Recovering 3D Shape and Light Source Positions from Non-Planar Shadows

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Abstract—Recently, Shadow Graph has been proposed for recovering 3D shapes from shadows projected on curved surfaces. Unfortunately, this method requires a large computational cost. In this paper, we introduce 1D Shadow Graph which can be used for recovering 3D shapes with much smaller computational costs. We also extend our method, so that we can estimate both 3D shapes and light source positions simultaneously under a condition where 3D shapes and light sources are unknown.

Keywords—Shape from Shadow; Light Source Estimation; Shadow Graph; Shortest Path Finding; Coarse-to-Fine

I. INTRODUCTION

If the light source moves in the 3D space, shadows of objects caused by the light are useful for recovering 3D shapes. Such methods are called Shape from Shadow [1], [2]. Recently, Shadow Graph [2] has been proposed for representing shadow information in images. It can represent the relative height between a pair of image pixels in a directed graph with costs, and is used for recovering 3D shapes from shortest path finding [3]. Unfortunately, shortest path finding in the Shadow Graph requires a large computational costs. We in this paper introduce 1D Shadow Graph which are extracted from circular motions of a light source. We show that the 1D Shadow Graph can be used for recovering 3D shapes with much smaller computational costs than before. We first derive a method for recovering 3D shapes from shadows caused by two orthogonal light motions, and then extend it to a method for two arbitrary light motions. Finally, we propose a method for recovering both 3D shapes and light source positions from shadows simultaneously under a condition where shapes and light sources are unknown.

II. SHADOW GRAPH

We first explain Shadow Graph proposed by Yu et al [2]. The Shadow Graph is a directed graph with costs which represent the relative height of points computed from the shadow and the orientation of light source. Let us consider a 3D surface as shown in Fig.1, and let $h(x)$ be the height of the surface at point $x$. The orientation of light source is $\phi$. First, considering a surface point in the shadow area $[x_c, x_a]$, the following inequality holds.

$$h(x) \leq h(x_a) - \tan \phi (x_a - x), \forall x \in [x_c, x_a]$$  \hspace{1em} (1)

This is called shadow constraints. The shadow constraints are represented by a directed graph, in which $-\tan \phi (x_a - x)$ is the cost from $x_a$ to $x$. Next, considering a surface point $x_1$ which is not in the shadow area, the following inequality holds.

$$h(x) \leq h(x_1) + \tan \phi (x - x_1), \forall x \in [x_1, x_c]$$  \hspace{1em} (2)

This is called anti-shadow constraints. The anti-shadow constraints are also represented by a directed graph, in which $\tan \phi (x - x_1)$ is the cost from $x_1$ to $x$. Thus, each cost in the Shadow Graph represents a relative height between a pair of pixels. Therefore, the 3D shape can be recovered by finding the shortest paths for all the pairs of nodes in the graph. However, the computational cost is very large because any pair of pixels may have edge in the Shadow Graph.

III. FAST 3D SHAPE RECOVERY FROM 1D SHADOW GRAPHS

In this section, we propose a method for recovering the 3D shape from a combination of 1D Shadow Graphs. In this method, we assume that the light source moves in two independent circular orbits.

We first explain the case, where a light source orbits along row and column directions as shown in Fig.2. In this case, the extracted Shadow Graphs are connected only in row or column direction. We call these graphs 1D Shadow Graphs. By finding the shortest path in each 1D Graph, we can recover a set of row and column slices of 3D surface. The computational cost is very small because the number of nodes in each 1D Graph is very small. However, each
Figure 2. Two independent circular orbits of light source motions

Figure 3. (a) Two light source orbits which are at an angle of $\theta_x$, $\theta_y$ to row and column directions. (b) Affine transformation of the image so that two orbits along row and column directions.

Figure 4. (a) The light source orbit inclined to zenith with an angle of $\omega$. (b) Original scene is inclined at an angle of $\omega$ to reference plane and light moves in the orbit which goes through zenith.

row and column slice of the surface has an ambiguity of vertical translation. Thus, we recover the whole shape by computing the vertical translation of each slice minimizing $\epsilon$ in the following equation:

$$\epsilon = \sum_{i,j} (h_{ij} + t_i - h_{ij}' - t'_j)^2$$  \hspace{1cm} (3)

where $h_{ij}$ and $h_{ij}'$ are height of row and column slices at pixel $(i, j)$, and $t_i$ and $t'_j$ are vertical translation of $i$th row slice and $j$th column slice.

We next explain the case, where two light source orbits are at an angle of $\theta_x$ and $\theta_y$ to row and column directions as shown in Fig.3 (a). In this case, we compute 2D affine transform $A$ which transforms these two orbits into row and column directions, and transform all images with $A$ as shown in Fig.3 (b). As a result, we can recover 3D shape by using the method mentioned above. But, the recovered shape is transformed by $A$. So, we obtain the final shape by transforming the recovered shape with the inverse transform $A^{-1}$.

We next consider the case, where the light source orbit is slanted with an angle of $\omega$ to zenith as shown in Fig.4 (a). In this case, shadows caused by moving light on the slanted orbit are equal to the shadows caused by the case where the original surface is inclined at an angle of $\omega$ to the reference plane and the light moves in the orbit which goes through zenith as shown in Fig.4 (b). Therefore, we can recover 3D shape with an affine ambiguity by using the method described above, since inclining the scene can be represented as a 3D affine transform.

IV. SIMULTANEOUS RECOVERY OF 3D SHAPE AND LIGHT SOURCE MOTIONS FROM SHADOWS

In this section, we consider the case where both 3D shapes and light source motions are unknown, and propose a method for estimating 3D shapes and light source motions simultaneously from shadows.

As described in section III, if we know the light source motions, we can recover the 3D shape of the scene, even if the light source orbits are slanted and are not perpendicular to each other. On the other hand, if we know the 3D shape of the scene, we can recover the light source motions. This can be achieved as follows.

We first discretize light source hemisphere with constant interval. Then, at each light source position we generate a shadow image from the light source position and the 3D shape of the scene, and check the consistency of generated shadow image with the real shadow image. After iterating this procedure for all the light source positions, we find the most suitable light source position by choosing the one which provides us the highest consistency.

Thus, if the 3D shape is known, we can estimate the light source position from shadows, and if the light source position is known, we can recover the 3D shape. Therefore, we estimate 3D shape and light source position simultaneously by iterating these two procedures alternately in the Coarse-to-Fine [5] framework. This is achieved as follows.

Suppose we have a set of images of an unknown 3D scene taken under unknown light source motions in two different orbits. We first recover an initial 3D shape from shadows in coarse images. Since light source motions are unknown, we assume $\theta_x = \theta_y = 0$, and generate 1D Shadow Graphs in which all the costs are equal to $-1$. By fixing the costs to $-1$, we simply represent the relationship of higher and lower between two points regardless of light source orientations. By using the slices recovered from these 1D graphs, we can obtain the initial 3D shape which represents a rough shape of the 3D surface. Based on this initial 3D shape, we next estimate the rough light source position of each image discretizing the light source hemisphere in
better resolution. The estimated light source positions are used for recovering the 3D shape in better resolution. By iterating these procedures alternately changing the resolution of images and light source positions from coarse to fine, we can estimate the 3D shape and the light source positions simultaneously in good accuracy avoiding local solution and reducing computational time. Note, the estimated 3D shape and light source positions include an affine ambiguity so-called Bas-Relief-Ambiguity [4].

V. EXPERIMENTS

A. Real image experiments

In this section, we show the results from the proposed method by using the real images. In this experiment, we use a face model as shown in Fig.5 (a) as an example of arbitrary curved surface. We obtained 34 images moving a light source around the object. The example of taken images are shown in Fig.5 (b). As shown in these images, we have very complex shadows on a curved surface. The initial shape recovered from the first iteration of the proposed method is shown in Fig.5 (c), and the final 3D shape recovered after the five iterations is shown in Fig.5 (d). From these results, we find that the initial result represents a rough 3D structure of the object, and the final result represents the 3D shape of the object properly. The recovered light source positions are shown in Fig.5 (e). The changes in coincidence ratio between the real shadow and the shadow generated from recovered shape and light source positions in each iteration is shown in Fig.5 (f). From this figure, we find the coincidence ratio increases as the number of iteration increases, and the proposed method can estimate accurate 3D shape and light source positions after few iterations.

B. Accuracy evaluation

In this section, we evaluate the accuracy of the proposed method by using synthetic images. In this experiment, we used a sinusoidal surface shown in Fig.6 (a). The camera is fixed in front of the object assuming orthogonal projection. The light source is at the infinity and moves around the object in two different orbits. The 3D shapes and light source positions recovered from the proposed method have affine ambiguity. In this experiment, we removed the affine ambiguity by computing an affine transformation between recovered 3D points and true 3D points. The accuracy of the recovered shape is evaluated by the difference $\Delta(\%)$ between the true height $h_{ij}$ and the recovered height $h'_{ij}$ as follows:

$$D = \frac{1}{N^2} \sum_{i,j} \frac{\|h_{ij} - h'_{ij}\|}{h_{\text{max}} - h_{\text{min}}} \times 100 \quad (4)$$

where, $h_{\text{max}}$ and $h_{\text{min}}$ are the maximum height and the minimum height of the true shape, and $N$ is the size of the image.

1) The accuracy of two light orbits which are inclined to row and column directions: We first evaluate the accuracy of the proposed method in the case where two light source orbits are at an angle of $\theta_x = 30^\circ$ and $\theta_y = -30^\circ$ to row and column directions respectively. The light source orientation in each orbit changes from $10^\circ$ to $170^\circ$ with interval of $10^\circ$. The final shape recovered from the proposed method is shown in Fig.6 (b). The coincidence ratio of shadows was $91.74\%$. The 3D shape and the light source positions after removing affine ambiguity from the final estimated results are shown in Fig.6 (c) and (d) respectively. The red points and the blue points represent the estimated positions and the real positions of the light source respectively, and the red line and the blue line represent the estimated orbit and the real orbit of the light source respectively. The error of height between the recovered shape and the real shape is $3.29\%$. In Fig.6 (d), the error of angle between the estimated positions and the real positions of the light source is $4.28^\circ$. From these results, we find that the proposed method can estimate both 3D shape and the light source positions accurately.

2) The accuracy of variable sampling interval: We next evaluate the accuracy of the proposed method in the case where the interval of light source orientation is not constant. We used the same experimental settings used in the previous
experiment except the interval of light source orientation. The light source orientations in the first orbit are 10°, 20°, 35°, 55°, 130°, 140°, 155°, 170°, and the orientations in the second orbit are 10°, 30°, 35°, 50°, 135°, 145°, 165°, 170° respectively. The coincidence ratio of shadows was 89.89%. The 3D shape and the light source positions after removing affine ambiguity are shown in Fig.7 (a) and (b) respectively. The error of the recovered shape was 5.05%, and the error of the estimated light source positions was 4.28°. Comparing these results with Fig.6 (c) and (d), we find the error of estimated results does not increase so much, and the proposed method can estimate both 3D shape and light source positions accurately even if the light source interval is not constant. Since we used only 16 images in this experiment, we also find that the proposed method works efficiently even if the number of input images is small.

3) The accuracy of light orbit which is inclined to zenith: We next evaluate the accuracy of the proposed method in the case where two light source orbits are at an angle of ω = 20° to zenith. The coincidence ratio of shadows was 89.16%. The 3D shape recovered after the final iteration is shown in Fig.8 (a), and the 3D shape after removing the affine ambiguity is shown in Fig.8 (b). From Fig.8 (a), we see that the recovered shape has an affine ambiguity. However, since the coincidence ratio of shadows is high, we find the reasonable shape is recovered from the proposed method. The error of the recovered shape in Fig.8 (b) was 7.38%, and the error of the estimated light source positions was 6.48°. From these results, the proposed method works accurately even if the light source orbit is inclined to zenith.

VI. Conclusion

In this paper, we proposed a method for estimating both 3D shape and light source positions simultaneously under a condition that 3D shape and light source motions are unknown. We first showed that the 3D shape can be recovered with small computational costs by using 1D Shadow Graphs, and the method can be extended for arbitrary light source orbits. We next proposed an iterative method for estimating both 3D shape and light source positions simultaneously from just shadow information. The efficiency of the proposed method was shown by real image experiments and synthetic image experiments. The future work includes the extension of the proposed method, so that it can be applied for non-orbital light source motions.

REFERENCES