PN code-aided timing estimation, channel estimation and signal compensation in OFDM systems

Jyh-Horng Wen, Gwo-Ruey Lee, Te-Lung Kung, Shu-Hong Lee

1. Introduction

The significant technology, orthogonal frequency division multiplexing (OFDM), with the capacity for a high data-rate transmission has been applied into many digital transmission systems such as European digital audio broadcasting (DAB) system, European digital video broadcasting terrestrial transmission (DVB-T) system, asymmetric digital subscriber line (ADSL), wireless local area network (WLAN), broadband wireless access (BWA) network, worldwide interoperability for microwave access (WiMAX) systems and ultra-wideband (UWB) systems [1–4]. However, one of the principal disadvantages of OFDM is its sensitivity to frame synchronization errors, represented by the so-called timing offsets. Timing offset results in the rotation of the OFDM subcarrier constellation. As a result an OFDM receiver cannot recover the transmitted signal without the knowledge of timing offset [5–9]. Hence, the task of frame synchronization algorithms is to provide the knowledge of frame timing offset in the OFDM systems.

In the OFDM systems, both data-aided and non-data-aided synchronization algorithms have been proposed previously [6–17]. Within the non-data-aided algorithms, the cyclic property of the guard interval proceeding of the OFDM symbol could be used for the frame synchronization without any training symbol [6,7]. Among these non-data-aided algorithms, the joint maximum likelihood (ML) symbol-time and carrier-frequency offset estimator proposed by J.J. Van de Beek applies the correlation of the cyclic prefix and the OFDM symbol to find the symbol timing [6]. The results show that the Beek’s estimator could have a lower error variance when the number of cyclic prefix samples is larger. Besides, the estimator exploits
the second-order cyclostationarity of the received signals and, then, it obtains the information of symbol-timing offset and carrier-frequency offset by the cyclic correlation [7]. The algorithm using the rank behavior of certain autocorrelation matrices constructed with the received signal is proposed to achieve the symbol synchronization [8]. The symbol timing could be estimated when the autocorrelation matrix with a minimum rank. Moreover, based on the complex conjugate symmetry in the fast Fourier transformation, the BPSK-OFDM symbol synchronization algorithm employing this conjugate character is proposed to obtain the symbol timing [9]. The proposed estimation algorithm has a better bandwidth efficiency than the other timing estimation schemes, without adding the training symbol.

In the data-aided scheme, the design of a pilot symbol is based on a repeated complex maximum length sequence for synchronization of OFDM packets [10,11]. The pilot symbol-based synchronization algorithms used to estimate channel including frequency offset, phase offset and timing offset has been presented. The lower bound variance on the channel estimation could be achieved by both of the optimal training sequences and pilot tones in OFDM systems [12]. Besides, using a pilot symbol, the performance of BPSK-OFDM system under an additive white Gaussian noise (AWGN) channel is derived and analyzed [13]. Besides, the symbol timing estimation and delay spread estimation using the subblock correlation is applied to estimate symbol timing and the length of delay spread [14]. Based on an OFDM frame format with adding the pseudo-noise code, the frame timing could be obtained by the proposed scheme with applying the regression method [15]. Based on the preamble defined in IEEE 802.16a, a timing algorithm is proposed with applying conjugate symmetry within the preamble structure [16]. The proposed scheme can reduce the estimation error significantly. In general, at the receiver, the frame synchronization could be implemented with the aid of the dedicated training symbols and the aid of the pilot signal, respectively.

Channel estimation is necessary before the demodulation of OFDM received signal since the radio channel is frequency selective and time-varying for wideband mobile communication systems. With the help of channel estimation, the system performance could be improved. Both blind channel estimation scheme and data-aided channel estimation scheme are proposed. Blind channel estimation techniques try to estimate the channel information without any knowledge of the transmitted signal. With the data-aided schemes, pilots or training sequences are inserted in the transmitted signal to estimate the channel response. In pilot-based schemes, the channel estimation can be performed by either inserting pilots into all of subcarriers in OFDM symbols with a specific periodicity or inserting pilots into reserved subcarriers in each OFDM symbol [17,18]. The first method is called as block-type pilot channel estimation [17]. The method is developed with the assumption that the channel transfer function is assumed to be not changing rapidly. Although block-type pilot channel estimation could estimate the overall channel response and provide a better performance, it suffers the bandwidth efficiency. The other is called as com-type pilot channel estimation. In the scheme, the pilots are inserted into the reserved subcarriers to obtain the partial channel response based on least square (LS), minimum mean square error (MMSE), or least mean square (LMS) algorithms [18]. Training sequence-based channel estimation is widely used in packet-based communications. In this paper, based on the property of pseudo-noise (PN) code, an algorithm is proposed to achieve the frame synchronization. Based on the scheme, the path delay time could also be estimated. With the information, the channel impulse response could be obtained. Besides, with the estimated channel impulse response, a frequency domain one-tap equalizer is proposed to realize the signal compensation and, then, the system performance could be improved in the OFDM systems. In the following section, the OFDM system and the multipath channel modeling are described. The proposed algorithm for frame synchronization is presented in Section 3. The channel estimation and the signal compensation scheme are proposed in Section 4. Simulation results are shown in Section 5. Finally, a conclusion is given in Section 6.

2. OFDM system description

Basically, an OFDM system contains a series-to-parallel converter, a signal mapping scheme, a fast Fourier transform (FFT)/inverse FFT (IFFT) module [1–4]. The series and parallel converter is considered to realize the concept of parallel data transmission to provide a high data rate transmission. \( M \)-ary phase shift keying (PSK) or \( M \)-ary quadrature amplitude modulation (QAM) mapping scheme is modulated in each parallel subcarrier to provide a different data rate service in the system. Besides, the IFFT and FFT are employed to replace the banks of sinusoidal generator for the modulation and demodulation with different carrier frequencies to reduce the complexity of OFDM modem implementation [2,4]. The block diagram of baseband OFDM system is shown in Fig. 1. The serial bit stream is transformed to a parallel form. Each parallel data is mapped with the PSK or the QAM scheme and, then, those data are modulated by means of an IFFT on \( N \)-parallel subcarriers. The resulting OFDM symbol extended with a cyclic prefix and the PN code is serially transmitted over a discrete-time channel. At the receiver, based on the property of the PN code, the frame synchronization and channel estimation schemes are applied. Besides, the receiver performs the inverse process of the transmitter, the data are demodulated by a FFT. One-tap equalizer is used to recover the channel effect. Then, the parallel data are demapped with corresponding PSK or QAM scheme to obtain the estimated bit stream.

Without timing and frequency offset, the baseband discrete-time data symbol \( x(n) \) is as

\[
x(n) = \sum_{k=0}^{N-1} X(k)e^{j\frac{2\pi nk}{N}},
\]  

(1)
where \( N \) denotes the number of subcarriers, \( T \) denotes the symbol duration, and \( X(k) \) is the complex modulated data of \( k \)th subcarrier. The data symbol extended with a cyclic prefix forms an OFDM symbol \( x_t(n) \),

\[
x_t(n) = \begin{cases} 
X(n - N_g + N), & 0 \leq n \leq N_g - 1, \\
X(n - N_g), & N_g \leq n \leq N + N_g - 1.
\end{cases}
\]  

(2)

where \( N_g \) denotes the sampling number of guard interval. The proposed frame format is shown in Fig. 2. The PN code is added in the OFDM symbol. The transmitted OFDM symbol is as

\[
s(n) = \begin{cases} 
c(n), & 0 \leq n \leq N_c - 1, \\
x_t(n - N_c), & N_c \leq n \leq N_g + N + N_c - 1.
\end{cases}
\]  

(3)

where \( c(n) \) denotes the PN code with the chip rate \( CR \), \( N_c \) is the sampling number of the PN code before spreading.

Multipath is the main effect that makes the distortion of received signal in wireless communications. In this paper, the multipath channel is referred to the OFDM multipath channel model in the 802.11 wireless LAN [19]. The multipath channel is expressed as

\[
h(n) = \sum_{l=1}^{L} h_l \delta(n - \tau_l),
\]  

(4)

where \( L \) denotes total path number, \( \tau_l \) is the delay time of \( l \)th path and \( \tau_1 \neq 0 \). The channel impulse response is composed of complex samples with random uniformly distributed phase and Rayleigh distributed magnitude with average power decaying exponentially. Hence, the \( l \)th channel impulse response is given by

\[
h_l = N \left( 0, \frac{1}{2} \sigma_l^2 \right) + j N \left( 0, \frac{1}{2} \sigma_l^2 \right),
\]  

(5)

where \( N(0, \sigma_l^2/2) \) is a Gaussian random variable with zero mean and variance \( \sigma_l^2/2 \), where \( \sigma_l^2 = \sigma_0^2 \cdot e^{-\frac{T_s}{T_{RMS}}} \) and \( \sigma_0^2 = 1 - e^{-\frac{T_s}{T_{RMS}}} \), where \( T_s \) is the sampling period and \( T_{RMS} \) is the delay spread of the channel. Besides, the maximum
delay spread value is assumed to be smaller than the cyclic prefix in this paper. Hence, the received signal \( y(n) \) could be expressed as

\[
y(n) = \sum_{l=1}^{L} h_l s(n - \tau_l) + w(n),
\]

(6)

where \( w(n) \) is an additive white Gaussian noise.

Before demodulating the received signal in OFDM system, the receiver has to work on the frame synchronization, frequency synchronization and channel estimation. On the OFDM link, the orthogonal subcarriers are required if the transmitter and receiver use the exact frequencies. Actually, the carrier frequency synchronization algorithm in [20], for instance, could be used to compensate the effect of frequency offset. In this study, the carrier frequency synchronization is not considered. However, the frame synchronization and channel estimation have to be considered. When an OFDM symbol is received, the first step is to retrieve the PN code to achieve the frame synchronization. Then, channel estimation also could be done based on the PN code. Finally, the signal with estimated channel response could be compensated through one-tap equalizer in frequency domain. In the following section, the frame synchronization is proposed to estimate the frame timing.

3. Proposed frame synchronization scheme

According to the proposed frame format, a PN code is inserted within each OFDM symbol. The frame timing could be achieved by using the autocorrelation properties of the PN code. In this study, the PN code is selected as Gold cold, the autocorrelation of Gold code is [21]

\[
\theta_{bb}(k) = \begin{cases} 
\frac{-1}{N_c} t(n), \\
\frac{-1}{N_c}, \\
\frac{1}{N_c} [t(n) - 2], 
\end{cases}
\]

(7)

where \( N_c \) denotes the code length, \( n \) is an integer, and

\[
t(n) = \begin{cases} 
1 + 2^{0.5(n+1)}, & \text{for } n \text{ odd}, \\
1 + 2^{0.5(n+2)}, & \text{for } n \text{ even}.
\end{cases}
\]

(8)

Based on the proposed frame format, the frame timing could be obtained according to maximum correlation between the desired PN code and received signal. Hence, at the receiver, the correlation between the received signal and PN code is obtained as

\[
R_{yc}(m) = \sum_{m=1}^{N_{OB}} \sum_{n=0}^{CR N_c - 1} y(n) c(k + m) = \sum_{m=1}^{N_{OB}} \sum_{n=0}^{CR N_c - 1} \left\{ \sum_{l=1}^{L} h_l c(n - \tau_l) + \sum_{l=1}^{L} h_l x(n - \tau_l) + w(n) \right\} \cdot c(n + m) \\
\approx \sum_{l=1}^{L} CR \cdot N_c \cdot h_l \cdot \delta(m - \tau_l),
\]

(9)

where \( N_{OB} \) denotes the length of the observation interval, \( CR \) denotes the chip rare of PN code \( c(n) \). In the proposed algorithm, the estimated frame timing is chosen by the value of \( \hat{m} \) to obtain the maximum of the cross-correlation between received signal and PN code in the observation interval \( N_{OB} \), i.e.,

\[
\hat{m} = \arg \max R_{yc}(m).
\]

(10)

Hence, the estimated frame timing, \( \hat{m} \), could be obtained. In the following section, the proposed channel estimation scheme is used to estimate the channel impulse response.

4. Channel estimation scheme and signal compensation

Based on the proposed synchronization scheme, the frame timing is obtained. Besides, the path delay time could also be estimated within the peak values of cross-correlation function under the multipath environment. Under the multipath environment, the received signal could be expressed as the linear combination of the transmitted signal with different delays in Fig. 3. When the path delay time is estimated, the channel impulse could be obtained with the proposed scheme. In order to simplify the expression, an example of channel estimation for the three-ray channel is shown in Fig. 4.
Thus, the received signal can be expressed as

\[
\begin{align*}
\forall 1 \leq i \leq k_1, & \quad y_i = h_0 \cdot c_i + w_i, \\
\forall k_1 + 1 \leq i \leq k_1 + k_2, & \quad y_i = h_0 \cdot c_i + h_1 \cdot c_i - r_1 + w_i, \\
\forall k_1 + k_2 + 1 \leq i \leq k_1 + k_1 + k_3, & \quad y_i = h_0 \cdot c_i + h_1 \cdot c_i - r_2 + h_2 \cdot c_i - r_1 - r_2 + w_i,
\end{align*}
\] (11)

where \( h_0, h_1 \) and \( h_2 \) are the channel tap gains, \( c_i \) is the spreading code, \( y_i \) is the received signal, and \( w_i \) is an AWGN noise.

In this paper, maximum likelihood criterion is used to obtain the channel tap gain, the likelihood function of the received signal could be given as

\[
\Lambda(h_0, h_1, h_2) = \frac{1}{(2\pi \sigma^2)^{N/2}} e^{-\frac{\sum_{i=1}^{N} |y_i - h_0 \cdot c_i|^2}{2\sigma^2}} \cdot e^{-\frac{\sum_{i=k_1+1}^{k_1+k_2} |y_i - h_0 \cdot c_i - h_1 \cdot c_i - k_1|^2}{2\sigma^2}} \times e^{-\frac{\sum_{i=k_1+k_2+1}^{k_1+k_1+k_3} |y_i - h_0 \cdot c_i - h_1 \cdot c_i - k_2 - h_2 \cdot c_i - k_1 - k_2|^2}{2\sigma^2}}. 
\] (12)

Hence, the log-likelihood function is expressed as

\[
\ln \Lambda(h_0, h_1, h_2) = -\frac{N}{2} \cdot \log(2\pi \sigma^2) - \frac{\sum_{i=1}^{N} |y_i - h_0 \cdot c_i|^2}{2\sigma^2} - \frac{\sum_{i=k_1+1}^{k_1+k_2} |y_i - h_0 \cdot c_i - h_1 \cdot c_i - k_1|^2}{2\sigma^2} - \frac{\sum_{i=k_1+k_2+1}^{k_1+k_1+k_3} |y_i - h_0 \cdot c_i - h_1 \cdot c_i - k_2 - h_2 \cdot c_i - k_1 - k_2|^2}{2\sigma^2}. 
\] (13)

Thus, the channel tap gain could be estimated by

\[
(\hat{h}_0, \hat{h}_1, \hat{h}_2) = \arg \max_{(h_0, h_1, h_2)} \left[ \ln \Lambda(h_0, h_1, h_2) \right]. 
\] (14)
With the derivation of log-likelihood function, the estimated channel tap gain, $\beta_0$, $\beta_1$ and $\beta_2$ could be obtained by

\[
\begin{align*}
\frac{\partial \ln \Lambda(h_0, h_1, h_2)}{\partial h_0} &= \frac{\sum^{k_1}_{i=1} c_i \cdot |y_i - h_0 \cdot c_i|}{\sigma^2} + \frac{\sum^{k_1+k_2}_{i=k_1+1} c_i \cdot |y_i - h_0 \cdot c_i - h_1 \cdot c_{i-k_1}|}{\sigma^2} \\
&\quad + \frac{\sum^{k_1+k_2+k_3}_{i=k_1+k_2+1} c_i \cdot |y_i - h_0 \cdot c_i - h_1 \cdot c_{i-k_1} - h_2 \cdot c_{i-k_1-k_2}|}{\sigma^2} = 0, \\
\frac{\partial \ln \Lambda(h_0, h_1, h_2)}{\partial h_1} &= \frac{\sum^{k_1+k_2}_{i=k_1+1} c_{i-k_1} \cdot |y_i - h_0 \cdot c_i - h_1 \cdot c_{i-k_1}|}{\sigma^2} \\
&\quad + \frac{\sum^{k_1+k_2+k_3}_{i=k_1+k_2+1} c_{i-k_2} \cdot |y_i - h_0 \cdot c_i - h_1 \cdot c_{i-k_1} - h_2 \cdot c_{i-k_1-k_2}|}{\sigma^2} = 0, \\
\frac{\partial \ln \Lambda(h_0, h_1, h_2)}{\partial h_2} &= \frac{\sum^{k_1+k_2+k_3}_{i=k_1+k_2+1} c_{i-k_1-k_2} \cdot |y_i - h_0 \cdot c_i - h_1 \cdot c_{i-k_1} - h_2 \cdot c_{i-k_1-k_2}|}{\sigma^2} = 0.
\end{align*}
\]  

(15)

In Eq. (15), the estimated channel gain could be obtained as below:

\[
\begin{align*}
\hat{h}_0 &= \frac{1}{k_1} \sum^{k_1}_{i=1} c_i \cdot y_i, \\
\hat{h}_1 &= \frac{1}{k_2} \sum^{k_1+k_2}_{i=k_1+1} c_{i-k_1} (y_i - \hat{h}_0 \cdot c_i), \\
\hat{h}_2 &= \frac{1}{k_3} \sum^{k_1+k_2+k_3}_{i=k_1+k_2+1} c_{i-k_1-k_2} (y_i - \hat{h}_0 \cdot c_i - \hat{h}_1 \cdot c_{i-k_1}).
\end{align*}
\]  

(16)

With the proposed scheme, the channel impulse response could be obtained. To avoid the ISI effect caused by multipath environment, an equalizer could be designed to compensate the received signal according the estimated channel response. Hence, the frequency domain one-tap equalizer is proposed to achieve the signal compensation. Based on the proposed scheme, simulations are given in the following.
5. Simulation results

At the beginning, the frame synchronization is evaluated by miss probability of frame timing and, then, compared with moving average (MA) scheme and Beek’s timing estimation scheme [6]. In each simulation, it is assumed the carrier frequency to be 5 GHz, required bandwidth to be 5 MHz, mapping scheme to be 16QAM, the number of the total subcarriers to be 1024, the sampling number of cyclic prefix to be 256, the sample duration to be 0.2 μs, the length of PN code before spreading to be 35, the chip rate to be 1 or 4, and the number of pilots to be 32. In Fig. 5, the missing probability of frame timing with the proposed algorithm performs better than that with the MA scheme and Beek’s algorithms. The results show that the frame timing could be obtained even at a lower SNR. Besides, the performance of channel estimation scheme is presented. Fig. 6 shows the estimator mean square error (MSE) with the proposed channel estimation scheme performs better than that with the least square (LS) and the MMSE algorithms using the frequency domain pilots and the linear
interpolation (LS + linear interpolation and MMSE + linear interpolation). Moreover, the bit error rate (BER) performance with the perfect channel estimation (Perfect Channel Est.), LS and MMSE with linear interpolation (LS + linear interpolation and MMSE + linear interpolation), and PN code based channel estimation scheme (Proposed scheme) are shown in Fig. 7. In the figure, the system performance with the proposed scheme performs better than that with the other algorithms.

6. Conclusion

In this paper, the PN code is inserted into the OFDM symbol to form a frame format. Based on the properties of the PN code, frame timing and the path delay time can be obtained with the proposed scheme. According to the information, the channel impulse response can be obtained in the proposed scheme. Then, based on the estimated channel response, a one-tap frequency domain equalizer is presented to achieve the signal compensation. The simulation results show that the frame timing could be obtained even at a lower SNR. Besides, the estimator mean square error of channel estimation with the proposed algorithm is lower than that with the LS and the MMSE algorithms using frequency domain pilots. Besides, the frequency domain one-tap equalizer is presented to realize the signal compensation and, then, the system performance can be improved in the OFDM system.

References


Jyh-Horning Wen received the Ph.D. degree in electrical engineering from National Taiwan University, Taipei, in 1990. From February 1991 to July 2007, he was with the Institute of Electrical Engineering, National Chung Cheng University, Chia-Yi, Taiwan, first as an Associate Professor and, since 2000, as a Professor. Since August 2007, he has been with the Department Head of Electrical Engineering, Tunghai University, Taichung, Taiwan. He is an Associate Editor of the Journal of the Chinese Grey System Association. His current research interests include OFDM system, multi-carrier system, personal communications, spread-spectrum techniques, wireless broadband systems, and gray theory.

Gwo-Ruey Lee received the B.S. degree in Department of Electronic Engineering from the Fu-Jen Catholic University, Taipei, Taiwan, in 2000, the M.S. degree from Department of Communications Engineering, National Chung Cheng University, Chia-Yi, Taiwan, in 2002 and the Ph.D. degree in Department of Electrical Engineering, National Chung Cheng University, Chia-Yi, Taiwan, in 2008. Since September 2009, he is a post-doctoral researcher at Taipei Municipal University of Education. His current research interests include OFDM system, multi-carrier system, personal communications, spread-spectrum techniques, wireless broadband systems, radar systems and its applications.

Te-Lung Kung received the M.S. degree from Department of Electrical Engineering, National Chung Cheng University, Chia-Yi, Taiwan, in 2003. His current research interests include OFDM system and wireless broadband systems.
Shu-Hong Lee received the B.S. degree in electronic engineering from Feng-Chia University, Taichung, Taiwan, ROC in 1994, the M.S. degree in electrical engineering in 1998 from National Yunlin University of Science and Technology, Yunlin, Taiwan and the Ph.D. degree in electrical engineering in 2008 from National Chung Cheng University, Chiayi, Taiwan, ROC. Since August 2002, he has been a Lecturer of Department of Electronic Engineering, Chienkuo Technology University, Changhua, Taiwan. His research interests include multiuser detection, orthogonal frequency division multiplexing and its applications.