Joint Maximum Likelihood Estimation of Channel and Preamble Sequence for WiMAX Systems

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Abstract—This paper examines the detection problem of the preamble sequence index in the WiMAX system. The mobile station receiver knows all the possible preamble sequences and should estimate which preamble sequence has been transmitted from the base station. Since the preamble in the orthogonal frequency division multiplexing (OFDM) transmission is usually the first received symbol, the channel is unknown to the receiver, which makes the problem of preamble sequence estimation complicated. In this paper, this problem is addressed by developing the joint maximum likelihood (ML) estimator of the preamble sequence and the channel. A simple decoupled estimator and a minimum mean square error (MMSE) estimator are also presented as benchmarks for the joint ML estimator. Then it is shown how the joint ML estimator can be used for the segment detection. Since the joint ML estimator can be computationally complex in its general form, low-complexity algorithms are developed depending on the type of pilot subcarrier locations for general OFDM systems including WiMAX. The simulation results show that the joint ML estimator detects the preamble sequence index very well in the absence of the channel knowledge.

Index Terms—Orthogonal frequency division multiplexing (OFDM), WiMAX, channel estimation, preamble detection, maximum likelihood estimation.

I. INTRODUCTION

RECENTLY, orthogonal frequency division multiplexing (OFDM) systems are widely used for wireless communication systems. The OFDM is used in the wireless local area network (LAN) based on the IEEE 802.11 standards, WiMAX based on the IEEE 802.16 standards, digital audio broadcasting, digital video broadcasting, etc. One of the primary reasons that OFDM is widely used is its simplicity in the system design due to the use of fast Fourier transform (FFT) instead of a complex equalizer.

Usually for OFDM systems, the first symbol of the frame is a preamble OFDM symbol. The preamble OFDM symbol is used mainly for synchronization and channel estimation. For example, the IEEE 802.11 standards define short and long sync (preamble) symbols that are known to the receiver to aid the synchronization and channel estimation. However, unlike the wireless LAN based on IEEE 802.11, the WiMAX system does not have a preamble OFDM symbol that is completely known to the receiver; the receiver has only partial information about the preamble OFDM symbol. The WiMAX system has 114 preamble sequences in total, to distinguish signals from multiple cells and segments in the cellular network. The base station transmitter chooses one of the 114 preamble sequences and transmits the corresponding preamble OFDM symbol. The mobile station receiver knows all 114 preamble sequences, but does not know which preamble sequence has been chosen by the base station.

Thus, this paper deals with the problem of the preamble sequence estimation for OFDM systems such as the WiMAX system, when only the candidate preamble sequences are known to the receiver. The preamble sequence detection problem is considered under the assumption that the channel is unknown. This assumption is essential for practical system design because the preamble OFDM symbol is usually the first OFDM symbol and there is no channel knowledge at the beginning. The preamble sequence detection with a known channel is quite straightforward. For example, the received signal can be equalized by the channel and then correlated with all preamble sequences to find the match. The channel estimation for a given preamble sequence is also researched heavily, and some of the widely used techniques can be found in [1]–[5]. In particular, the maximum likelihood (ML) channel estimator is presented in [5], with known preamble pilots assumed. This paper, however, considers the preamble sequence detection problem when the channel is unknown to the receiver, and derives the joint ML estimator of the preamble sequence and the channel. This joint ML estimator is calculated in a simple way when the pilot location for the preamble sequence has some regularity. The estimator is applied to WiMAX system based on IEEE 802.16e, and the simulation results are provided.

The rest of the paper is organized as follows. Section II describes an OFDM system model and defines the preamble sequence estimation problem. Section III provides a simple estimator that decouples the preamble sequence estimation and the channel estimation. Section IV presents the joint ML estimator in the general form and its low-complexity versions. A minimum mean square error (MMSE) estimator is also provided to highlight the distinction of the joint ML estimator. The joint ML estimator is shown to have additional capability of segment detection in the WiMAX system. Then, Section V evaluates the performance of the joint ML estimator by simulation, comparing to the simple decoupled scheme and the MMSE estimator. Section VI summarizes the paper.

II. SYSTEM MODEL

An OFDM system transmits information as a series of OFDM symbols, and an OFDM symbol can be generated...
as follows. The inverse discrete Fourier transform (IDFT) is performed on \( N \) information symbols \( X[k] \) for \( k = 0, 1, \ldots, N - 1 \) and cyclic prefix samples of length \( N_g \) are prepended. Here \( k \) represents the subcarrier index. Then the \( n \)-th time-domain\(^1\) sample \( x[n] \) can be expressed as follows:

\[
x[n] = \left\{ \begin{array}{ll}
\frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j \frac{2\pi kn}{N}} & \text{for } -N_g \leq n \leq N - 1 \\
0 & \text{elsewhere.}
\end{array} \right.
\]

The OFDM symbol \( x[n] \) is transmitted through a channel \( h[n] \), which is assumed to be block-stationary, i.e., time-invariant during each OFDM symbol. It is also assumed that the channel has finite duration such that \( h[n] = 0 \) for \( n < 0 \) and for \( n > N_g \). With these assumptions, the output \( y[n] \) of the channel can be represented as

\[
y[n] = \sum_{r=0}^{N_g} h[r] x[n-r] + z[n],
\]

where \( z[n] \) is the additive white complex circularly symmetric Gaussian noise with zero mean and variance \( \sigma_Z^2 \).

The received preamble OFDM symbol in the frequency domain can be represented as follows:

\[
Y[k] = H[k] X_i[k] + Z[k],
\]

where \( H[k] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N_g} h[n] \exp \left( -j \frac{2\pi kn}{N} \right) \), \( X_i[k] \) is the \( k \)-th value of the \( i \)-th preamble sequence, and \( Z[k] \) is the Gaussian noise. It is assumed that all the possible preamble sequences are known to the receiver, but the receiver does not know which preamble sequence has been transmitted. It is further assumed that the subcarriers consist of only pilot subcarriers and null subcarriers. Null subcarriers, by definition, have zero values, whereas the pilot subcarriers are assumed to have magnitude of one with arbitrary phase \( \theta_i[k] \):

\[
X_i[k] = \left\{ \begin{array}{ll}
0, & k \in N_i \\
\exp(j \theta_i[k]), & k \in P_i
\end{array} \right.
\]

where \( N_i \) and \( P_i \) are the sets of null subcarrier indices and pilot subcarrier indices, respectively, for the preamble sequence index \( i \). Thus, the pilot subcarrier indices and the null subcarrier indices can be different depending on the preamble sequence. This is general enough to include the pilot pattern of the IEEE 802.16e-based WiMAX system, which defines three distinct sets of pilot locations.

In a vector-matrix notation, the received signal can be expressed as

\[
Y = X_i H + Z.
\]

Since \( H[k] \) is a discrete Fourier transform of \( h[n] \) and the channel is of finite duration, the received signal can be represented in terms of \( \tilde{h} = [h[0] \ h[1] \cdots h[N_g]]^T \):

\[
Y = X_i \tilde{F} h + Z,
\]

where \( \tilde{F} \) is a submatrix of size \( N \times (N_g + 1) \) of a discrete Fourier matrix \( F \) whose \((m, n)\)-th element is

\[
F_{m,n} = \frac{1}{\sqrt{N}} \exp \left( \frac{-2\pi (m-1)(n-1)}{N} \right).
\]

For notational convenience, \( I_{m,n} \) is defined as follows. For \( m > n \),

\[
I_{m,n} = \begin{bmatrix} I_n \\ O_{m-n,n} \end{bmatrix},
\]

where \( I_n \) is an identity matrix of size \( n \times n \) and \( O_{m,n} \) is a zero matrix of size \( m \times n \), respectively. On the other hand, for \( m < n \),

\[
I_{m,n} = \begin{bmatrix} I_m \\ O_{m,n-m} \end{bmatrix}.
\]

Then, the Fourier submatrix \( \tilde{F} \) can be represented as

\[
\tilde{F} = F I_{N,N_g+1}.
\]

With the received signal represented as (9), the objective is to estimate both \( i \) and \( h \) from the received signal \( Y \). The difficulty of this problem arises from the fact that both the channel \( h \) and the preamble sequence index \( i \) are unknown. The detection of the preamble sequence index without channel knowledge is quite challenging. So is the channel estimation without the knowledge of the preamble sequence. However, the estimation of the preamble sequence index in the absence of the channel knowledge may not be as difficult as it looks because usually the number of preamble sequences is much smaller than the number of pilot subcarriers. For example, in IEEE 802.16e systems with 1024 FFT size, the number of preamble sequences is 114, whereas the number of pilot subcarriers used for each preamble sequence is 852. The total of 852 symbols are used for the transmission of \( \log_2(114) \approx 6.8 \) bits of information, so that the preamble OFDM symbol can be considered as being heavily coded. Thus, it can be expected that the preamble sequence index can be estimated reasonably well in the absence of channel knowledge as long as the number of the preamble sequences is much smaller than that of the pilot subcarriers.

III. Simple Decoupled Estimator

In this section, a simple decoupled estimator of the preamble sequence index and the channel is first derived before the joint ML estimator is developed. This estimator decouples the preamble sequence index and channel estimation at the cost of performance degradation. It will serve as a benchmark for the performance comparison of the joint ML estimator.

First, we consider the preamble sequence index estimation when the channel is known to the receiver. In this case, the ML estimate of the preamble sequence index \( i \) with the observation of \( Y[k] \) is

\[
\hat{i} = \arg \min_i \left\{ \sum_{k=0}^{N-1} |Y[k] - H[k] X_i[k]|^2 \right\}.
\]
Since $X_i[k]$ is assumed to have the same magnitude regardless of the preamble sequence, the ML estimator can be simplified as follows:

$$\hat{i} = \arg \max_i \left\{ \Re \left( \sum_{k=0}^{N-1} Y[k]H^*[k]X_i^*[k] \right) \right\}.$$  

(15)

Now we consider the case when the channel has the same magnitude and the unknown phase common for all subcarriers, i.e., $H[k] = |H| \exp(j \theta)$. In this case, it can also be derived that the joint ML estimator of the preamble sequence index $i$ and the phase $\theta$ is given by the following:

$$\hat{i} = \arg \max_i \left\{ \sum_{k=0}^{N-1} Y[k]X_i^*[k] \right\},$$  

(16)

and

$$\hat{\theta} = \angle \left( \sum_{k=0}^{N-1} Y[k]X_i^*[k] \right).$$  

(17)

It will be inefficient to use this preamble sequence index estimator (16) in a frequency selective channel. In case of a frequency selective channel, the estimator (16) can be modified as follows.

Consider a frequency selective channel that can be modeled as a flat channel within a band consisting of the fixed number of subcarriers, i.e., $H[k] = |H|_i \exp(j \theta)$ for $bN/B \leq k \leq (b+1)N/B - 1$, where $b = 0, 1, \cdots, B - 1$ and $B$ is the number of distinct frequency bands that divide the whole $N$ subcarriers. The correlation is calculated separately within a band and then the magnitudes of the correlations is summed over the whole bands, i.e.,

$$\hat{i} = \arg \max_i \left\{ \sum_{l=0}^{B-1} \sum_{k=1(l/N/B)}^{(l+1)(N/B)-1} Y[k]X_i^*[k] \right\}. $$  

(18)

Note that (16) is the special case of (18) with $B = 1$, i.e., without band division. And in general, $B$ should be increased for more frequency selective channels.

It can be seen that the number of bands, $B$, needs to be determined carefully based on the channel it operates. For the best performance, the number of bands may need to be determined adaptively based on the channel knowledge. However, in the absence of the channel knowledge, on the long-term statistics of the channel may be used.

Once the preamble sequence is estimated with the above estimator, channel estimation can be done with an estimated preamble sequence using various methods such as MMSE and ML channel estimators in [1]–[5].

IV. JOINT MAXIMUM LIKELIHOOD ESTIMATION OF PREAMBLE SEQUENCE AND CHANNEL

Given the received signal $Y$, the joint maximum likelihood estimation of the channel $h$ and the preamble sequence index $i$ can be represented as follows:

$$(\hat{h}, \hat{i}) = \arg \max_{(h,i)} f(Y|h,i),$$  

(19)

where $f(Y|h,i)$ is a conditional probability density function of $Y$ given $h$ and $i$. Since the background noise $Z$ is a circularly symmetric complex Gaussian vector whose elements have means of zero and variances of $\sigma_z^2$, the conditional probability density function is given by

$$f(Y|h,i) = \frac{1}{(\pi \sigma_z^2)^N} \exp \left( -\frac{||Y - X_i \tilde{F}h||^2}{\sigma_z^2} \right).$$  

(20)

Since the exponential function $e^{-x}$ is monotonically decreasing with respect to $x$, maximizing $f(Y|h,i)$ is equivalent to minimizing the following square error:

$$S(h,i) = ||Y - X_i \tilde{F}h||^2.$$  

(21)

Then the problem of finding $(h,i)$ that minimizes $S(h,i)$ can be solved in two steps. First, for each possible $i$, $\tilde{h}_i$ that minimizes $S(h,i)$ is found. Second, among the $\tilde{h}_i$'s found in the first step, $i$ that minimizes $S(\tilde{h}_i, i)$ is chosen. And the estimate for $h$ is selected as $\tilde{h}(i)$. In other words,

$$\hat{h}_i = \arg \min_h \{S(h,i)\},$$  

(22)

$$\hat{i} = \arg \min_i \{S(\tilde{h}_i, i)\},$$  

(23)

and

$$\hat{h} = \tilde{h}(i).$$  

(24)

The channel estimate $\hat{h}_i$ for given $i$ that minimizes $S(h,i)$ can easily be derived by noticing that this is a well-known least-squares estimation problem. This least-squares estimation has been investigated in [5], the result of which is summarized as follows. For notational convenience, define $A_i = X_i \tilde{F}$. Assuming that $A_i$ is a full rank matrix, we have a solution to (24), given as $\hat{h}_i = A_i^H Y$, where $A_i^H$ represents a pseudo-inverse of $A_i$, i.e., $A_i^H A_i = I$. The pseudo-inverse $A_i^H$ can be represented as

$$A_i^H = (A_i^H A_i)^{-1} A_i^H.$$  

(25)

The corresponding square error $S(\hat{h}_i, i)$ for the channel estimate $\hat{h}_i$ is

$$S(\hat{h}_i, i) = ||Y - A_i \hat{h}_i||^2.$$  

(26)

For every $i$, the square error (26) is evaluated and the preamble sequence index $i$ that minimizes the square error (26) is chosen with the corresponding channel estimate $\hat{h}_i$.

As can be seen above, the joint ML estimation of channel and preamble sequence can be performed with little difficulty in theory. However, the blind application of a pseudo-inverse does not produce a computationally efficient algorithm. Moreover, it does not give much insight on this problem of joint channel and preamble sequence estimation. Especially, the solution that was presented above requires the calculation of the pseudo-inverse of $A_i$ for each $i$.

We now examine the channel estimation and the square error calculation in detail and present low complexity algorithms. We consider four different cases depending on the pilot pattern. We start with the simplest case when all subcarriers are pilot subcarriers, and then move on to the case when the pilot subcarriers are regularly spaced. The third case is when the pilot subcarriers are arbitrarily located but the pilot
locations are fixed regardless of the preamble sequence index, and the fourth is when the position of the pilot subcarriers are different for different preamble sequence indices. The last one is the practical pilot pattern for WiMAX, and is also used for our simulation results. Each of other cases is presented with its own algorithm to give some insight to the variation of the algorithm based on the different preamble type. The segment selection problem, which is related to the preamble sequence and channel estimation, is also addressed at the end of this section.

A. All Pilot Subcarriers

In this subsection, we consider the case when all the subcarriers are pilot subcarriers with the same energy. In this case, the diagonal matrix \( X_i \) representing the preamble sequence is a unitary matrix, i.e., \( X_i^H X_i = I_N \). Then the square error (21) can be represented as follows:

\[
S(h, i) = \| X_i^H Y - \hat{F} h \|^2 .
\]  

(27)

Since the columns of \( \hat{F} \) are orthogonal to each other with unit magnitude, the channel estimate \( \hat{h}_i \) that minimizes the square error for given preamble sequence index \( i \) can be shown to be

\[
\hat{h}_i = \hat{F}^H \hat{H}_i,
\]  

(28)

where \( \hat{H}_i = X_i^H Y \) is an initial frequency-domain channel estimate. Basically, the time-domain channel estimate can be obtained by calculating the first \( N_g + 1 \) IDFT samples of the initial frequency-domain channel estimate. As is shown in Appendix, the corresponding square error can be expressed in terms of \( \hat{h}_i \) as follows:

\[
S(\hat{h}_i, i) = \| Y \|^2 - \| \hat{h}_i \|^2 .
\]  

(29)

Essentially, this square error expression shows that choosing the preamble sequence index that minimizes the square error is equivalent to choosing the preamble sequence index that maximizes the squared norm of the final channel estimate. Thus, the preamble sequence index estimate can be simply found as

\[
\hat{i} = \arg \max_{i} \| \hat{h}_i \|^2
\]  

(30)

with the corresponding channel estimate \( \hat{h}_i \). Note that this channel estimation algorithm does not require the calculation of a pseudo-inverse of \( A_{i} \) for every \( i \) as in (25).

B. Regularly-Spaced Pilot Subcarriers

Here we explore the problem of joint ML estimation of channel and preamble sequence when pilot subcarriers are regularly spaced with null subcarriers between them. Define \( W = X_i X_i^H \), which is a diagonal matrix whose diagonal element has a value of 1 for a pilot subcarrier location and 0 for a null subcarrier location. The square error \( S(h, i) \) in (21) can be expressed as follows:

\[
S(h, i) = \| W ( Y - X_i \tilde{F} h ) \|^2 + \| ( I - W ) ( Y - X_i \tilde{F} h ) \|^2 .
\]  

(31)

Since \( ( I - W ) X_i = 0 \), the second term in (31) is simplified to \( \| ( I - W ) ( Y - X_i \tilde{F} h ) \|^2 = \| ( I - W ) Y \|^2 \), which does not depend on \( h \). Thus, we define a modified square error \( \hat{S}(h, i) \) with only the first term of (31):

\[
\hat{S}(h, i) = \| W ( Y - X_i \tilde{F} h ) \|^2 .
\]  

(32)

Since \( W = X_i X_i^H \) and \( W X_i = X_i \), the modified square error can be represented as

\[
\hat{S}(h, i) = \| X_i^H Y - W \tilde{F} h \|^2 = \| X_i^H Y - W F I_{N_g,N_g+1} \|^2 ,
\]  

(33)

where (13) is applied.

To further simplify (33), we consider the case when the even numbered subcarriers are pilots and the odd numbered subcarriers are nulls. Note that we can extend to other cases in a straightforward manner as long as the pilot subcarriers are regularly spaced. For this case, we have

\[
W F = \frac{1}{2} F \begin{bmatrix} I_{N/2} & I_{N/2} \\ I_{N/2} & I_{N/2} \end{bmatrix} .
\]  

(34)

Thus, \( \hat{S}(h, i) \) can also be represented as

\[
\hat{S}(h, i) = \| X_i^H Y - \frac{1}{2} F B h \|^2 ,
\]  

(35)

where

\[
B = \frac{1}{2} \begin{bmatrix} I_{N_g+1} \\ O_{N/2-N_g-1,N_g+1} \\ I_{N_g+1} \\ O_{N/2-N_g-1,N_g+1} \end{bmatrix} .
\]  

(36)

Since \( F \) is a unitary matrix, the modified square error is

\[
\hat{S}(h, i) = \| F^H X_i^H Y - B h \|^2 .
\]  

(37)

It can be easily shown that a pseudo-inverse of \( B \) is \( B^\dagger = 2 I_{N_g+1,N} \). We remind that the definition of \( I_{m,n} \) is given in (11) and (12).

The channel estimate \( \hat{h}_i \) that minimizes the modified square error for given preamble sequence index \( i \) is

\[
\hat{h}_i = 2 I_{N_g+1,N} F^H \tilde{H}_i = 2 F^H \hat{H}_i,
\]  

(38)

with the corresponding channel estimate \( \hat{h}_i \). Thus, the time-domain channel estimate is given by calculating the first \( N_g + 1 \) IDFT samples of the initial channel estimate \( \hat{H}_i \) and then multiplying them by the scaling factor of 2.

In the similar manner as in (29), it can be shown that the corresponding modified square error can be expressed as

\[
\hat{S}(\hat{h}_i, i) = \| W Y \|^2 - \| \hat{h}_i \|^2 .
\]  

(39)

Thus, the preamble sequence index estimate is given as

\[
\hat{i} = \arg \max_{i} \| \hat{h}_i \|^2
\]  

(40)

with the corresponding channel estimate \( \hat{h}_i \). Again, as in subsection IV-A, this channel estimation algorithm does not require the calculation of a pseudo-inverse of \( A_{i} \) for every \( i \) as in (25).
C. Arbitrarily Located But Fixed Pilot Subcarriers

Now, we assume that the positions of the pilot subcarriers are arbitrary but fixed for all preamble sequences. As in the last subsection, the modified square error can be represented as

$$\tilde{S}(h, i) = ||X_i^HW - W\tilde{h}||^2.$$  \hfill (41)

Let $\tilde{W} \in \mathbb{C}^{N_p \times N}$ be a matrix that contains only nonzero rows of $W$. Then the modified square error is expressed in terms of $\tilde{W}$ as

$$\tilde{S}(h, i) = ||\tilde{W}X_i^HW - \tilde{W}\tilde{h}||^2.$$  \hfill (42)

For notational convenience, let $A = \tilde{W}\tilde{F}$. Then the channel estimate for given $i$ can be expressed as

$$\hat{h}_i = A^\dagger\tilde{W}\tilde{H}_i,$$  \hfill (43)

where the pseudo-inverse $A^\dagger$ for a full rank matrix $A$ is $A^\dagger = (A^H A)^{-1}A^H$. Note that $A$ does not depend on the preamble sequence index $i$, so that the calculation of the pseudo-inverse of $A$ is necessary only once.

By noting that $W = W^HW$, the channel estimate for given $i$ can also be expressed as follows:

$$\hat{h}_i = (\tilde{F}^HW\tilde{F})^{-1}\tilde{F}^WH_i.$$  \hfill (44)

This equation shows that the time-domain channel estimate can be obtained by computing the $N_g + 1$ IDFT samples of the initial frequency-domain channel estimate and then filtering with a matrix $(\tilde{F}^HW\tilde{F})^{-1}$. In many cases, the frequency-domain channel estimate needs to be calculated rather than the time-domain channel estimate. The frequency-domain channel estimate is

$$\hat{H}_i = \tilde{F}\hat{h}_i = \tilde{F}(\tilde{F}^HW\tilde{F})^{-1}\tilde{F}^WH_i.$$  \hfill (45)

The frequency-domain channel estimate at pilot subcarriers is

$$\hat{H}_{i,\text{pilot}} = W\hat{H}_i = GH_i,$$  \hfill (46)

where

$$G = W\tilde{F}(\tilde{F}^HW\tilde{F})^{-1}\tilde{F}^H.$$  \hfill (47)

The channel estimation for a given preamble sequence is done by calculating an initial frequency-domain channel estimate at pilot subcarriers and filtering the initial estimate with a filter $G$ to obtain the final frequency-domain channel estimate. Here, the filter $G$, given in (47), is a time-domain low pass filter by removing all the sample points at $n > N_g$, and thus, it is a smoothing filter in the frequency domain. Figure 1 compares the initial and final channel estimates in frequency (i.e., before and after smoothing by the use of filter $G$). The figure shows that the final channel estimate is closer to the original channel response than the initial one, due to our assumption of finite channel duration within the cyclic prefix length $N_g$.

The modified square error can be expressed as

$$\tilde{S}(\hat{h}_i, i) = ||\hat{H}_i - \hat{H}_{i,\text{pilot}}||^2.$$  \hfill (48)

This shows that the criterion for the preamble sequence estimation is to find the preamble sequence index that has the least difference between the initial channel estimate and the final channel estimate.

![Figure 1. Comparison of initial and final channel estimates, i.e., $\hat{H}$ and $\tilde{H}$](image)

In the similar manner as in (29) and (39), it can be shown that the modified square error can be expressed as

$$\tilde{S}(\hat{h}_i, i) = ||WY||^2 - ||\hat{H}_{i,\text{pilot}}||^2,$$  \hfill (49)

where $\hat{H}_{i,\text{pilot}} = GH_i$. Thus, the preamble sequence index estimate is given as

$$\hat{i} = \arg\max_i ||\hat{H}_{i,\text{pilot}}||^2$$  \hfill (50)

with the corresponding channel estimate $\hat{H}_{i,\text{pilot}}$.

In this algorithm, the calculation of the frequency-domain channel estimate at the pilot subcarriers takes the most amount of computation. There are $N_p^2$ nonzero elements in matrix $G$. Thus, when the number $N_p$ of pilots used is not much larger than $N_g + 1$, the complexity of this direct frequency-domain approach can be less than that of the time-domain approach that requires to do inverse fast Fourier transform (IFFT), to filter it with a matrix of size $(N_g + 1) \times (N_g + 1)$, and to do FFT again.

Eq. (50) shows that finding the preamble sequence index that has the largest magnitude for the final channel estimate is a criterion equivalent to finding the preamble sequence index that has the least difference between the initial channel estimate and the final channel estimate. Both criteria make intuitive sense. Basically, if an incorrect preamble sequence is chosen, the initial channel estimate for each subcarrier can have a significantly different value from the channel estimates for adjacent subcarriers, even though the underlying channel varies smoothly over the frequency. The channel estimate after filtering can have a significantly different value from the channel estimate before filtering. However, if a correct preamble sequence is chosen, the channel estimate after smoothing will have little difference from the channel estimate before smoothing. Thus, the square error $S(\hat{h}_i, i)$ in (26), which essentially represents the difference between the channel estimate before smoothing and after smoothing, will be small for the correct preamble sequence index.

In addition, if an incorrect preamble sequence is chosen, the channel estimate error can be spread over all time-domain
samples. The smoothing filter removes the frequency-domain error components that do not lie on the space spanned by the complex exponential functions represented by the first \(N_g + 1\) time-domain samples. Thus, the channel estimate after filtering will have small values for an incorrect choice of preamble sequence. On the other hand, if a correct preamble sequence is chosen, the magnitude of the channel estimate after filtering will not be much smaller than that of the channel estimate before filtering because the time-domain channel estimate will be confined to the first \(N_g + 1\) samples in the absence of the background noise. The magnitude reduction due to filtering results only from the background noise components at \(n > N_g\).

This intuitive explanation of the joint ML estimator leads us to the possibility of devising other estimators. Basically, the ML channel estimator can be replaced by a plain channel estimator based on the locally weighted averages of pilot subcarriers or any other channel estimator. Here we present an MMSE channel estimator, so that we can highlight the distinction of the joint ML estimator, such as capability of segment detection and no need of channel statistics. Under the condition of receiving an unknown preamble sequence among multiple candidates, the MMSE estimator works as follow.

For every \(i\), the MMSE estimator calculates the channel estimate \(\hat{H}_i\) as a linear function of \(H_i = X_i^H Y\), such that
\[
\hat{H}_i = R_{HH}(R_{HH} + \sigma^2 Z I_N)^{-1} \hat{H}_i, \tag{51}
\]
where \(R_{HH} = E[H H^H]\) is the covariance matrix of channel \(H_i\), and can be calculated if the power delay profile of the channel is given. The MMSE estimator needs a priori knowledge on the channel covariance and noise variance. In practice, however, the channel statistics should be estimated from observation and may not be perfectly accurate. In the simulation results, we will show that the performance of the MMSE estimator is vulnerable to the incorrect knowledge of the channel statistics.

The criterion for preamble sequence selection can be either i) choosing the smallest difference between the initial channel estimate and the final or ii) choosing the largest magnitude of the final channel estimate. It is reminded that the ML estimator minimizes \(S(h, i) = ||Y - X_i \tilde{F} h||^2\) in (21), which leads to minimizing \(||\hat{H}_i - \hat{H}_{i,\text{pilot}}||^2\), and equivalently maximizing \(||\hat{H}_{i,\text{pilot}}||^2\). This equivalence does not necessarily hold for the MMSE estimator or the local averaging estimator. Since the metric used for optimization is different for each estimator, the estimate \(\hat{H}_i\) is different, too. Nevertheless, either of these two can be considered as a good criterion as the intuitive explanation above suggests.

D. Variable Pilot Subcarriers

Next, we consider the case when the positions of the pilot subcarriers are different for different preamble sequence indices. We define \(W_i = X_i^H X_i\). Let \(W\) be a diagonal matrix whose \((k, k)\)-th element is
\[
W_{k,k} = \begin{cases} 1, & \text{if } k \in \bigcup_{i=0}^{J-1} P_i \\ 0, & \text{otherwise}, \end{cases} \tag{52}
\]
where \(J\) is the total number of preamble sequence candidates (e.g., 114 in the WiMAX system) and \(P_i\) is the set of pilot subcarrier indices for the preamble sequence index \(i\). Note that the previous subsection considered the case of identical \(P_i\) for every \(i\), but here \(P_i\) can be different for each preamble sequence.

As in the previous subsection, we can define a modified square error \(\tilde{S}(h, i)\) as follows:
\[
\tilde{S}(h, i) = ||W(Y - X_i \tilde{F} h)||^2. \tag{53}
\]
This modified square error is further partitioned to two terms:
\[
\tilde{S}(h, i) = ||W_i(Y - X_i \tilde{F} h)||^2 + ||W(W - W_i)(Y - X_i \tilde{F} h)||^2. \tag{54}
\]
The first term can be simplified to \(||(Y - X_i \tilde{F} h)||^2 = ||X_i^H Y - W_i \tilde{F} h||^2\), whereas the second term can be simplified to \(||(W(W - W_i))Y - X_i \tilde{F} h||^2 = ||W(W - W_i)Y||^2\). Thus, unlike the previous subsection with \(W = W_i\), the modified square error now has the second term as follows:
\[
\tilde{S}(h, i) = ||X_i^H Y - W_i \tilde{F} h||^2 + ||W(W - W_i)Y||^2. \tag{55}
\]
Thus, the preamble sequence index \(i\) that minimizes the first term of (55) can be expressed as
\[
\hat{i} = (\tilde{F}^H W_i \tilde{F})^{-1} \tilde{F}^H X_i^H Y. \tag{56}
\]
The frequency-domain channel estimate is given as
\[
\hat{H}_i = \tilde{F}(\tilde{F}^H W_i \tilde{F})^{-1} \tilde{F}^H \hat{H}_i. \tag{57}
\]
The frequency-domain channel estimate at pilot subcarriers is
\[
\hat{H}_{i,\text{pilot}} = W_i \hat{H}_i = G_i \hat{H}_i, \tag{58}
\]
where
\[
G_i = W_i \tilde{F}(\tilde{F}^H W_i \tilde{F})^{-1} \tilde{F}^H W_i. \tag{59}
\]
Unlike the previous subsection, here \(G_i\) depends on the preamble sequence index \(i\). However, since the number of distinct pilot sets is small, the number of distinct \(W_i\) and \(G_i\) is small as well. For example, WiMAX has three distinct pilot subcarrier sets. Thus, \(G_i\)’s needs to be calculated only for three different \(i\)’s that represent different preamble subcarrier sets. Although the channel estimate that minimizes the first term of (55) can be obtained similarly as in the previous subsection, the modified square error needs to be calculated differently. The corresponding modified square error from (53) can be expressed simply in terms of \(H_i\) as follows:
\[
\tilde{S}(\hat{h}_i, i) = ||W_i Y - X_i \hat{H}_{i,\text{pilot}}||^2, \tag{60}
\]
which can also be expressed as
\[
\tilde{S}(\hat{h}_i, i) = ||W_i Y||^2 - ||\hat{H}_{i,\text{pilot}}||^2, \tag{61}
\]
where \(\hat{H}_{i,\text{pilot}} = G_i \hat{H}_i\). Thus, the preamble sequence index estimate is given as
\[
\hat{i} = \arg \max_i ||\hat{H}_{i,\text{pilot}}||^2 \tag{62}
\]
with the corresponding channel estimate \(\hat{H}_{i,\text{pilot}}\).
E. Segment Detection

In the WiMAX system, a set of available subcarriers can be partitioned into three segments and a base station can transmit data in frequency reuse 3 within a cell. This can reduce interference to users especially in the cell boundary at the cost of the total network capacity loss. Each base station transmits a preamble that uses only every third subcarrier corresponding to its segment. Then, a mobile station can differentiate the signal from interference transmitted by adjacent base stations, to its segment. Then, a mobile station can differentiate the preamble sequence, and then choosing the maximum value among them.

\[ \Pi_s = \{3n + s | n = 0, 1, 2, 3, \cdots \} \text{ for } s = 0, 1 \text{ or } 2, \] (63) where \( s \) represents a segment and the corresponding subcarrier set.

Now we show that the joint ML estimation can provide a convenient way to detect a segment as well as a preamble. From (61) the joint ML estimator has a property of

\[ \hat{S}(\hat{h}_i, i) + \|\hat{H}_{i, \text{pilot}}\|^2 = \|WY\|^2, \] (64)

where \( \|WY\|^2 \) is a constant for every \( i \). Minimizing \( \hat{S}(\hat{h}_i, i) \) is then equivalent to maximizing \( \|\hat{H}_{i, \text{pilot}}\|^2 \) for the joint ML estimator.

One widely used method for segment detection is to compute the total signal power of each segment in the preamble and then choose the segment that has the maximum total power. In our analysis, the preamble total power is given in \( \|\hat{H}_{i, \text{pilot}}\|^2 \). Due to the equivalence shown above, we conclude that the joint ML estimator that chooses the preamble sequence with the maximum \( \|\hat{H}_{i, \text{pilot}}\|^2 \) provides segment detection as well.

We briefly remark that there can be an unknown integer carrier frequency offset (CFO), which causes the same phase shift as using a different segment. The integer CFO estimation can also be done jointly with preamble detection, by calculating cross-correlations of the shifted received signal with the preamble sequences, and then choosing the maximum value among them.

V. SIMULATION RESULTS

In this section, the algorithms developed in the previous section are applied to IEEE 802.16e-based WiMAX systems and simulation results are obtained. The joint ML, MMSE in (51) and simple decoupled estimators in (18), all of which have been developed in this paper, are compared. The simulation results will highlight the distinct performance of the estimators in various channel conditions (Vehicular A and Pedestrian B) and assumptions (channel or SNR mismatches).

The channel models used for simulation are Pedestrian B and Vehicular A channels of the ITU channel model [7]. The Pedestrian B channel has a root mean square (rms) delay spread of approximately 0.633 microsecond with the maximum delay spread of 3.7 microsecond, whereas the cyclic prefix duration used for simulation is approximately 11.4 microsecond. Thus, the channel frequency selectivity is quite moderate. The Vehicular A channel has an rms delay spread of approximately 0.370 microsecond with the maximum delay spread of 2.51 microsecond. The channel frequency selectivity of Vehicular A is lower than that of Pedestrian B. The simulations are performed for the case of variable pilot subcarriers (subsection IV-D), using 512-FFT and 5 MHz bandwidth. Table I summarizes simulation parameters used in this Section.

The total of 114 preamble sequences, designed for the WiMAX system, are used for the simulations. Figure 2 shows the normalized cross-correlation of all pairs of different WiMAX preamble sequences.

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exact values of $E\{|h(m,n)|^2\}$ (where $m$ is a symbol index, $n$ a sample index, and $E\{}$ an ensemble average operator) are known for $0 \leq n \leq N_g$, where $N_g$ is the length of the cyclic prefix. Note that the joint ML estimator assumes only $h(m,n) = 0$ for $n < 0$ and $n > N_g$ and does not assume anything about the probability density function of $h(m,n)$ for $0 \leq n \leq N_g$, which is much weaker assumption than that of the MMSE estimator.

Figure 5 and 6 show the probability of the false detection of the preamble sequence index for Pedestrian B and Vehicular A channels respectively, when another preamble sequence is transmitted from a different segment with power 5 dB lower than the desired segment preamble. Thus, this simulation shows the capability of segment detection discussed in subsection IV-E. Again, the joint ML shows better performance of segment detection than the simple decoupled estimator, but worse than the ideal MMSE.

We emphasize that the good performance of MMSE is only with the perfect knowledge of channel statistics, $R_{HH}$ and $\sigma_Z^2$ in (51), which represent channel correlation and noise variance respectively. When the channel information for MMSE is not perfect, the performance of the MMSE estimator degrades as is shown in Figure 7 and 8. While it is quite robust to the SNR mismatch caused by incorrect noise estimation up to $\pm 10$ dB, the MMSE estimator is very vulnerable to channel mismatch, which has been simulated by intentionally providing the estimator with inaccurate second-order channel statistics. In this simulation, the power delay profile of a Pedestrian B channel is used for the estimator in a Vehicular A channel, and vice versa. Though correct channel statistics can lead to very good performance, it will increase the system complexity in practice for long channel observation and/or adaptive estimation. The corresponding performance loss is significant especially in the high SNR region since the difference resulted from incorrect match is especially noticeable at the low noise level. The joint ML estimator does not require a priori channel statistics and
Fig. 7. Probability of the false detection of the preamble sequence index for a Pedestrian B channel

Fig. 8. Probability of the false detection of the preamble sequence index for a Vehicular A channel

Fig. 9. Probability of the false detection of the preamble sequence index for a Pedestrian B channel in the presence of carrier frequency offset

Fig. 10. Probability of the false detection of the preamble sequence index for a Pedestrian B channel in the presence of co-channel interference

thus, is more flexible for channel estimation.

Up to now, all the simulation was done under the ideal condition of zero carrier frequency offset (CFO). However, in reality, there could be non-zero CFO because the carrier frequency synchronization that is usually performed before channel estimation and preamble detection is non-ideal. It is well known that the OFDM system performance is sensitive to CFO because the carrier frequency synchronization that is usually performed before channel estimation and preamble detection is non-ideal. It is well known that the OFDM system performance is sensitive to CFO, which damages subcarrier orthogonality and results in intercarrier interference (ICI). So, the performance of the preamble estimators in the presence of CFO is shown in Figure 9 for Pedestrian B channel. In the figure, it was assumed that the CFO is uniformly distributed between ±5% of the subcarrier spacing. The corresponding signal-to-interference-ratio (SIR) is larger than 20 dB due to ICI introduced by CFO. Since the SNR range of our interest is very low, the resulting ICI is dominated by high noise level and effects little degradation in Figure 9. The performance loss is severe in the SNR range above 15 dB, which is not shown here.

Figure 10 shows the performance of the estimators in the presence of co-channel interference in the Pedestrian B channel. We assume another preamble signal in the same channel with power 5 dB lower than the desired preamble signal. All the estimators suffer performance degradation. While the simple decoupled estimator is very vulnerable to co-channel interference, we see that the performance loss of joint ML and MMSE is acceptable, partly because of good orthogonality of the WiMAX preamble sequences (shown in Figure 2).

VI. SUMMARY

This paper explored the problem of the preamble sequence detection when the multipath channel gain is unknown. The primary difficulty of this problem lies in the fact that both the preamble sequence and the multipath channel are unknown. To overcome this difficulty, this paper derived the joint ML estimator of the preamble sequence and the channel. A simple decoupled estimator and an MMSE estimator were provided...
and compared with the joint ML estimator. To mitigate computational complexity of the joint ML estimator, low complexity algorithms were suggested for the practical cases, including the WiMAX system whose pilot subcarrier locations are not fixed. It was shown that the joint ML estimator can provide the segment detection as well in the WiMAX system. Simulation results for IEEE 802.16e-based WiMAX systems showed the superb performance of the joint ML estimator: 1) better preamble sequence detection than the simple decoupled estimator and 2) more robustness to the inaccurate knowledge/estimation of the channel statistics (such as power delay profile) than the MMSE estimator especially in the high SNR region. Finally, we remark that this joint ML estimator is generic, and thus applicable to general wireless OFDM systems.

APPENDIX

We give the proof of (29) as follows:

\[ S(\hat{h}_i, i) = \| \tilde{X}_i^H Y - \tilde{F} \hat{h}_i \|^2 \]  
\[ \quad = (X_i^H Y - \tilde{F} \hat{h}_i)^*(X_i^H Y - \tilde{F} \hat{h}_i) \]  
\[ \quad = Y^H X_i \tilde{F}_i^H Y - \hat{h}_i^H \tilde{F} \tilde{F}_i^H X_i^H Y \]  
\[ \quad \quad \quad - \bar{Y}^H X_i \tilde{F} \hat{h}_i + \hat{h}_i^H \tilde{F} \tilde{F}_i^H \tilde{F} \hat{h}_i \]  
\[ \quad = \| Y \|^2 - \bar{H}_i^H \bar{H}_i - \tilde{F} \tilde{F}_i^H \tilde{F} \hat{h}_i + \| \hat{h}_i \|^2 \]  
\[ \quad = \| Y \|^2 - \| \tilde{h}_i \|^2, \]  

where \( \bar{H}_i = X_i^H Y = \tilde{F} \hat{h}_i \) is used in (68) and \( \| \tilde{H}_i \|^2 = \| \hat{h}_i \|^2 \) in (69). The similar proof can be shown for deriving (39), (49) and (61) as well.

REFERENCES


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