Performance of FH/MFSK systems with readable erasures

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Abstract

We propose a readable-erasure channel for frequency-hopping spread-spectrum multiple access systems with MFSK modulation scheme (FH/MFSK). In contrast to the conventional erasure-only channel having $M+1$ output symbols, the proposed readable-erasure channel has $2M$ output symbols. Half of the symbols are related to hit-free transmissions, and the rest of the symbols, referred to as weak symbols, are related to transmissions that were hit. The essence of the readable erasure is in the information hidden inside it. One may explore such information by using channel coding. Our analysis shows that in conjunction with channel coding, the readable-erasure channel may enjoy performance advantage over the erasure-only channel. By taking the spectrum efficiency into consideration, we point out that using RS codes over large channel alphabet size in FH/MFSK systems is inefficient. Thus, for FH/MFSK systems, we suggest BCH codes over GF(4) for use with the readable-erasure channel.

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1. Introduction

Frequency-hopping (FH) spread-spectrum multiple access communication systems, which is one kind of code division multiple access system, have been receiving intensive attention. A variety of FH spread-spectrum systems using different modulation schemes have been proposed. Among those systems, the one that uses $M$-ary frequency shift keying (MFSK) modulation scheme has been widely studied [1–6]. It is the FH/MFSK system with which we are concerned in this paper.

For the FH/MFSK spread-spectrum communication systems, the entire available bandwidth is divided into a number of frequency slots (or bands), with each slot containing $M$ tone positions, shared by a number of sender-receiver pairs simultaneously operating in the system. Signals from users are hopped from slot to slot by changing the carrier frequencies at certain points in time, called hop epochs [2], while the symbol in each hop is represented by a tone pertaining to that frequency slot. The sequence of carrier frequencies of a signal is known as its frequency-hopping pattern.

We may classify the frequency-hopping patterns into two types: random patterns or deterministic patterns. One model for a random pattern is a sequence of independent random variables, each of which is uniformly distributed over a set of frequency slots [1]. Many well-constructed deterministic patterns are known; a detailed description of coded sequences for deterministic patterns can be found in [7]. For performance analysis, the random hopping patterns are usually adopted due to its simplicity. Whenever two or more users hop on the same frequency slot, a hit occurs. The probability of the occurrence of hits primarily dominates the performance of the frequency-hopping system.

For digital communication systems, error control codes are used to protect signals against the errors caused by the channel. Thought a variety of codes exist, the Reed-Solomon (RS) code is one of the favorite codes due to its maximum-distance property. The performance of the FH/MFSK system employing RS codes was studied in
In those studies, RS codes were used as erasure-correction codes. To do so, the demodulator declares an erasure when the corresponding symbol was hit. However, under a hit, certain useful information still exists. We believe that if the demodulator can output a decision in conjunction with the erasure, then an improvement in performance is possible. This kind of erasures are referred to as readable erasures in [9]. In addition, for a regular RS code over Galois field $M$, $GF(M)$, the code length $n$ is $M - 1$. Hence, for an RS code with reasonable length, a large $M$ is needed. This will lead to poor utilization of channel bandwidth because, for the MFSK modulation scheme, the bandwidth efficiency decreases as $M$ increases. Of course, with small $M$, we may use RS codes over a large Galois field $Q (Q = M^r, r \geq 1)$ to obtain the desired code length [6]. In this method, a codeword symbol is represented by $r$ channel symbols. It is obvious that for a codeword symbol to be correctly estimated, all the $r$ channel symbols must be correctly estimated. However, we in this paper consider only the case in which a code symbol is represented by a channel symbol. In addition, we also consider BCH codes, with which the code length can be arbitrarily large with a moderate value of $M$ [10].

In the following, we shall refer to the channel with readable erasures as readable-erasure channel and the channel with conventional erasures as erasure-only channel. Performance evaluation of the readable-erasure channel employing RS codes or BCH codes is the primary subject of this paper. Performance of the erasure-only channel is also evaluated for comparison purposes.

The rest of this paper is organized as follows. Section 2 gives system description. Section 3 deals with the channel-symbol error probability of the readable-erasure channel. The codeword error probabilities for RS and BCH codes are derived in Section 4. Section 5 presents some simulation results to validate the effectiveness of the mathematical analysis. Finally, conclusions are drawn in Section 6.

### 2. System description

The model of a transmitter using a channel code of length $n$ and dimension $k$ is shown in Fig. 1. Every block of $\log_2 M$ bits constitutes an $M$-ary symbol, and $k$ symbols are encoded to obtain an $n$-symbol codeword. Then every codeword is transmitted over the frequency-hopping channel using MFSK modulation. At the receiver, the demodulator estimates the received symbols, and then $n$ estimated symbols are decoded to obtain $k(\log_2 M)$ bits, which are passed to the data sink.

We assume that the random hopping pattern is used. Thus, the number of users occupying any frequency slot is independent from hop to hop. We also assume that one symbol is transmitted during one hop interval. By this we mean that the system under consideration is a slow frequency-hopping system. Also, we assume a synchronous system; that is, the hop epochs of all users are the same. Hence, there is no partial hit.

We assume that the signal strengths from different users are the same, and that perfect channel side information on whether the channel symbols are hit or not is available for the demodulators. The assumption of perfect side information was made in several previous studies on the frequency-hopping systems [2,4,8]. We may get such side information using some means, as described in [8,11], for example. In the FH/MFSK system, there are a number of simultaneous users. We choose one of them as our desired user and focus our concerns on the desired user. In conventional erasure-only channel, an erasure is output in response to a hit symbol. In the readable-erasure channel, a hit symbol is demodulated as a weak symbol, i.e., readable erasure. If the weak symbol is different from the one actually transmitted, a channel-symbol error occurs. In making a weak decision, the demodulator makes a decision in favor of the tone position being occupied by the most tones. Under our assumption, this rule is equivalent to choosing the bin of the strongest power. If two or more bins happen to have the same largest number of tones, one of them is randomly chosen. Under the decision rule, the probability that the bin position the desired user occupies is the same as the bin chosen is no less than $1/M$, which corresponds to the probability that any of the $M$ bins is randomly chosen. Fig. 2 shows the state-transition diagram of a readable-erasure channel, where $p_h$ denotes the probability of a hit and the underlines aside the symbols denote weak

![Fig. 1. Model of a transmitter with a channel code of length $n$ and dimension $k$.](image1)

![Fig. 2. State-transition diagram of the readable-erasure channel.](image2)
symbols. Note that by integrating all the weak states into an erasure state, one may obtain a model of an erasure-only channel.

Suppose there are \( K \) simultaneous users operating in the system and there are \( q \) frequency slots. Given the desired user hopping on a specific frequency slot, the probability that \( u \) of the other \( K - 1 \) users hop on the same slot has the following binomial distribution:

\[
P_{C}(u; K - 1) = \binom{K - 1}{u} \left( \frac{1}{q} \right)^{u} \left( 1 - \frac{1}{q} \right)^{K-1-u}.
\]

It is clear that \( u = 0 \) means that the transmission of the desired user was not hit. Thus, the probability of a hit is

\[
p_{h} = 1 - \left( 1 - \frac{1}{q} \right)^{K-1}
\]

and the probability that \( j \) of the \( n \) symbols of a transmitted codeword were hit is

\[
P_{H}(j) = \binom{n}{j} p_{h}^{j} (1 - p_{h})^{n-j}.
\]

3. Symbol error probability of readable-erasure channel under hits

In the analytical approach developed in this section and the next, we focus our attention on the multiple-access capacity of the system. Thus, background noise, channel fading and the other factors that might corrupt transmissions are ignored for simplicity. By this we mean that the only factor that affects transmissions is the interference arising from other users [1–5]. Hence, when not hit, transmissions are considered to be error free for both readable-erasure and erasure-only channels. Later in Section 5, we shall take the noise and channel fading into account by conducting computer simulations.

According to the decision rule described above, we derive the symbol error probability of the readable-erasure channel as follows. Let \( p_{\text{er}} \) be the probability of an erroneous channel symbol given \( z \) users involved in a hit and \( p_{\text{er}} \) be the complementary probability of \( p_{\text{er}} \). Thus, we may derive \( p_{\text{er}} \) by way of \( p_{\text{er}} \).

Before we can proceed with the derivation, we first define a two-variable probability mass function (pmf) \( P_{ij}(x; z; M) \) as the probability that there are \( i \) bins containing \( j \) users’ tones and each of the remaining \( (M - j) \) bins containing at most \( (j - 1) \) tones, resulting from \( z \) users’ tones involved in the same \( M \)-ary frequency band.

To specify the pmf, we further define the variable \( v_{k} \), \( k = 1, \ldots, M \), as the number of users’ tones on the \( k \)-th bin. Thus, we have \( v_{1} + v_{2} + \cdots + v_{M} = z \). By definition, \( j \) corresponds to \( \max(v_{1}, \ldots, v_{M}) \) and \( i \) corresponds to the number of variables \( v_{1}, \ldots, v_{M} \) taking value \( j \). It is clear that \( 1 \leq i \leq M \) and \( \lceil z/M \rceil \leq j \leq z \), where \( \lceil x \rceil \) denotes the smallest integer equal to or greater than \( x \). Expressing \( P_{ij}(x; z; M) \) in a closed-form for general values of \( M \) and \( z \) seems very difficult, but through a series of combinatorial exercises, we can get recursive expression for it.

Consider a grid containing \( M \) rows and \( z \) columns. One \( O \) is placed in each column by randomly choosing the row location. There are \( M^{z} \) distinct patterns of \( O \)’s that can arise. Obviously the number of \( O \)’s in the \( k \)-th row corresponds to the number of users transmitting tones on the \( k \)-th bin. We define \( N_{i,j}(x; z; M) \) as the number of distinct patterns in which there are \( i \) rows each containing \( j \) \( O \)’s and the remaining \( (M - i) \) rows each containing at most \( (j - 1) \) \( O \)’s. Thus, we may compute \( P_{ij}(x; z; M) \) by

\[
P_{ij}(x; z; M) = \left( 1/M^{z} \right) N_{i,j}(x; z; M).
\]

For any feasible \( j \) in the range \( \lceil z/M \rceil \leq j \leq z \), variable \( i \) should satisfy the conditions \( z - ij \leq (M - i)(j - 1) \) and \( i \leq \lfloor z/j \rfloor \), where \( \lfloor x \rfloor \) denotes the integer part of \( x \). These constraints are needed because \( j \) cannot exceed \( z \) and, by definition, any one of the remaining \( (M - i) \) rows cannot have \( O \)’s more than or equal to \( j \). For \( i \) and \( j \) that cannot satisfy these constraints, \( P_{ij}(x; z; M) = 0 \). To simplify the notation, we denote the set of feasible values for \( i \), given \( j \), \( M \) and \( z \), by \( \{i\mid j; z; M \} \).

With the constraints for \( j \) and \( i \), we may express \( N_{i,j}(x; z; M) \) as

\[
N_{i,j}(x; z; M) = \binom{M}{i} \sum_{j=M-i}^{j} \sum_{y=1}^{\lceil z/j \rceil} N_{y,x; M-i,j-1}.
\]

Eq. (5) shows that there are \( \binom{M}{i} \) ways to choose the \( i \) rows. For the first row there are \( \binom{z}{j} \) ways to distribute the \( j \) \( O \)’s, the second row \( \binom{z-j}{j} \) ways, and so on. The last term in (5) denotes that there are \( (M - i)z - ij \) ways to place the remaining \( O \)’s (the number is definitely less than \( j \)) in the remaining \( (M - i) \) rows. In the case \( i < \lfloor z/j \rfloor \), since the number of the remaining \( O \)’s to be placed in the remaining \( (M - i) \) rows is larger than \( j \), we must eliminate those patterns containing \( j \) or more \( O \)’s in any of the remaining \( (M - i) \) rows. The recursive term in the square brackets on the right-hand side of (6) accounts for this manner.

With (4)–(6), \( P_{ij}(x; z; M) \) may be exactly determined. Note that for the special case \( M = 2 \), i.e., the BFSK scheme, no recursive call is needed and the expression is reduced to
\( P(j, i; z, 2) = \binom{2}{i} \binom{z}{j} \left( \frac{1}{2} \right)^z \), \hspace{1cm} (7)

for \( i = 2, \ j = z/2 \) or \( i = 1, \ z/2 < j \leq z; \) otherwise, \( P(j, i; z, M) = 0 \).

After specifying the pmf, we now determine \( p_{\text{ej} | z} \). Since the probability that the desired tone occupies the bin chosen is \( 2^{-1} \), \( p_{\text{ej} | z} \) may be expressed as

\[
p_{\text{ej} | z} = \sum_j \sum_i P(j, i; z, M) \frac{j}{z}.
\]

Thus, we may readily obtain \( p_{\text{ej} | z} \) by \( p_{\text{ej} | z} = 1 - p_{\text{ej} | z} \). Fig. 3 shows the values of \( p_{\text{ej} | z} \) for \( M = 4 \) and \( M = 64 \) with \( z \) ranging from 2 to 100. Two horizontal lines representing \( 1/4 \) and \( 1/64 \) are also shown. These two lines correspond to \( p_{\text{ej} | z} \) in the case where the receiver randomly chooses one of the \( M \) symbols as its output when the received symbol was hit. The potential strength of choosing the bin with the strongest power can be readily seen by examining the difference between \( p_{\text{ej} | z} \) and \( 1/M \). This figure shows that when \( z \) is small, the discrepancy of \( p_{\text{ej} | z} \) and \( 1/M \) is noticeable. The discrepancy gradually disappears as \( z \) increases. The curves shown in this figure presenting the simulation results will be discussed later in Section 5.

By averaging (8) over \( z \), we get \( p_{\text{ej}} \); that is,

\[
p_{\text{ej}} = \frac{\sum_{z=1}^{K-1} p_{\text{ej} | z+1} P_{E} (z; K-1)}{P_h}.
\]

4. Codeword error probability and channel throughput

4.1. Codeword error probability

Since RS codes are maximum distance separable (MDS) codes, an RS code of length \( n \) and dimension \( k \), denoted by \( (n, k) \), has a minimum distance \( d_{\text{min}} \) equal to \( (n - k + 1) \). And the code is capable of correcting up to \( t = \lfloor (d_{\text{min}} - 1)/2 \rfloor \) errors or \( d_{\text{min}} - 1 \) erasures. For BCH codes the scenario differs because they are not MDS codes. For a BCH code to be capable of correcting \( t \) errors, \( k \) is determined by \( (n - s) \), where \( s \) is the degree of the generating polynomial of the code. Thus, for a given code length \( n \), \( s \) needs to be determined before \( k \) can be determined. The \( d_{\text{min}} \) of the BCH code is, therefore, \( (2t + 1) \). We list various pairs of \( k \) and \( t \) for BCH codes of length 63 over GF(4) in Table 2, which will be further discussed later in the section.

Depending on the requirements of applications, a received word may be decoded using one of the following two manners: incomplete (bounded-distance) decoding or complete decoding [10]. The incomplete decoding algorithm assigns every received word to a codeword within distance \( t \), if there is one, or otherwise declares the received word to be unrecognizable. The complete decoding algorithm always decodes every received word as a closest codeword. When high reliability performance is required, one may prefer using an incomplete decoder in conjunction with an automatic-repeat-request protocol (see [12,13], for example), whereas when retransmissions are not feasible due to delay constraints, a complete decoder is suitable.

In the following we derive the codeword error probabilities for readable-erasure and erasure-only channels. For channel coding, both RS and BCH codes with complete decoding algorithm are considered. Since the codes to be considered are linear, different codewords are equally likely to be corrupted. Without loss of generality, we assume that the all-zero codeword is transmitted. Accordingly, we may calculate the codeword error probability \( P_{E} \) caused by error patterns.

**Case 1**: Reed-Solomon codes

- \( P_{E} \) of erasure-only channel

Usually, \( P_{E} \) of an erasure-only channel is approximately estimated to be the probability that the number of erased symbols of a received word equals or exceeds \( d_{\text{min}} \) [1-4]. However, the weight distribution of RS codes can help us calculate \( P_{E} \) exactly. The weight distribution of an \((n, k)\) RS code is known to be [10]:

\[
A_l = \binom{n}{l} (q - 1) \sum_{j=0}^{l-d_{\text{min}}} (-1)^{l-j} \binom{l-1}{j} q^{l-d_{\text{min}}-j}
\]

for \( l \geq d_{\text{min}} \), \( A_l = 0 \), for \( 1 \leq l \leq d_{\text{min}} - 1 \), and \( A_0 = 1 \). The exact expression of \( P_{E} \) for an erasure-only channel is derived as follows.

Suppose that a word containing \( v \) erasures is received. We denote the \( v \) positions containing erasures as \( E \). Since we have assumed that the zero codeword was transmitted, the symbols of the word in un-erased positions, i.e., positions not in \( E \), are everywhere zeros. When decoding the received word, any legal codeword with positions not in \( E \) everywhere zeros is a candidate for the decoded word. If \( v < d_{\text{min}} \), the zero codeword is the only candidate because no other codeword can have symbols not in \( E \) everywhere zeros. Thus, for \( v < d_{\text{min}} \), \( P_{E} = 0 \). When \( v \geq d_{\text{min}} \), a
codeword error might occur. To evaluate the $P_E$, we define $N(v)$ as the number of non-zero codewords of which the positions not in $E$ are everywhere zeros. Obviously, only the codewords with weight equal to or less than $v$ can contribute to $N(v)$. Thus, we may get $N(v)$ by

$$N(v) = \sum_{i=d_{\text{min}}}^{v} \binom{v}{i} \frac{A_i}{\binom{n}{i}}. \tag{11}$$

Since the decoded word is randomly chosen from among the $N(v)$ non-zero codewords and the zero codeword, a correct decision occurs only when the zero codeword is chosen; thus the codeword error probability may be expressed as

$$P_E = \frac{N(v)}{N(v) + 1}. \tag{12}$$

By averaging over $v$, we get $P_E$; that is,

$$P_E = \sum_{v=d_{\text{min}}}^{n} P_E(v)P_H(v), \tag{13}$$

where $P_H(v)$ is given in (3).

- $P_E$ of readable-erasure channel

With a specific erasure pattern, the possible error symbols of a received word are constrained in those erased symbols, and the un-erased components are error free. Thus, a codeword error may occur only when there are at least $d_{\text{min}}$ erased symbols and at least $v+1$ of the erased symbols are in error. Deriving an exact expression of $P_E$ seems somewhat complicated. Instead, we express $P_E$ as a function of $v$ as

$$P_E(v) \leq \sum_{i=v+1}^{n} \binom{v}{i} (p_{eq})^v (1-p_{eq})^{i-v}, \quad v \geq d_{\text{min}}, \tag{14}$$

where $p_{eq}$ is the complementary probability of $p_{eq}$ given in (8). Similar to (13), we have

$$P_E \leq \sum_{v=d_{\text{min}}}^{n} P_E(v)P_H(v). \tag{15}$$

In our illustrative examples shown in the following figures, we set $q$ to 100. Fig. 4 shows the $P_E$'s as a function of $K$ for both channels using (63,30) RS code. The curves present performance for (63,11) BCH code shown in this figure will be discussed later. We observe from this figure that under the constraint of $P_E \leq 10^{-2}$, the maximum achievable numbers of users of readable-erasure and erasure-only channels are 51 and 49, respectively. As expected, the readable-erasure channel indeed has a larger user capability than the erasure-only channel.

**Case 2: BCH codes**

For the MFSK modulation scheme, given a source data rate $R_b$, the channel symbol rate is $R_s = R_b/\log_2 M$. To have $M$ orthogonal tones, the bandwidth required for an $M$-ary frequency slot is given by

$$W = \frac{R_b}{\log_2 M}. \tag{16}$$

With a fixed $R_b$, $M = 2$ and $M = 4$ require the same amount of bandwidth, and in the cases where $M > 4$, $W$ is a monotonically increasing function of the channel alphabet $M$. The value of $n$ for a regular $(n,k)$ RS code over GF($M$) is $(M-1)$. Thus, when $n$ is needed to be large, so is $M$. By (16), we may evaluate the ratio of bandwidth required for a $Q$-ary system ($Q = M^4$) to an $M$-ary system by

$$\mu = \frac{1}{\log_2 M}. \tag{17}$$

For example, the bandwidth required for a $Q = 64$ system is 16/3 times that required for an $M = 4$ system. Thus, using a larger channel alphabet size to accommodate the code symbols required for RS codes seems very inefficient in spectrum utilization. One alternative is to use a small $M$ and employ BCH codes. Since BCH codes are no longer maximum distance codes, the weight distributions are not known for most of them. Thus, an exact analysis of the codeword error probability for the erasure-only channel seems very troublesome. Thus, the following upper bound is used:

$$P_E \leq \sum_{v=d_{\text{min}}}^{n} P_E(v). \tag{18}$$

By this expression we mean that as long as $d_{\text{min}}$ or more code symbols are erased, a decoding error occurs. As for readable-erasure channels, $P_E$ may be evaluated by using (14) and (15). The $P_E$'s for the (63,11) BCH code are previously shown in Fig. 4. From that figure, we see that the maximum numbers of users within $P_E \leq 10^{-2}$ for readable-erasure and erasure-only channels are 50 and 42, respectively. Clearly, we achieve a significant improvement in user capacity by using the readable-erasure channel.
4.2. Channel throughput

To account for the effects of the code rate on performance and the bandwidth required for different modulation schemes, we define the normalized channel throughput by

\[ \eta = K \left( 1 - P_E(K, n, k, q) \right) \frac{r}{wq}, \]  

(19)

where \( r = k/n \) is the code rate and \( w = M/(\log_2 M) \) being obtained by normalizing (16) with respect to \( R_b \). For a given \( q, \eta \) is a function of \( K \) and \( r \). Thus, for a given code rate, it is believed that an optimum \( K \) exists which maximizes \( \eta \). Fig. 5 shows the normalized channel throughput as a function of \( K \) for (63,\( k \)) RS codes with readable-erasure channels obtained using (19). As shown, the optimum values of \( K \) for \( k = 46, 34, \) and 22 are 29, 53 and 88, respectively. The values of \( P_E \) for \( k = 46, 34, \) and 22 when the optimum throughput appears are \( 1.6 \times 10^{-1}, 1.08 \times 10^{-1} \) and \( 8.74 \times 10^{-2} \), respectively. Unfortunately, these values are too large to suit for a practical system. For this reason, we impose a constraint of \( P_E \leq 10^{-2} \) on (19). The maximum numbers of allowable users and the corresponding throughputs obtained thereby are denoted by \( K^* \) and \( \eta^* \), respectively. Table 1 lists these values for various (63,\( k \)) RS codes. We observe from this table that for all values of \( k \), the corresponding values of \( K^* \) for readable-erasure channel are all larger than those for erasure-only channel by at least one. It can be also seen that the optimum throughput for readable-erasure channel is \( 2.4 \times 10^{-2} \), which is achieved at \( k = 22 \).

Similar to Table 1, Table 2 lists the values of \( K^* \) and \( \eta^* \) for various (63,\( k \)) BCH codes over GF(4). By carefully examining the results, we see that the improvement of readable-erasure channel over erasure-only channel is quite noticeable. On the other hand, the maximum throughput for readable-erasure channel is \( 5.41 \times 10^{-2} \), which is achieved when \( k = 27 \). This value is 2.14 times that of the corresponding value obtained in RS coding case. Thus, the performance advantage of readable-erasure channel with BCH codes over that with RS codes is quite noticeable. Since \( M = 2 \) case is as spectrally efficient as \( M = 4 \) case, we also evaluated the performance for various (63,\( k \)) BCH codes over GF(2). The results show that the maximum value of \( \eta^* \) is slightly smaller than \( M = 4 \) case, so those results are not shown. Consequently, we suggest BCH over GF(4) for use with the readable-erasure channel in FH/MFSK systems.

5. Simulation results

In the above analysis, we assumed that noise is negligible relative to the signal power. We also assumed that the bin occupied by the most tones always has the strongest power. This is equivalent to assuming that all tones arrive at the base station with the same phase. In order to get an insight into the effects of noise and different arrival phases on performance, we also conducted simulations to evaluate \( p_\text{c,v} \) for various signal to noise power ratios (SNRs). In the simulations we assumed that the phase of each tone is randomly distributed over 0 to \( 2\pi \).

From the simulation results shown in Fig. 3, for \( \text{SNR} = 10 \) and 20 dB, we may make two observations. One is that when \( z \) is small, the effect of different phases is small, whereas when \( z \) is large, there is a little difference between the analysis and simulation results. The other observation is that the difference in performance between

![Fig. 5. Maximum throughputs obtained directly using (19) for various (63,\( k \)) RS codes with a readable-erasure channel.](image)

---

**Table 1**

<table>
<thead>
<tr>
<th>( k )</th>
<th>Readable-erasure ( K^* ) ( \eta^* )</th>
<th>Erasure-only ( K^* ) ( \eta^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>88</td>
<td>2.33 \times 10^{-2}</td>
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<tr>
<td>22</td>
<td>74</td>
<td>2.4 \times 10^{-2}</td>
</tr>
<tr>
<td>30</td>
<td>51</td>
<td>2.25 \times 10^{-2}</td>
</tr>
<tr>
<td>34</td>
<td>42</td>
<td>2.1 \times 10^{-2}</td>
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<td>1.85 \times 10^{-2}</td>
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</tr>
<tr>
<td>46</td>
<td>20</td>
<td>1.35 \times 10^{-2}</td>
</tr>
</tbody>
</table>

**Table 2**

<table>
<thead>
<tr>
<th>( k ) ( (i) )</th>
<th>Readable-erasure ( K^* ) ( \eta^* )</th>
<th>Erasure-only ( K^* ) ( \eta^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8(20)</td>
<td>81</td>
<td>5.09 \times 10^{-2}</td>
</tr>
<tr>
<td>11(15)</td>
<td>50</td>
<td>4.32 \times 10^{-2}</td>
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<td>20(12)</td>
<td>36</td>
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<td>5.41 \times 10^{-2}</td>
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<td>36(6)</td>
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<td>39(15)</td>
<td>11</td>
<td>3.37 \times 10^{-2}</td>
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<tr>
<td>45(4)</td>
<td>8</td>
<td>2.83 \times 10^{-2}</td>
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</tbody>
</table>
SNR = 10 and 20 dB is not significant. As the probability distribution of the number of users involved in a hit is dominated by the first few integers, our observations suggest that the analysis may obtain acceptable results when averaging over the distribution.

The second part of simulations were devoted to gaining insight into the effects of noise and channel fading on the codeword error probability $P_E$ of the readable erasure channel. The received sinusoidal signal is modeled as

$$y(t) = \sqrt{2E_sR} \cos[2\pi(f_c + f_r)t + \phi] + n(t),$$

where $E_s$ is the effective power, $f_c$ is the carrier frequency, $f_r$ is the offset frequency, $\phi$ is the random phase, and $R$ is a random variable obeying the Nakagami distribution; that is,

$$f_R(r) = \frac{2}{\Gamma(m)} \left( \frac{m}{\Omega} \right)^{2m} r^{2m-1} \exp\left( -\frac{m}{\Omega} r^2 \right) \quad r \geq 0,$$

$$m \geq \frac{1}{2},$$

where $\Gamma(m)$ is the Gamma function and $E[r^2] = \Omega$ is the second moment of the variable $R$. The distribution becomes Rayleigh one when $m = 1$. As $m$ approaches infinity, the non-fading case is approached.

To evaluate $P_E$, a complete Monte Carlo simulation run needs to take the encoding and decoding processes into account. However, the decoding process requires a list look-up procedure to determine which codewords were actually transmitted. This process is quite time-consuming. Thus, in order to reduce the run time, we in our simulations repeatedly transmitted the all-zero codeword. At the receiver site, the estimated symbols were recorded. From every $n$ estimated symbols, a codeword decision was made. It is worth noting that in the simulation, the transmitted symbols are subject to background noise and channel fading. Therefore, the hit-free symbols are no longer error free. Suppose there are $e_1$ erroneous symbols and $e_2$ erased symbols of which $e_3$ symbols are in error. For erasure-only channels, a codeword error may occur if

$$e_2 \geq d_{\text{min}} - 2e_1$$

and for readable-erasable channels, the condition is

$$e_2 \geq d_{\text{min}} - 2e_1 \text{ and } e_3 > \left( d_{\text{min}} - 2e_1 - 1 \right)/2.$$

As a rule of thumb suggesting that codewords simulated should be more than $10/P_E$, we simulated $10^5$ codewords in each simulation run.

As shown in the last section, the use of BCH codes with $M = 4$ provides some capacity advantage over the use of RS codes. Thus, in our simulation, we considered only the BCH coding case. Fig. 6 depicts some simulation results of $P_E$ obtained from using the $(63,11)$ BCH code over GF(4) with SNR = 20 dB and Nakagami fading parameter $m = 5, 2, 1$. The curve presenting the analysis result previously shown in Fig. 4 is redrawn for comparison purposes. We observe from this figure that when $m = 5$, both the curves presenting simulation and analysis results almost overlap. As $m$ decreases to 2, $P_E$ slightly increases. The deterioration is more evident in Rayleigh fading case ($m = 1$). This suggests that the analysis, which assumes negligible noise, is effective, if the SNR is at least 20 dB and $m$ not less than 5. While channel fading may cause degradation in $P_E$, the potential advantage of the readable-erasable channel over erasure-only channel is still observed.

6. Conclusions

We proposed a readable-erasable channel for FH/MFSK system. The user capacities of the readable-erasable channel using RS codes and BS codes were evaluated. The conventional erasure-only channel was also considered for comparison purposes. By mathematical analysis, we showed that in conjunction with channel coding, the readable-erasable channel may enjoy performance advantage over the erasure-only channel. In addition, we showed that using RS codes with large channel alphabet size is inefficient in spectrum efficiency and thus we suggested BCH codes over GF(4) for use with the readable-erasable channel. Computer simulations were conducted to validate the effectiveness of the analysis approach by taking into account the system noise and channel fading. Results showed a good agreement between the analysis and simulation results for non-fading channels with an SNR of 20 dB.

References


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