PAPR REDUCTION IN OFDM SYSTEM USING DIFFERENTIAL EVOLUTION-BASED PARTIAL TRANSMIT SEQUENCES SCHEME

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Abstract:
This paper proposes a partial transmit sequence (PTS) scheme with applying a stochastic optimization technique for peak-to-average power ratio (PAPR) reduction in the orthogonal frequency division multiplexing (OFDM) system. PTS technique combining can improve the PAPR statistics of an OFDM signals, but the considerable computational complexity for the required search through a high-dimensional vector space is a potential problem for implementation in the practical systems. Differential evolution (DE) is an efficient and powerful population-based stochastic search technique for solving optimization problems over continuous space, which has been widely applied in many scientific and engineering fields. Thus, to reduce the complexity for searching phase weight vector and to improve the PAPR statistics, we introduce the DE, an effective algorithm that solves various combinatorial optimization problems, to search the optimal phase weight factors. The simulation results show that the proposed DE-based PTS obtains the excellent PAPR performance with a low computational complexity.

Keywords:
Orthogonal frequency division multiplexing; Peak-to-average power ratio reduction; Partial transmit sequence and differential evolution

1. Introduction

Orthogonal frequency division multiplexing (OFDM) has been widely used in several digital transmissions such as digital video/audio broadcasting, digital subscriber lines, and wireless local area networks (WLAN), worldwide interoperability for microwave access (WiMAX), ultra-wideband MB-OFDM systems and long term evolution (LTE), because of its ability to combat with the frequency selective fading of wideband communication with reasonable complexity [1-4]. However, one major problem for OFDM applied system is its time-domain transmitted signal with a high PAPR especially for a large number of subcarriers. In addition, when a high PAPR signal passes through a power amplifier (PA), the PA may be pushed to a saturation region and, then, both in-band and out-of-band distortions are occurred to degrade the system performance. To alleviate the PAPR effect in the OFDM system, many techniques have been proposed including clipping techniques, coding techniques and multiple signal representation techniques such as partial transmit sequence (PTS) and selected mapping (SLM) [5-17]. The SLM and PTS schemes were proposed to lower the PAPR with a relatively small increase in redundancy but without any signal distortion [6-9, 12-13]. In the SLM scheme, an input symbol sequence is multiplied by each of the phase sequences to generate alternative input symbol sequences. Each of these alternative input symbol sequences is modulated by an inverse fast Fourier transform and, then, the corresponding signal with a lower PAPR is selected for transmission. The conventional PTS scheme is an efficient and a distortionless phase optimization technique for PAPR reduction by optimally combining signal subblocks [6]. In the scheme, the input data is divided into smaller disjoint subblocks. Each subblock is then multiplied by rotating phase factors, where the phase factor could be chosen freely within [0; 2π). Subsequently, the subblocks are added to form the OFDM symbol for transmission. Accordingly, the objective of the PTS scheme is to implement an optimal phase factor for the sub-block set that minimizes the PAPR. PTS scheme significantly improves PAPR performance, but unfortunately, it is a complex, non-linear optimization problem for finding the optimal phase factors. However, the conventional PTS requires an exhaustive search from all combinations from the all allowed phase weight factors. It turns out that search complexity increases exponentially with the number of subblocks. To reduce complexity for phase weight searches, some stochastic search techniques have recently been proposed because they could obtain the desirable PAPR reduction with a low computational complexity [14-20]. The famous stochastic techniques for PAPR reduction include simulated annealing (SA) algorithm [14], Cross-Entropy (CE) method [15-16], Genetic algorithm (GA) [17] and particle...
System Model and Problem definition

In the conventional OFDM system, inverse fast Fourier transform is usually implemented to modulate multiple sub-band signals within an OFDM symbol. Consider an OFDM system with L subcarriers, the complex baseband signal could be written as,

\[ x_i = \frac{1}{\sqrt{L}} \sum_{t=0}^{L-1} X e^{j2\pi nt/L}, 0 \leq t \leq L - 1 \]  

where \( X \) is the data symbol within \( i \)-th subcarrier, \( j = \sqrt{-1} \), \( X = [X_0, X_1, \ldots, X_{L-1}]^T \) is \( n \) input symbol sequence, and \( r \) stands for a discrete time index. The PAPR of the transmitted signal in Eq.(1) could be defined as

\[ \text{PAPR} = 10 \log_{10} \frac{\max_{0 \leq t \leq L - 1} |x_t|^2}{E[|x_t|^2]} \]  

where \( \max_{0 \leq t \leq L - 1} |x_t|^2 \) is the maximum values of the OFDM signal power, and \( E[\cdot] \) denotes the expected value operation. In principle, PAPR reduction techniques are concerned for reducing \( \max_{0 \leq t \leq L - 1} |x_t|^2 \). For the discrete-time signal, the amplitude of samples of \( x_t \) is usually processed by PAPR reduction techniques. By applying the central limit theorem with assuming that the number of sub-channels is sufficiently large, the time domain symbol is approximately zero-mean complex Gaussian distributed and the power distribution becomes a central chi-square distribution with two degrees of freedom.

The conventional PTS technique, an input data a block of \( N \) symbols is partitioned into disjoint subblocks. The subcarriers in each subblock are weighted by a phase weighting factor for the subblock. The phase weighting factors are selected such that the PAPR of the combined signal is minimized. Besides, for PAPR reduction, the PTS scheme can be easily combined with the corresponding symbol transform scheme. In the scheme, it requires to divide data block into subblocks or clusters and then, multiply the appropriate phase weighting factors to each sub-blocks for PAPR reduction. The data block is defined as a vector \( \hat{x} = [x_1, x_2, \ldots, x_n]^T \). Then, \( X \) is partitioned into \( M \) disjoint sub-blocks represented by the vector \( \hat{x} \) such that

\[ x = \frac{1}{M} \sum_{i=1}^{M} X_i \]  

Here, it is assumed that the clusters \( X_i \) consist of a set of sub-blocks with equal size. Then, the weighted sum combination of the \( M \) subblocks which could be written as

\[ \hat{x}(W) = \sum_{i=1}^{M} W_i X_i \]  

where \( W_i, i = 1, 2, \ldots, M \) is the phase weighting factor with phase factor \( \varphi_i = 0 \) or \( 2\pi \) commonly. In general, the selection of the phase weighting factors is limited to a set with finite number of elements to reduce the search complexity. After transforming to the time domain, the new time domain vector becomes

\[ x = \text{IFFT} \left( \sum_{i=1}^{M} W_i X_i \right) = \sum_{i=1}^{M} W_i \text{IFFT} \{X_i\} \]  

The optimization process is to find phase weighting factor that minimize the PAPR. The optimal phase weighting factor \( W_i \) that minimizes the PAPR can be obtained from a comprehensive simulation of all possible \( W^{M-1} \) combination. The objective of the proposed method is to choose a phase weighting vector \( W = \{W_1, W_2, \ldots, W_M\} \) to reduce the PAPR of \( \hat{x} \), and the cost function is defined as

\[ \hat{w} = \arg \min_w \left\{ \max \left| \sum_{i=1}^{M} W_i X_i \right| \right\} \quad \text{subject to} \quad W \in \{-1, +1\}^M \]
The optimum solution of (6) is to perform an exhaustive over all combination of $2^L$ in order to determine the optimal sign sequence. The performance of PAPR reduction is direct proportional to the factor of the number of phase weighting factor. However, when the number of phase weighting factor is a large one, the number of parallel addition processor and the number of phase weighting factor patterns need a complex computation to find the optimum set of phase weighting factors and it leads to a heavy load for the system. However, the assumption of i.i.d. symbols is not hold for the candidate signals using in the PTS scheme. The PAPR reduction performance in PTS may be degraded by the correlation among those candidate signals. Those correlations could be determined by two phase weighting factors. One is the sub-block partition style as shown in [6].

3. The Proposed Differential Evolution-Based PTS scheme

3.1. The Differential evolution (DE) Algorithm

In this section, we proposed a novel implementation of the PTS scheme based on the differential evolution (DE) algorithm [19-20]. In Section 3, we first briefly summarize the idea of DE algorithm, and then elaborate the step-by-step procedure of the proposed DE algorithm for PTS scheme to reduce the PAPR. Differential evolution (DE) is a simple yet powerful evolutionary algorithm (EA) for global optimization introduced by Price and Storn [19-20]. The DE algorithm has gradually become more popular and has been used in many practical cases, mainly because it has demonstrated good convergence properties and is principally easy to understand [20]. The DE method starts with an initial solution set, and an attraction-repulsion mechanism is then used iteratively to move those particles towards optimality. The DE method usually has three main procedures: initialization, mutation operation, crossover operation, and selection operation. These procedures are shown as follows. The first procedure, initialization, is used to sampling $K$ points randomly form the feasible region. The next procedure, mutation operation of DE applies the vector differentials between the existing population members for determining both the degree and direction of perturbation applied to the individual subject of the mutation operation. Once the mutation phase is complete, the crossover process is activated. The crossover operator is applied to the primary and secondary parents, resulting in the offspring vector. Finally procedures, the selection scheme of DE also differs from that of other EAs. The population for the next generation is selected from the individual in current population and its corresponding vector according to the select rule. The general scheme for the DE algorithm is shown in Algorithm 1, which consists of four main procedures: Initialization, Mutation operation, crossover operation, and selection operator, respectively.

ALGORITHM 1 Differential Evolution algorithm

1. Initialize population ( ); Evaluate fitness
2. while termination criteria are not satisfied do
3. Mutation ( )
4. Crossover ( );
5. Selection ( )
6. end while

3.2. DE Based PTS Scheme for PAPR Reduction

In principle, the DE algorithm is a population-based search method in which a set of potential solutions to the problem is evolved. In the following, the DE method is applied to search the optimal phase weight factor for the PTS technique in order to reduce the PAPR and called as DE-based PTS Scheme. The procedure of the proposed DE-based PTS could be described as follows:

Step 1. Initial population at $G=0$: According to the DE, let us assume that the objective is to minimize a cost function $F(W)$ with respect to the vector of parameters, $W = \{W_1, W_2, \ldots, W_L\}$, where $N$ is the dimension of the solution space. Evolutionary optimization algorithm are base on a population of candidate solution $W_k, k=1,2, \cdots, P, P$ is the population size), that are updated iteratively searching the solution space for the global minimum of $F(W)$. Then, new generations are produced iteratively by applying mutation, crossover, and selection operators to the current set of candidate solution. As we are interested in the values of phase factors in the range of 0 to $2\pi$, the upper bound and lower bound are set to 0 and $2\pi$, respectively. After a point is sampled from the space, the objective function value for the point is calculated. Given a point (i.e., phase factor vector b) $F(W)$ objective function, defined as the amount of PAPR reduction, can be expressed as

$$\hat{W} = \arg \min_{W} \left\{ \max_{r} \left| \sum_{i=1}^{N} W_i X_i \right| \right\}$$

Evaluate the objective values of all sample point. When the M points are all identified, the point with the best objective function value is stored.

Step 2. Mutation operator: during mutation, for each individual solution $W_k$. For each primary, a mutant vector $M_k^r$ is generated by

$$M_k^r = W_k + \delta(W_k - \bar{W})$$

where $r1, r2, and r3 \in \{1, 2, \cdots, K\}$ are selected randomly

1416
and are mutually different, as well as different form $k$. The $\delta$ is the mutation factor and positive and tunes the impact of the difference $(W_{r2} - W_{r3})$ on the construction of $M_k$.

Step 3. **Crossover operator:** the crossover operator is applied to the primary and secondary parents, resulting in the generation of the final offspring $u^G_k = [u_{k0}, u_{k1}, \ldots, u_{k(N-1)}]$.

The components of $u^G_k$ are inherited to a probabilistic scheme. To ensure that the offspring is different from the primary parent, the offspring inherits at least one component from the secondary parent. The index $l$ of this particular component is selected random from the set $\{0,1,\ldots,N-1\}$.

Thus, $u^G_k = M^G_k$, whereas the rest components of the offspring are given by the following scheme:

$$u^G_{kn} = \begin{cases} M^G_{kn}, & \text{if } h_n \leq H \\ W^G_{kn}, & \text{if } h_n > H \end{cases} \quad \text{for } n \neq l \quad (8)$$

where $n = 1, 2, \ldots, N$, $h_n$ is a random number uniformly distributed within $[0,1]$ and $H \in (0,1)$ is a predefined crossover probability. It should be mentioned that $u^G_k$ has to inherit at least one component from $M_k$.

Step 4. **Select operator:** Finally, during selection, the offspring, $u^G_k$, competes with the initial solution candidate, $W_k$, and if it is fittest with respect to the cost function, it replaces $W_k$ in the next generation ($G+1$), i.e.,

$$W^G_{k+1} = \begin{cases} u^G_k, & \text{if } F(u^G_k) \leq F(W^G_k) \\ W^G_k, & \text{if } F(u^G_k) > F(W^G_k) \end{cases} \quad (9)$$

Form Equation (9), we conclude that the best candidate solution of the new generation performs at least as good as the best candidate of the previous generation. Hence the cost function is monotonically decreasing with respect to the number of generation.

Step 5. Repeat step 2 to step 4 for $G=G+1$ until the maximum number of iteration is hold.

4. **Results and Discussions**

As above analyzed, we find most of existing solutions still have some drawbacks and the obvious one is the trade-off between PAPR reduction and some factor such as sub-block. In this section, the system performance with applied proposed PTS scheme is evaluated based on the PAPR Complementary Cumulative Distribution Function (CCDF) and the bit error rate (BER) by computer simulation.

The effectiveness of the DE is evaluated by using a challenging set of cost function in comparison to differential to particle swarm optimization algorithm (PSO) [20], genetic algorithm (GA) [19] and simulation annealing algorithm (SA)[14-15]. The modulation is chosen as QPSK scheme and the number of sub-carriers is assumed to be $N=256$ and $128$. In the simulations, the random sub-blocks partitioning is used both in the conventional PTS and the proposed PTS schemes.

The complexity of those three techniques with several of number of sub-blocks $V$ is also considered in this paper. The number of DFT points is equal to $128\times4$, it is meaning that signals are oversampled by a factor of four (i.e., $S=4$). The parameters, the number of clusters and the number of allowed phase weighing factors $W$ for transmit sequences, are also considered in the simulations. It is assumed that the $N$ time domain signal samples are mutually independent and uncorrelated. In the proposed method, the population size is assumed to be $P=20$; the maximum number of iterations is $G=100$, and the corresponding maximum number of iterations are $G=20, 40, 60, 80$, and $100$, respectively.

Fig. 1 and 2 show the variation in CCDF with the PAPR CCDF of the different numbers of the maximum number of iterations, $G$, with $N = 512$ and $N = 128$, respectively. In the DE method, the population size is assumed to be $P = 20$ and the corresponding maximum number of iterations are $G = 20, 40, 60, 80$, and $100$, respectively. In addition, the exhaustive search algorithm mentioned in [4] is presented to compare the performance of PAPR reduction with that of the DE searching method.
In the PTS, the selection of the phase factors was limited to a set of finite number of elements \( W \). The PTS was then employed to find the best phase factor. Here, four allowed phase factors \( \pm 1 \), and \( \pm j (W = 4) \) are used for the PTS, and the PAPR reduction performance is achieved by a Monte Carlo search with a full enumeration of \( WV \) (\( 4^8 = 65,536 \)) phase factors. As shown in Fig. 2 and 3, as the maximum number of iterations is increased and, then, the CCDF of the PAPR has a better performance. When \( P_c[PAPA > PAPR_0] = 10^{-3} \), it shows that the performance of the proposed DE method provides an approximate PAPR reduction as with that of the conventional PTS.

Fig. 3 evaluates the performance of the DE algorithm as a function of the number of iterations, \( G \). Increasing \( G \) beyond 100 seem to bring very little improvement in performance. As expected, PAPR performance improves with an increase in the number of iterations for the DE scheme. Of course, this occurs at an increasing level of complexity.

Next, the proposed DE-based PTS scheme with other existing stochastic optimization-based PTS approaches for the same number of samples, \( Sam = 2000 \) is shown. Fig. 4 shows the CCDFs of the PAPR of the OFDM system using the GA, the SA method, PSO, the proposed DE method, and the original OFDM with the number of subcarriers \( N=128 \). It shows that the PAPR of the original OFDM signal at \( P_c[PAPA > PAPR_0] = 10^{-3} \) for \( N=128 \) is 12 dB, which indicates a large PAPR. For \( N=128 \), the suppressed PAPRs of the GA, the SA method, PSO, and the proposed DE method at \( P_c[PAPA > PAPR_0] = 10^{-5} \) are 7.86, 7.0, 7.0, and 6.5 dB, respectively. With the same complexity, Fig. 4 shows the superiority of our proposed DE-based PTS scheme. By decreasing the complexity, the proposed DE method with \( Sam = 1000 \) obtains almost the same PAPR reduction as that of the GA with \( Sam = 2000 \). This means GA requires more samples (i.e., higher complexity) to obtain the similar PAPR reduction performance as the proposed method. Therefore, the proposed DE-based PTS method can offer better PAPR reduction while keeping a low complexity.
techniques to reduce PAPR have been analyzed, all of which have the potential to provide substantial reduction in PAPR at the number of iterations increase, BER performance improved, computational complexity reduction, and so on. The computer simulation results show that the proposed DE method obtained the desirable PAPR reduction with low computational complexity when compared with the various stochastic search techniques. For its flexibility, i.e. the number of sub-blocks and the number of admitted angles, the resulting peak power reduction can be achieved with several levels of the required PAPR. Besides, the proposed scheme is more suitable for the high data rate multi-carrier transmission systems because the computational complexity reduction ratio is increased with an increasing number of sub-carriers.

6. Acknowledgment

This work is partially supported by National Science Council, Taiwan, under Grant NSC 98-2221-E-029-011-MY2 and NSC 99-2218-E-029-001.

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