq (\geq) R(\tilde{b}_i), \quad i = 1, 2, \ldots, m \]

\[ \Rightarrow \sum_{j=1}^{n} R(\tilde{a}_{ij}) x_j^n \leq (\geq) R(\tilde{b}_i), \quad i = 1, 2, \ldots, m \]

\[ \Rightarrow \sum_{j=1}^{n} a'_{ij} x_j^n \leq (\geq) b'_i \]

\[ \Rightarrow x \in M_2 \]

So,

\( M_1 = M_2 \)

Hence the optimal solutions of P₄ and P₈ are equivalent.
4. Proposed Method (Main Result)

To solve the fuzzy non linear programming problem of case-1 we propose the following method:

Step-1: Assume the cost coefficient \( c_i \) and coefficients of matrix \( a_{ij} \) and \( b_i \) to be fuzzy triangular numbers in \( P_4 \).

Step-2: Use the ranking function on each fuzzy number of \( P_4 \).

Step-3: Use the KKT Conditions to determine the stationary points of \( P_4 \).

Step-4: Check the optimality of \( P_4 \) at these stationary points.

Step-5: Determine the optimal solution of \( P_4 \).

5. Numerical Examples

Consider the following FLNPP:

\[ P_9: \quad \text{Minimize} \quad z = 3x_1^2 + 2x_2^2 \]
\[ \text{subject to} \quad \begin{align*}
1x_1 + 3x_2 & \leq 4 \\
2x_1 - 1x_2 & \leq 2
\end{align*} \]

and \( x_1, x_2 \geq 0 \)

Using the fuzzy triangular numbers we get \( P_9 \) as

\[ P_{10}: \quad \text{Minimize} \quad z = (1, 3, 4)x_1^2 + (1, 2, 3)x_2^2 \]
\[ \text{subject to} \quad \begin{align*}
(0, 1, 3)x_1 + (2, 3, 5)x_2 & \leq (3, 4, 6) \\
(1, 2, 4)x_1 - (0, 1, 2)x_2 & \leq (1, 2, 5)
\end{align*} \]

and \( x_1, x_2 \geq 0 \)

Using the ranking function on the above problem we get \( P_9 \) as;

\[ P_{11}: \quad \text{Minimize} \quad z = R(1, 3, 4)x_1^2 + R(1, 2, 3)x_2^2 \]
\[ \text{subject to} \quad \begin{align*}
R(0, 1, 3)x_1 + R(2, 3, 5)x_2 & \leq R(3, 4, 6) \\
R(1, 2, 4)x_1 - R(0, 1, 2)x_2 & \leq R(1, 2, 5)
\end{align*} \]

and \( x_1, x_2 \geq 0 \)

which is equals to

\[ P_{12}: \quad \text{Minimize} \quad z = (11/4)x_1^2 + 2x_2^2 \]
\[ \text{subject to} \quad \begin{align*}
5x_1 + 13x_2 & \leq 17 \\
9x_1 - 4x_2 & \leq 10
\end{align*} \]

and \( x_1, x_2 \geq 0 \)

We define the Langrangian function [7]:

\[ L(x_1, x_2, \lambda_1, \lambda_2) = (11/4)x_1^2 + 2x_2^2 - \lambda_1(5x_1 + 13x_2 - 17) - \lambda_2(9x_1 - 4x_2 - 10) \]

The necessary KKT Conditions for the \( (P_{11}) \) are:

\[
\begin{align*}
\left(\frac{11}{2}\right)x_1 - 5\lambda_1 - 9\lambda_2 &= 0 \\
4x_2 - 13\lambda_1 + 4\lambda_2 &= 0 \\
5x_1 + 13x_2 - 17 &= 0 \\
9x_1 - 4x_2 - 10 &= 0
\end{align*}
\]

which yield,

\[
\begin{align*}
x_1 &= 1.445, \\
x_2 &= 0.75125, \\
\lambda_1 &= 0.4294, \\
\lambda_2 &= 0.644.
\end{align*}
\]
Here the principal minors of the Hessian matrix corresponding the objective function are \((11/2, 0\ 0\ 4)\) whose determinants are positive so, the objective function is convex and all the constraints are linear. The constraint equations are concave (since linear functions are both concave and convex).

Hence

\[
\begin{align*}
x_1 &= 1.445, \\
x_2 &= 0.75125, \\
\lambda_1 &= 0.4294, \\
\lambda_2 &= 0.644
\end{align*}
\]

is the optimal solution of the problem and the optimal value of the objective function is 6.870823.

In case-2 and case-3 we take the variables to be fuzzy numbers and use the following method to determine fuzzy optimal solution.

6. Proposed Method to Find the Fuzzy Optimal Solution of FNLP

In this section a new method is proposed to find the fuzzy optimal solution to the following FLNP problem\(P_3\) and \(P_6\):

The steps of the proposed method are as follows:

(i) The variables in the objective function are changed from \(\tilde{X}^n\) into \((\tilde{x}_j^n)\) in the problem by expressing them as fuzzy triangular numbers.

(ii) The Langrangian function \(L\) is obtained involving the objective function and constraints in fuzzy form.

(iii) Use the ranking function transforms the problem from fuzzy to crisp form.

(iv) Use KKT necessary conditions to determine the stationary point.

(v) Check the optimality condition at the stationary point \(s^{x_j}\).

(vi) The optimal value is determined by putting \(x_j\) in the objective function.

Next we prove the following proposition:

**Proposition 6.1:** If there exists a non-negative fuzzy number \(\tilde{X}^n\) such that \(\tilde{A} \otimes \tilde{X}^n = \tilde{b}\) then

\[
R(\tilde{C}^T \otimes \tilde{X}^n) > R(\tilde{C}^T \otimes \tilde{X}^\sigma) \quad \text{for a maximization problem and}
\]

\[
R(\tilde{C}^T \otimes \tilde{X}^n) < R(\tilde{C}^T \otimes \tilde{X}^\sigma) \quad \text{for a minimization problem.}
\]

**Proof:**

To prove that the fuzzy number \(\tilde{X}\) is optimal solution of the FNLP problem we show that it satisfies the following conditions:

For \(\tilde{X}, \tilde{A} \otimes \tilde{X}^n \leq \tilde{b}\)

Let there be another fuzzy number \(\tilde{X}'(\neq \tilde{X})\) such that \(\tilde{A} \otimes \tilde{X}' \leq \tilde{b}\) with

\[
R(\tilde{C}^T \otimes \tilde{X}') > R(\tilde{C}^T \otimes \tilde{X}^n) \quad \text{for maximization and}
\]

\[
R(\tilde{C}^T \otimes \tilde{X}') < R(\tilde{C}^T \otimes \tilde{X}^n) \quad \text{for minimization.}
\]

We assume that in the maximization problem

\[
R\left(\tilde{C}^T \otimes \tilde{X}^n\right) < R\left(\tilde{C}^T \otimes \tilde{X}^\sigma\right)
\]

\(\Rightarrow \tilde{C}^T \otimes \tilde{X}^n < \tilde{C}^T \otimes \tilde{X}^\sigma\) \quad (\because \tilde{X}\ and \tilde{X}'\ are non negative fuzzy numbers) \quad [10]\ so \(\tilde{X}'\ is the optimal solution of FNLP Problem which contradicts the result that \(\tilde{X}\ is the optimal solution.

Hence

\[
R\left(\tilde{C}^T \otimes \tilde{X}^n\right) > R\left(\tilde{C}^T \otimes \tilde{X}^\sigma\right)
\]
7. Numerical Example

Consider the following FNLP problem:

\[ P_{13}: \text{Maximize } x_1^2 + x_2^2 \]
\[ \text{subject to } \]
\[ (0, 1, 2) \otimes x_1 \oplus (1, 2, 3) x_2 \leq (1, 10, 27) \]
\[ (1, 2, 3) \otimes x_1 \oplus (0, 1, 2) x_2 \leq (2, 11, 28) \]

and \(x_1, x_2\) are non-negative fuzzy triangular numbers.

Let \(\tilde{x}_1 = (x_1, y_1, z_1)\) and \(\tilde{x}_2 = (x_2, y_2, z_2)\).

So we obtain

\[ P_{14}: \text{Maximize } (x_1, y_1, z_1) \oplus (x_1, y_1, z_1) \oplus (x_2, y_2, z_2) \]
\[ \text{subject to } \]
\[ (0, 1, 2) \otimes (x_1, y_1, z_1) \oplus (1, 2, 3) \otimes (x_2, y_2, z_2) \leq (1, 10, 27) \]
\[ (1, 2, 3) \otimes (x_1, y_1, z_1) \oplus (0, 1, 2) \otimes (x_2, y_2, z_2) \leq (2, 11, 28) \]

and \((x_1, y_1, z_1)\) and \((x_2, y_2, z_2)\) are non negative fuzzy triangular numbers.

The modified program is

\[ P_{15}: \text{Maximize } (x_1^2, y_1^2, z_1^2) \oplus (x_2^2, y_2^2, z_2^2) \]
\[ \text{subject to } \]
\[ (x_2, y_1 + 2y_2, z_1 + 3z_2) \leq (1, 10, 27) \]
\[ (x_1, 2y_1 + y_2, 3z_1 + 2z_2) \leq (2, 11, 28) \]

and \((x_1, y_1, z_1)\) and \((x_2, y_2, z_2)\) are non negative fuzzy triangular numbers.

The Langrangian function is [7],

\[ L(x_1, y_1, z_1, \lambda_1) = (x_1^2, y_1^2, z_1^2) \oplus (x_2^2, y_2^2, z_2^2) \]
\[ \lambda_1 ((x_1, y_1 + 2y_2, z_1 + 3z_2) - (1, 10, 27)) \]
\[ \lambda_2 ((x_1, 2y_1 + y_2, 3z_1 + 2z_2) - (2, 11, 28)) \]

Using ranking function the Langrangian function [7] becomes

\[ L(\tilde{x}, \tilde{y}, \tilde{z}, \tilde{\lambda}) = \frac{x_1^2 + x_2^2 + 2y_1^2 + 2y_2^2 + z_1^2 + z_2^2}{4} \]
\[ -\lambda_1 (\frac{x_2 - 27 + 2y_1 + 4y_2 - 20 + 2z_1 + 3z_2 - 1}{4}) \]
\[ -\lambda_2 (\frac{x_1 - 28 + 4y_1 + 2y_2 - 22 + 3z_1 + 2z_2 - 2}{4}) \]

Using the KKT Conditions [7] we get,

\[ X_1 = 4/3, \ y_1 = 4, \ z_1 = 6 \]
\[ X_2 = 1, \ y_2 = 4, \ z_2 = 17/3 \]
\[ \lambda_1 = 2, \ \lambda_2 = 8/3 \]

which implies \(\tilde{x}_1 = (4/3, 4, 6)\) and \(\tilde{x}_2 = (1, 4, 17/3)\).

The optimal value is \( Z = (25/9, 32, 613/9) \).

Note: If the constraints are of equality type then the Langrangian method [7] may be used to determine optimal solution after converting the fuzzy non linear programming problem into crisp form with the help of Ranking function.

8. Conclusion

We have obtained the fuzzy optimal solution of the non-linear programming problems with linear constraints by using the proposed method in this paper and verified the correctness of the proposed method through the numerical examples.
9. References


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