Robust and Optimal Locations for Sustainable Environment and Systems (ROLSES)

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Abstract

This research project aims at presenting some new methodological approaches and recent results to solve the problem of optimal facility location for several demand points in the geographical space. More precisely, It is an interdisciplinary research project, funded by the French National Research Agency (ANR). The poster provides interesting points of view and perspectives on discrete facility optimal location problems in isotropic spaces. All the centers and methods to fix them are original because they focus on robustness and sustainability purposes, and also because they provide to planners a certain degree of freedom to locate the center on the territory. This work opens to fruitful applications on transport networks.

Keywords: optimal location ; robust centers ; spatial metrics ; Lp norms ; efficiency ; equality ; equity ; sustainability.

Résumé

L’objectif de ce projet de recherche est de présenter de nouvelles approches et des résultats récents dans la localisation optimale de centres (facilité) servant plusieurs points de demande dans l’espace géographique. Il s’agit d’un projet interdisciplinaire financé par l’Agence Nationale de la Recherche. Le poster proposé fournit notamment des points de vue originaux et des perspectives intéressantes sur les problèmes de localisation optimale de facilités discrètes dans des espaces isotropes. Tous les centres et toutes les méthodes pour les fixer sont originaux car ils soutiennent des objectifs de robustesse et de durabilité, et offrent également la possibilité aux aménageurs de disposer d’un espace de liberté dans leur choix. Ces travaux ouvrent sur des applications fructueuses dans le domaine des réseaux de transport.

Mots-clés : Centres robustes, métriques spatiales, normes Lp, efficacité, égalité, équité, durabilité.
1. Introduction

Geographical space is a plane where population is not randomly located. Urban structures are aggregated and allow to efficiently supply resources used by the population demand. These facilities can be schools, health centers, shopping centers, rail stations, airports, wastes collection centers, etc. They all should need to be optimally located according to a certain metric. Furthermore, their important building costs should require their location to be robust and their function to remain sustainable on the long term, and as far as possible, independent from population density and location changes.

This tackled issue is called the K-facilities optimal location problem and can be solved by different ways, using different mathematical $L_p$-norms, for instance: (i) the $L_1$-norm minimizes the sum of the distances from the demand points to the $K$ facility(s), called the $K$-median, often used to locate logistic centers, economically efficient, (ii) the $L_2$-norm minimizes the sum of the squared distances, called the center of gravity, often used to locate geometrical centers in Geographical Information Systems, for equality purpose, (iii) the $L_{\alpha}$-norm minimizes the maximum distances, called $K$-center, often used to locate health centers to apply an objective of equity.

However, these well-known metrics have limitations:

- We do not know their effective robustness when they are faced to observed demands
- They provide crisp accurate location(s) in X an Y, but not area(s) of «good» possible solutions
- They are static and do not take into account any possible change of the spatial configuration
- The solutions provided do not explicitly support territorial sustainability

Funded by the French ANR, the project ROLSES aims at providing location methods which remain robust in different spatial configurations of demands or in a long term. The inter-disciplinary team (geography and mathematics) proposes several approaches that can help decision makers to find optimal (areas of) centers for an objective of territorial sustainability. The aims are the following:

- To deeply understand the current metrics properties and behaviour;
- To propose renewed robust metrics to find optimal centers;
- To enrich the information about the center (area of possible solutions around the center, maps of influence of the demand points, behaviour and asymptotic properties of the center, etc.);
- To provide a methodological framework for users and urban planners to choose an optimal center in practice.

The tackled methodological domains are the following: (1) muti-criteria analysis and Pareto principle, (2) robust M-estimators in space, (3) possibility theory and fuzzy set (4) sensitivity analysis of the centers, (5) new $L_p$-norms with peculiar properties ($p=1.5, p=3, p=4$, etc.).

All the proposed solutions succeed in providing a more supple and reliable analysis of sustainable centers provided for $k$-facilities optimal location problems (Blanke et al, 2012), including some degree of freedom for planners to choose the appropriate center according to their objective and needs.

2. Methodological approaches and results

2.1. Multi-criteria analysis to set a wastes collection center

For this part of the research, we explored existing methods to find an optimal center. We applied different metrics on the problem of wastes collection and found different optimal locations. Multicriteria analysis was used and accessibility on the network was one of the different constraints considered to optimize the center (Gourion & Josselin, 2012). Six sites were candidates. Although two of them seemed to be more appropriate to set the center location for waste collection, we noticed with the stakeholders that the location was highly depending on the criteria combinations and on the metrics chosen for accessibility assessment. Two examples are depicted in the figures 1 and 2.
Fig. 1. A site chosen using the K-median according to a set of demand points.

Fig. 2. A site chosen using the K-median according to a set of points weighted by population.
2.2. Weighted m-estimators for clustered data

For this particular statistical approach of robust spatial estimators (Blanke et al., 2012), the motivation was to study the M-estimators for dependent simulated data and to analyse the properties of Weighted M-estimators (consistency, asymptotic normality, efficiency...). Some of the mathematical elements are presented in the figure 3 (El Asri, to appear).

- **Definition**: $X_{ij}, \ldots, X_{i,n}, i = 1, \ldots, n$ clustered random variables in $\mathbb{R}^d$ with distribution $P_\theta$, where $\theta \in \Theta \subset \mathbb{R}^d$ and $X_{ij}$ represents the $j^{th}$ element in the $i^{th}$ cluster. A weighted M-estimator is defined by:

$$\hat{\theta}_n = \arg\min_{\theta \in \Theta} \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{m_i} w_{ij} \rho(X_{ij}, \theta) \left( \sum_{i=1}^{n} \sum_{j=1}^{m_i} w_{ij} \psi(X_{ij}, \hat{\theta}_n) \right) = 0$$

Where $\rho$ is a measurable function, $\psi$ is its derivative and $w_{ij}$ are positive weights.

- **Results**: Under some conditions of regularity for $\rho$ function and some assumptions for the weights:

1. Consistency: $\hat{\theta}_n \xrightarrow{n \to \infty} \theta$.
2. Asymptotic normality: $\sqrt{n} (\hat{\theta}_n - \theta) \xrightarrow{d} N \left( 0, V^{-1}(\theta) \left( C_{\psi} + C_{\rho} \right) V^{-1}(\theta) \right)$.

with $V_{\theta} = E_{X \sim P_\theta} \left( \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{m_i} w_{ij} \rho' (X_{ij}, \theta) \psi (X_{ij}, \theta) \right)$,

$C_{\psi} = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{m_i} w_{ij} \psi (X_{ij}, \theta) \psi' (X_{ij}, \theta)$,

$C_{\rho} = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{m_i} w_{ij} \rho' (X_{ij}, \theta) \rho (X_{ij}, \theta)$.

Fig. 3. Some mathematical formalism and properties of weighted M-estimators

The output of this research was the following: whatever the spatial configuration of clusters, the Weighted M-estimators provide an accurate location of the theoretical center. This means that they are robust, as illustrated in the figure 4. They can be used to locate centers that will be sustainable in space, due to their capacity to resist urban density changes. This methodology is applied on an isotropic space but could be transferable to networks.

![Fig. 4. Location of different M-estimators in a plane and effect of the weights related to the spatial statistical law](image)

2.3. Fuzzy representation of gravity centers of fires

Sometimes, due to a certain difficulty to locate some equipment in space, crisp centers found using deterministic metrics are not really appropriate. Indeed, planners may need to draw a region that represents a kind of space (of possibility) to locate more or less acceptable centers. This was applied to design the center of a set of fire clusters
(Rojas-Mora et al, 2011, 2012). This center aims at minimizing the on-fly distance to watch over a large territory particularly sensitive to fire risks, thanks to smoke detection improvement. In the example provided in the figure 5, this center corresponds to a smoothed area of "acceptable" centers of gravity to summarize the fire starts. Moreover, we proposed to model the center fuzzy location using both marginal density and magnitude (number of cases) fires along a series of census in several years. We used a trapezoidal fuzzy number as presented in the fig. 6. This approach can easily be used considering the network and, in this case, it minimizes the sum of the squared accessibility from the fires to the center (of gravity). But it could be applied on any type of center.

Fig. 5. Location of fuzzy gravity centers of forest fire starts in the PACA region (France)

Fig. 6. A fuzzy center with two trapezoidal fuzzy numbers in X and Y based on densities
2.4. Sensitivity analysis of the center and points influence

Another output of this research program leads to generate maps of the demand influence for a given optimal center. These maps are obtained by infinitesimally moving each point and calculating the center location sensitivity, that gives an influence assessment to the demand point (figure 7). The mathematical process is explained in the figure 8. Three different maps, resulting from three different metrics, are shown in the figure 9. Using this method, we can assess how much a demand point participated in the optimal center location (Josselin & Ciligot-Travain, 2013).

Fig. 7. Moving a little each demand point to assess the center location sensitivity

\[
\begin{align*}
\text{Demand} & \quad (x, m) = (x_1, x_2, \ldots, x_n, m_1, \ldots, m_n) \\
\text{Center position} & \quad c(x, m) = (x_c, m_c)
\end{align*}
\]

- Consider one feature of the demand: for example, position \( x_i \) of one point, fix the others;
- variation (in some infinitesimal way) of the feature: \( x_i \rightarrow x_i + dx_i, dx_i \rightarrow 0 \);
- \( \Rightarrow \) variation of the center:
  \[
c(x_1, \ldots, x_i + dx_i, \ldots, x_n, m) = c + dc =
\]
  \[
c(x, m) + L(dx_i) + o(dx_i);
\]
- \( L \) is the influence (in some sense) of \( i \)-th point of the demand relatively to the position.

Fig. 8. Processing the demand point influence according to the center sensitivity (\( m \) is the mass at the point)

Fig. 9. Influence of the demand points on the center \( C \). ▼ using three \( L_p \)-norms (left: \( I \)-median \( p=1 \), middle: gravity center \( p=2 \), right: \( I \)-center \( p=\alpha \))
3. Conclusion and perspectives

The project “ROLSES” allowed to assess current methods of optimal location. It also proposed original, efficient and robust methods for defining sustainable centers including fuzzy representation, weighted M-estimators, Multi-Criteria Analysis, points influence mapping with sensitivity analysis and also new Lp norms with peculiar properties (optimized value of $p=1.5$, $p=3$ and $p=4$, etc.). In further works, these new metrics will be processed on road networks and generalized for all $L_p$-norms.

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References


