Abstract

The visual servoing technique has been studied extensively as a robust method for navigating robots toward goal positions and orientations reliably. Unfortunately, the existing visual servoing methods require the calibration of cameras and robots, which is time-consuming. Thus, in this paper, we propose a visual servoing method which does not require the calibration of cameras and robots. In particular, we show that we can navigate uncalibrated robots to goal positions properly by using the projective reconstruction based on the abstract projection of control values.

1 Introduction

Recently, the visual servoing has been studied extensively for navigating robots to goal positions reliably. In particular, a linear control method based on interaction matrices [4] has been studied. For generating optimum paths, the epipolar geometry is also applied for robot navigations [2]. These existing visual servoing methods require calibration of cameras and robots [4, 2, 1, 3]. However, in general, it is very difficult to calibrate cameras and robots accurately and reliably.

Thus, in this paper, we propose a method for navigating uncalibrated robots from uncalibrated cameras properly by using the projective reconstruction of control values. In this method, we use uncalibrated robots which have 3 DOF in translational motions, and no rotations. Since the robots are uncalibrated, we do not know the direction of motions and the amount of motions caused by unit translational commands. The only assumption on the robots is that a fixed amount of control value causes a fixed amount of robot motion. Since cameras fixed on the robots are also uncalibrated, we do not know parameters of the cameras.

For navigating the uncalibrated robots properly under these conditions, we consider a control space defined by control values. Since the robot has three translational axes, its control space is three dimensional. A control value for a robot motion can be considered as a point in this space. If we send a control value \( Y \) to the robot, the viewpoint of the camera after a robot motion is projected to a point \( m \) in the image before the robot motion. Then, the image point \( m \) can be considered as a projection of the control value \( Y \). Thus, we can compute control values to navigate the robots properly by using the projective reconstruction of control values \( Y \) from points \( m \) in images without calibrating cameras and robots. In this paper, we propose the visual servoing method for uncalibrated robots from uncalibrated cameras based on this abstract projection of control values.

In section 2, we describe the epipolar geometry which plays an important role for considering the abstract projection of control values. We next show that the projective reconstruction can be applied for computing control values for navigating uncalibrated robots to goal positions precisely. Finally, we show the results of some visual servoing experiments.

2 Information from Uncalibrated Cameras

Let us consider a current viewpoint \( C \) and a goal viewpoint \( C' \) under visual servoing as shown in Fig. 1. In this case, a pair of corresponding points \( m = [u, v]^T \) and \( m' = [u', v']^T \) in two images at two viewpoints \( C \) and \( C' \) holds the epipolar equation as follows:

\[
\tilde{m}'^T \tilde{F} \tilde{m} = 0
\]  

(1)

where, \((\cdot)^T\) denotes homogeneous coordinates, and \( \tilde{F} \) denotes the fundamental matrix. Since we assume that the robots have only translational motions, camera motions are considered as pure translations, and the matrix \( \tilde{F} \) has only 2 DOF [7]. Since we can derive one constraint on \( \tilde{F} \) from a pair of corresponding points and (1), we can compute the matrix \( \tilde{F} \) from at least two corresponding points.

Then, we find that the epipole \( e \) in the current image \( \pi \) shows the direction of the goal viewpoint \( C' \) as shown in Fig. 1. Thus, if we send the robots a control value for navigating the robots toward the direction of the epipole \( e \), we can navigate the robots toward goal positions. Since we can compute the epipole \( e \) from \( \tilde{F} \) even if the cameras are uncalibrated, we can extract the information of the goal positions from uncalibrated cameras.
3 Control of Uncalibrated Robots

As shown in section 2, we need to control the robots toward the direction of epipoles for navigating robots to goal positions. However, since the robots are uncalibrated, we do not know how much control values are required for each translational axes for controlling the robots toward the direction of epipoles. In this section, we show that the epipoles can be considered as the projections of control values. Then, we propose a method for computing the control values for navigating the robots to goal positions from epipoles by using the projective reconstruction technique.

3.1 Projection of Control Values

To show the relationship between control values and epipoles, we consider a control space defined by control values. Since the robot has three translational axes, its control space is three dimensional. The single control value for a robot motion can be considered as a point in the control space. If the robots move to the points \( C = [X, Y, Z]^\top \) from \( C_0 \) by the control values \( Y \) in the control space, the relationship between \( Y \) and \( C \) can be described by a \( 4 \times 4 \) matrix \( A \) for 3D affine transformations as follows:

\[
\tilde{C} \sim A \tilde{Y} \tag{2}
\]

where, \( \sim \) denotes equality up to a scale. However, since the robot is uncalibrated, we do not know both the affine matrix \( A \) and the viewpoints \( C \).

Although we do not know the actual positions of viewpoints \( C \), their projections can be available as epipoles \( e = [u, v]^\top \) in the image \( \pi \) at the viewpoint \( C_0 \) as shown in section 2. Since the epipoles \( e \) are projections of the viewpoints \( C \), the relationship between \( C \) and \( e \) can be described by a \( 3 \times 4 \) projective camera matrix \( P \) as follows:

\[
\tilde{e} \sim PC \tag{3}
\]

However, since the epipoles are available, the projective camera matrix \( P \) is unknown.

Although both the affine matrix \( A \) and the projective camera matrix \( P \) are unknown, the relationship between the control values \( Y \) and the epipoles \( e \) can be derived from (2) and (3) as follows:

\[
\tilde{e} \sim P'\tilde{Y} \tag{4}
\]

where, \( P' = PA \). Since \( P' \) in (4) is a \( 3 \times 4 \) projective camera matrix, the epipoles \( e \) can be considered as the projections of control values \( Y \) by a projective camera as shown in Fig. 2. Thus, if we know the projective camera matrix \( P' \), we can directly compute the control values \( Y \) for navigating robots to goal positions from epipoles which correspond to goal positions. However, since cameras and robots are uncalibrated, the affine matrix \( A \) and the projective camera matrix \( P \) are unknown. Thus, in the next section, we show a method for computing the control values for navigating the robots to goal positions using the projective reconstruction technique by sending four basis control values in advance.

3.2 Computation of Control Values from Projective Reconstruction

For computing control values of uncalibrated robots, we first send the robots four arbitrary basis control values \( Y_1, Y_2, Y_3 \) and \( Y_4 \). These four control values \( Y_k (k = 1, \ldots, 4) \) and \( Y_0 = [0, 0, 0]^\top \) can be considered as five basis points in the control space. Note, any four of five points \( Y_k (k = 0, \ldots, 4) \) must not be coplanar, and any three of five points must not be colinear. As shown in (4), these five control values are projected to images as follows:

\[
\tilde{e}_{0k} \sim P'_0\tilde{Y}_k \tag{5}
\]
\[
\tilde{e}_{1k} \sim P'_1\tilde{Y}_k \tag{6}
\]

where, \( e_{ij} \) denotes an epipole of \( j \)th camera in \( i \)th image, and \( P'_i \) denotes a camera matrix \( P' \) of \( i \)th camera. Thus, the relationship between control values and epipoles can be considered as camera projections by projective camera matrices \( P'_0 \) and \( P'_1 \) as shown in Fig. 3.

Then, if we know the epipolar geometry between the image \( \pi_0 \) and the image \( \pi_1 \), the camera matrices \( P'_0 \) and \( P'_1 \) can be described as follows[6, 5]:

\[
P'_0 = [I \ 0] \tag{7}
\]
\[
P'_1 = [e_{10}]_x F \ e_{10} \tag{8}
\]
Therefore, we can compute \( Y'_k \) from two projective camera matrices \( P'_0 \) and \( P'_1 \) and epipoles \( \tilde{e}_{0k} = [u_{0k}, v_{0k}, w_{0k}]^T \) and \( \tilde{e}_{1k} = [u_{1k}, v_{1k}, w_{1k}]^T \) in image \( \pi_0 \) and \( \pi_1 \) by solving the following linear equations:

\[
\begin{bmatrix}
  u_{0k} P_{03} - v_{0k} P_{01} \\
  u_{0k} P_{03} - v_{0k} P_{02} \\
  u_{1k} P_{13} - v_{1k} P_{11} \\
  u_{1k} P_{13} - v_{1k} P_{12}
\end{bmatrix}
\begin{bmatrix}
  Y'_k
\end{bmatrix}
= 0 \tag{9}
\]

where, \( p_{ij} \) denotes a \( j \)th row of \( P'_i \). However, we have to note that there is the following relationship of 3D projective transformations between \( Y'_k \) and the control values \( Y_k \):

\[
Y'_k \sim H Y_k \tag{10}
\]

Since the projective transformation \( H \) can be computed from five corresponding points, we can compute \( H \) from given basis control values \( Y_0, \cdots, Y_4 \) and their reconstructions \( Y'_0, \cdots, Y'_4 \). Then, by using the matrix \( H \), we can compute the control values \( Y'_k \) from \( Y'_k \) for navigating the robots to the positions defined by epipoles \( e_{0k} \) and \( e_{1k} \). Thus, we can, from (9) and (10), compute the control values \( Y_g \) for navigating robots to goal positions by extracting the epipoles of goal viewpoints \( e_{0g} \) and \( e_{1g} \), in the image \( \pi_0 \) and \( \pi_1 \). Since each element of \( Y_g \) is the control value for each translational axes of the robots, we can navigate the robots to goal positions by sending the control values \( Y_g \) to robots. In this way, even if cameras and robots are uncalibrated, we can compute control values for navigating robots to goal positions properly from the projective reconstruction based on the abstract projection of control values.

4 Visual Servoing Experiments

We next show some results from the visual servoing experiments by using the proposed method.

4.1 Real Time Visual Servoing

We first show the results from real time visual servoing. Fig. 4 shows a robot arm with a camera used in our experiments. Since the robot arm and the camera are uncalibrated, we do not know the relationship between control values and robot motions, and we do not know internal and external parameters of the camera. The feature points in images are tracked by simple correlation trackers. Although the proposed method requires only two feature points, additional feature points can also be exploited for computing epipoles more accurately if they are available.

We first send four control values \( Y_1, \cdots, Y_4 \) to the robot. Next, we compute epipoles \( e_{g1}, e_{g2}, e_{g3}, e_{g4} \) and \( e_{10}, e_{12}, e_{13}, e_{14} \) from images taken before and after robot motions by using the method described in section 2. Then, we compute a control value \( Y_g \) from epipoles \( e_{0g} \) and \( e_{1g} \) of goal positions by using a method described in section 3.2. We can navigate the robot to the goal position by sending the computed control value \( Y_g \) to the robot.

Fig. 5 shows basis control values \( Y_1, Y_2, Y_3 \) and \( Y_4 \) given to the robot, and Fig. 6 shows epipoles in image \( \pi_0 \) and \( \pi_1 \) computed from images taken before and after robot motions. In this figure, white round dots show two of six feature points used for computing the epipolar geometry. The lines show epipolar lines, and square dots show extracted epipoles. Fig. 7 shows camera images and robot positions during visual servoing. Fig. 7(a) shows those at an initial position, and Fig. 7(b) shows those at a goal position. Fig. 7(c) shows those at a robot position after sending the control value \( Y_g \) computed by the proposed method. The camera image shown in Fig. 7(b) is almost identical with the goal image shown in Fig. 7(b). Also, as shown in Fig. 7(c), the robot was navigated to the goal position accurately.

4.2 Stability

We next show the stability of the proposed method. We navigate the robot from several different start positions, and evaluated the uncertainty bound of robot positions after visual servoing.

Fig. 8(a) shows the stability of servoing from minimum number of feature points, i.e. two points. The square dots show initial positions of the robot, and white dot is the goal position. The lines show loci of robot motions in the visual servoing, and the ellipsoid shows an uncertainty bound for \( 3\sigma \). X axis and Y axis show the horizontal and the vertical directions, and Z axis shows the direction of camera axis at
the goal position. The unit of these axes is [mm]. Fig. 8(b) shows the results from seven feature points. As shown in Fig. 8(a) and (b), the result is drastically improved by using a few more feature points than required. This is because the computation of the epipolar geometry is drastically improved by using more feature points than required.

5 Conclusion

In this paper, we proposed a method for navigating uncalibrated robots from uncalibrated cameras by using projective reconstruction of control values.

We first showed that goal positions can be identified by using the epipolar geometry, even if the cameras are uncalibrated. We next showed that the relationship between control values and epipoles can be described by a projective camera matrix, and showed that control values for navigating robots to goal positions can be computed without ambiguity from epipoles which show goal positions, even if the robots are uncalibrated. The proposed method was implemented and tested by navigating an uncalibrated arm robot with an uncalibrated camera to goal positions in real time.

References