New approach to 5-axis flank milling of free-form surfaces: Computation of adapted tool shape

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ABSTRACT

This paper develops a new approach to solve the problem of interferences during the flank milling of a non-developable ruled surface. Many articles propose to modify the tool path to reduce this problem. A novel approach is proposed here, Computation of Adapted Tool Shape (CATS), which computes and optimizes the tool shape to reduce these interferences. The aim of this CATS method is to maintain a standard CAM system thanks to the tool shape modification. This method is presented for the machining of an industrial part and for a numerical experimental design of nine surfaces.

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1. Introduction

Nowadays, manufacturing of complex workpieces is still difficult, although the 5-axis machining proposes a lot of new possibilities and advantages [1]. Impellers are an example of these workpieces; Young [2] presents an example of a 5-axis machining process of an impeller and its associated problems. Impellers, inductors and fans are the main parts of turbo-machines and the efficiency of an engine is directly linked to the quality of its components. These elements are mainly composed of blades composed of an intrados and an extrados. Industrially, these two surfaces are frequently modeled by Non-Developable Ruled Surfaces (NDRS). Moreover, the production of a blade is not limited to the machining of an isolated ruled surface. Indeed, part geometry and functional requirements impose supplementary constraints for the machining of the surface. Furthermore, the ruled surface is necessarily connected to the part by other surfaces. These linking surfaces, defined by Duc [3], constitute the blade shank in the case of a blade. Therefore, machining must reduce interferences, both on the ruled surface and on the blade shank. Moreover, the quality of the workpiece is not only dependent on the geometry during machining; also it will depend on mechanical and hydraulic constraints on the blade, which must have a minimal thickness to resist functional constraints, a good surface quality to reduce flow turbulence, a precise geometry to acquire the efficiency obtained in simulation and good machinability. An efficient tool path is important to obtain high-quality machining. In 5-axis machining, the tool position is given by one point of the tool (generally the tool tip) and a vector which defines the tool axis. All the positions of the tool constitute the tool path. This computation is carried out by the CAM program within the coordinate system of the workpiece [4] to obtain the required quality.

The machining of free-form surfaces can be carried out using two strategies: point milling and flank milling. These two strategies differ in the computation of their tool path and the result of their machining.

Point milling: the tool path is computed taking into account the fact that the contact between the tool and the machined surface is a point. 5-axis machining allows the tool axis to be modified, which is used to avoid collisions and obtain a better consideration of the cutting parameters. This does not require an accurate computation. The main advantage of point milling is the ability to machine all types of free-form surfaces, with a reduced form defect (Provided that the curvature radius of the surface is larger than the tool radius and that the tool does not interfere with the surface [5]). But the material removal rate of point milling is low. Machining is achieved by a set of tool paths, and the lateral step between successive paths is directly linked to the required maximal scallop height. After machining, this method frequently requires a polishing operation in order to diminish the height of the machined scallop on the part. Such operations considerably increase the cost of the workpiece [6].

Flank milling: the tool path is computed taking into account the fact that the contact between the tool and the machined surface is a curve. In this case, tool orientation enables a positioning of the
Fig. 1 presents a ruled surface and its associated notations. This type of surface can be generated by the displacement of a segment $[P_0P_1]$ along two directrix curves $C_0(u)$ and $C_1(u)$. Eq. (1) is the parametric equation of a ruled surface.

\[
S(u, v) = (1 - v) \cdot C_0(u) + v \cdot C_1(u) \quad \forall (u, v) \in [0, 1] \times [0, 1]
\]

Notation: $[P_0P_1]$ is the rule; $C_0(u)$ and $C_1(u)$ are the directrix curves; $N_0$ and $N_1$ are the unit vectors normal to the surface, respectively at $P_0$ and $P_1$; $h$ is the length of the rule; $\alpha = \text{angle}(N_0, N_1)$ is the twist of the surface along the rule $[P_0P_1]$.

Box 1

machined surface in relation to the nominal surface. This requires an accurate computation of the tool orientation. Flank milling is therefore very interesting for the machining of developable surfaces, because this method is able to machine the surface accurately with a high level of productivity. But difficulties appear for the machining of NDRS, because interferences are created between the nominal surface and the tool [7], although high productivity is preserved. These interferences are created by the non-developability of the surface. Such interferences generate a deviation between the nominal surface and the machined surface. This deviation modifies significantly the surface shape and the hydraulic/aerodynamic properties of the part.

Several studies have been carried out on the flank milling of NDRS and its interference problems [8–15]. These studies modify the tool positioning and then the tool path. Subsequently, positionings are presented using a cylindrical cutter although some of these are able to use conical tools. The software positioning is frequently implemented in industrial CAM software. This method puts the tool axis collinear to the rules and the tool is tangent to one chosen directrix curve (details at Section 3.1). It allows an analytical and robust computation, but the interferences generated are very large. The displacement of the contact point between the tool and the surface is an important improvement of this positioning. If this point is located in the middle of the rule, positioning is the Single Point Offset (SPO), which is the first positioning, defined by Liu [8]. This method conserves the advantages of software positioning and enables a reduction of interferences. The standard positioning developed by Rubio [9] is an improvement of SPO positioning. This positioning is able to calculate the tangency point between the cutter and the surface to balance maximum interference values on each directrix curve and is more successful than the previous two examples, but a numerical resolution is necessary. Other methods not only use the modification of the tangency point of the tool but also the tool axis orientation. For example, the Double Point Offset (DPO) method positions the tool at a tangent at two points of the surface, located at 1/4 and 3/4 of the rule, which induces a rotation of the tool axis, and then a reduction of interferences [8].

This method gives interesting results, preserving an analytical computation. Another method using tool axis orientation is improved positioning, elaborated by Redonnet [10]. With this method, the tool is tangent to the two directrix curves and the rule. This method almost cancels out the interferences but requires the numerical resolution of a system of seven equations with seven unknowns. This method engenders the problem of computing the maximum tool diameter able to machine the surface with the required tolerance. Senatore proposes a method to determine this diameter [11]. A positioning approach proposed by Menzel is very similar to improved positioning, but with a simplified computation algorithm [12]. Another positioning, the Three Points Offset (TPO), puts the tool axis in contact at three points on the offset surface. This positioning was created by Gong [13] and gives good results, but the resolution is still numerical. The positioning proposed by Lartigue [14] and developed by Pechard [15] covers the whole surface. This method is different from the other methods presented previously. Here the tool is not positioned at each point of the surface but on the whole surface. This method ensures the smoothness of the tool path, but this computation is numerical.

Research on NDRS flank milling is extensive and the major problem outlined is the interference which appears between the tool and the surface. The majority of proposed solutions optimize tool positioning by modifying the tool path. However, to compute this tool path it is necessary that this algorithm be implemented in the CAM system used; whereas, in industrial CAM, the positioning is often imposed and generally does not perform well. That why a new investigation approach, based on the optimization of the tool shape, is proposed. The name given to this new method is Computation of Adapted Tool Shape (CATS). The tool shape is optimized for given positioning, surface and industrial functional requirements. This method is used after the CAM system, and therefore, it is not in competition with positionings presented previously. CATS method is a new approach which is usable with all types of positioning, and thus with all CAM systems types, and it is able to improve the machining efficiency. However, the efficiency of CATS method depends on the employed positioning and the surface regularity. In effect, if the employed positioning is not performing well, the efficiency of CATS method may be great; whereas if the positioning is competitive, the CATS method gain will be limited. Moreover, the adaptation of the tool shape allows a better control of the interference distribution along the rule, for a better consideration of functional requirements.

This method uses the degrees of freedom offered by the tool shape, which are so far under-developed. In fact, currently only the tool type (cylindrical or conical) and tool diameter are exploited. As a result, in order to reduce and better control generated interferences, a new approach which optimizes the tool shape is proposed. Furthermore, CATS method opens new investigations such as the machining of a no ruled surface in flank milling using multi-path strategies.

In this paper, the steps of the CATS method will be presented. Next, the method will be applied with software positioning using a cylindrical cutter (this positioning is imposed by our CAM system) for the machining of an inductor’s extrados, presented in Fig. 2. The studied surface is used in the turbo pump of a space vehicle engine. This surface presents a twist of $5^\circ$ to $12.8^\circ$ and a rule length of 22.4 mm to 59.9 mm. To optimize the tool shape the CATS method proposes four optimization criteria. Three out of the four criteria are the expressions of the industrial requirements in the mechanical, hydraulic and machining fields. To evaluate the geometric efficiency of a machining method three performance indicators are created with respect to industrial requirements. Using these performance–indicators, a tool shape optimized with the CATS method will be compared with a standard tool for the machining of an industrial workpiece. To conclude a numerical experimental design is performed to study the influence of the machined surface on the efficiency of the CATS method.

2. New tool shape optimization method for flank milling: CATS

CATS method is defined in four steps:

1. *Computation of the tool path.* This step can be carried out with all types of CAM system and a standard tool (cylindrical or conical).
Fig. 1. Ruled surface. Twist is the main parameter of the ruled surface. It enables us to determine if the surface is developable or not. If the twist is equal to zero along the entire surface, then the ruled surface is developable, otherwise the surface is not developable.

Fig. 2. Example of industrial part modeling using NDRS.

(2) Calculation of the distance profiles between the tools axes and the surface. At each point of the tool path, the profile of distances between the tool axis and the surface is calculated. Note that these distances are computed orthogonal to the tool axis. This profile corresponds to the ideal tool shape (at this point) to obtain an infinite number of tangency points between the tool and the surface.

(3) Association of a tool profile. The goal of this step is to associate a tool shape with a given nominal surface, positioning and functional requirements. This step can be performed using two different methods:
- The barrel cutter method starts by associating a tool shape (e.g. a barrel cutter form) with each distance profile (found in step 2). Next, a tool is computed for the whole surface, according to the local tool and functional requirements. This method is described by Chaves-Jacob [16].
- The shaped-tool method is a direct association. All the local distance profiles must be stored with this method. Subsequently, a tool shape is associated with all these local distances thanks to a criterion.

After the computation of the theoretical optimal tool profile, the profile can be smoothed to respect sharpening requirements. The advantage of the barrel cutter method is that the obtained profile is smooth.

(4) Computation of generated interferences. This step verifies the usefulness of this method for the considered surface/positioning couple. Then interferences obtained with the optimized and standard tools (cylindrical or conical) are compared, to evaluate the advantages of this method.

The benefit of the CATS method is strongly determined by the machined surface regularity/employed positioning pair. To quantify the surface regularity, two parameters are proposed:
- Evaluation of rule length variation, Eq. (2).
  \[
  \Delta h = \frac{(\max(h) - \min(h))}{\text{mean}(h)}
  \]  
- Quantification of surface twist variation, Eq. (3).
  \[
  \Delta \alpha = \frac{(\max(\alpha) - \min(\alpha))}{\text{mean}(\alpha)}
  \]

3. Application to the machining of a blade using software positioning

Subsequently, the CATS method is applied to software positioning. The first step concerns the computation of the tool path.

3.1. Tool path computation using software positioning

Computation is based on the selection of one of the two directrix curves as a tangent guide curve. In this paper, the blade shank directrix curve is chosen as the tangent guide curve, \( C_0(u) \) shown in Fig. 3. Tool positioning is computed to obtain the tool tangent to the selected directrix curve and the tool axis parallel to the rule. Eq. (4) determines point \( P_{out} \) of the tool axis.

\[
\text{OP}_{out} = \text{OP}_0 + r \cdot N_0(u).
\]  
where \( r \) is the radius of the cylindrical tool.

Such positioning produces a large interference located on the second directrix curve, \( C_1(u) \). In this example, Fig. 3 presents interferences obtained using software positioning with a cylindrical cutter.

The maximum value of interferences can be evaluated using the following hypothesis:
- The tool is a perfect mathematical cylinder;
- The directrix curves are approximated by their tangents computed at the extremity of the considered rule. The tangents of $C_0(u)$ and $C_1(u)$ are respectively called $C'_0(u)$ and $C'_1(u)$.

Eq. (5) is obtained using the previous hypothesis, summarized in Fig. 3. This equation estimates the maximum value of the generated interferences, noted $\varepsilon$, with this positioning.

$$\varepsilon = r \cdot (1 - \cos(\alpha)). \tag{5}$$

To illustrate Eq. (5), a numerical application is performed with $r = 6\,\text{mm}$ and $\alpha = 14^\circ$; these values are common for industry. The obtained value is $\varepsilon = 0.18\,\text{mm}$.

In conclusion, the software positioning method is very fast and robust, but produces significant interferences. Thus speed and robustness are obtained, thanks to the analytical computation of the positioning. The interferences have many consequences:

- Modification of the machined surface geometry relative to the theoretical surface. This modification reduces the hydraulic efficiency of the surface.
- Modification of the surface position. With software positioning, the interferences are always overcut. If software positioning is used to machine a blade composed of an extrados and an intrados, each one defined by a NDRS, the thickness of this blade will be reduced due to the overcut. This reduction in thickness entails a deterioration of the mechanical properties of the blade;
- Modification of the blade shank by the interferences. This problem appears in software positioning if the tangency curve is not chosen on the link between the surface and the blade shank. In the case of the tangency curve of the tool being chosen on this link, the blade shank is considered but the head of the blade will have all the interferences.

3.2 Calculation of the distance profiles between the tool axes and the surface

This distance profile provides the ideal tool profile at one point in order to avoid interferences. This computation is frequently numerical and computation time can be long. But if the tool axis is collinear to the rule, an approximation can be done to maintain an analytical and fast computation: the surface is approximated by a tangent surface. This surface is illustrated on the extrados surface of the inductor in Fig. 4; this figure also presents the notations used to create this surface. The tangent surface is obtained in five steps by:

1. Considering a rule on the surface and its extremities $P_0$ and $P_1$;
2. Computing $PL_0$ and $PL_1$, the two normal planes of the rule at $P_0$ and $P_1$;
3. Computing $T_0$ and $T_1$, the two tangent vectors of $C_0$ and $C_1$ at $P_0$ and $P_1$;
4. Projecting $T_0$ and $T_1$ on $PL_0$ and $PL_1$ respectively to obtain $T'_0$ and $T'_1$;
5. Computing a tangent surface using $T'_0$ and $T'_1$ as the directrix curve of a NDRS. Note that a linear variation of the twist along the rule is imposed at the tangent surface.

Now the tangent surface is used to approximate the distance profiles between the tool axis and the surface. Fig. 5 presents the notations used to compute this distance profile. Note that these distances are the smallest distances calculated orthogonally to the tool axis to give the optimal tool radius value at each point of the tool axis. The distance profile is obtained taking into account a point $M$ of $C_0$ curve. At this point, the tool axis is computed using a software positioning (cf. Section 3.1). A point $M_1(t_1)$ of the tool axis is calculated, located at $t_1$ distance from vector $N_0$. Now the distance $d(t_1)$ between point $M_1(t_1)$ and the nominal surface is estimated by the distance between $M_1(t_1)$ and the tangent surface. Thanks to this approximation the computation of this distance is simple and given in Eq. (6).

$$d(t_1) = r \cdot \cos(\alpha - t_1/h). \tag{6}$$

Note that if the radius of the tool is equal to the value of $d(t_1)$ at point $M_1(t_1)$, then no interference appears at point $M_1(t_1)$ and the tool is tangent to the surface. Therefore, the computation of a set of radii for a value of $t_1$ that varies along the entire rule allows the positioning of the tool with infinite points of tangency on the surface. It ensures the elimination of interferences in this position.

The tangent surface approximation induces an error in the distance profiles computation. Fig. 6 represents studied surface where colors depend on the difference of the distance profiles computed using this approximation comparatively to the exact method. This error is mainly caused by the hypothesis of linear variation of the twist along the rule of the tangent surface. This approximation can be realized for surfaces with little twist. The use of this approximation enables an analytical computation, which reduces computation time. In the examples at Section 4 of this paper, this approximation is not done.

3.3 Association of a tool profile

The goal of this association is to use a single tool to prevent the appearance of machining marks. A single tool, and therefore a single tool profile, must be chosen, according to various functional criteria. In this paper, four criteria are presented, but others can be created for a specific functional requirement. Each of the criteria 2, 3 and 4 is created subject to a specific requirement.
- Criterion 1 is an average criterion. It roughly averages the generated interferences. This is a standard criterion, which can be applied in the absence of specific requirements;
- Criterion 2 cancels the overcut. This method ensures, in the case of a blade, the minimum thickness value. It was created to respect the requirements of the mechanical profession;
- Criterion 3 balances the maximum and minimum values of the interferences. This method optimizes the machining, respecting a centered tolerance interval for the workpiece. It was created to respond to requirements express by the machining profession;
- Criterion 4 balances the volume of overcut and undercut machining of the part. This method ensures that the surface geometry is well respected, in accordance with the requirements expressed by the hydraulic profession.

Subsequently, these criteria will be used to optimize the tool profile with the half-barrel and shaped-tool methods.

3.3.1. Barrel cutter method

With this method it is necessary to select a tool profile. For the software positioning we chose to approximate the distance profile by a circle arc. This gives rise to a tool in half-barrel form. Fig. 7 presents interferences obtained with a cylindrical cutter and their associated half-barrel. The main idea of the CATS method is to reduce the diameter of the tool where a cylinder cutter overcuts.

Fig. 8 presents the half-barrel cutter and its associated generative profile. Mathematically, the shape of the active part of this tool is defined by the rotation of a circular arc around the tool axis. This is the definition of a torus, defined by two radii:

- \( r \): is the maximum value of the tool radius
- \( R_{\text{barrel}} \): is the value of the barrel radius

Note that in Fig. 8 a corner radius is added at the tool tip to machine the blade shank. Therefore, such tool proposes a supplementary degree of freedom: \( R_{\text{barrel}} \), which allows the adaptation of the tool profile. Parameter \( r \) is chosen by the CAM user to avoid collisions between the tool and the part; for example, in the case of the impeller, the space between two consecutive blades limits the tool diameter.

Determining a half-barrel at one point of the tool path is equivalent to calculating \( R_{\text{barrel}}(i) \), where parameter \( i \) defines the number of a considered point along the tool path. Fig. 9 presents for one value of \( i \) the distance \( d(t_i) \) and the associated circular approximation. This circular approximation defines, with only one parameter, \( R_{\text{barrel}} \), the tool shape. The value of \( R_{\text{barrel}}(i) \) will be determined by three constraints, which are chosen according to functional requirements:

- The circular arc passes through 2 points, A and B, represented in Fig. 9. The circle passes through point A to respect the directrix curve of the blade shank and through point B to respect the shape of the distance profile \( d(t_i) \);
- The center of the circle is positioned at \( N_0 \) in order to respect the shape of the distance profile \( d(t_i) \).

---

Fig. 6. Error generated by the tangent surface approximation.

Fig. 7. Main idea of the new approach.

Fig. 8. Geometrical definition of half-barrel cutters.

Fig. 9. Computation of half-barrel radius.
These constraints impose the value of the half-barrel radius, which is given by Eq. (7):

\[ R_{\text{barrel}}(i) = \frac{-h(i)^2 + (r \cdot \cos(\alpha(i)) - r)^2}{2 \cdot (r \cdot \cos(\alpha(i)) - r)}. \quad (7) \]

Circular approximation induces a small deviation, \( e \): the distance between the surface and the tool, depending on parameter \( t_1 \) and presented in Fig. 9. An analytical estimation of \( e \) is computed between the half-barrel cutter profile, \( r_{\text{tool,CATS}}(t_1) \), and the tangent surface, Eq. (8).

\[ e = r_{\text{tool,CATS}}(t_1) - d(t_1). \quad (8) \]

Eq. (9) is obtained by changing \( r_{\text{tool,CATS}}(t_1) \) to the value of the half-barrel tool radius at distance \( t_1 \). This is obtained with the circle equation whose radius value is \( R_{\text{barrel}} \):

\[ e = \left[ r - R_{\text{barrel}}(i) + \sqrt{R_{\text{barrel}}^2(i) - t_1^2} \right] - \left[ r \cdot \cos(\alpha(i)) - t_1 / h(i) \right]. \quad (9) \]

This deviation value is insignificant in comparison with errors produced during the optimization of the tool over the whole surface. For example, the maximum value of \( e \) on an industrial surface is smaller than 0.2 \( \mu \text{m} \).

For the complete machining of the surface, a single tool and therefore a single value of \( R_{\text{barrel}} \) must be chosen. The goal is to use a single tool to avoid to create machining marks. This choice is made using a criterion, which responds to different functional requirements. In this paper, the four presented criteria (cf. Section 3.3) will be used.

- Criterion 1 is the mean of all the values of the half-barrel radius \( R_{\text{barrel}}(i) \). The value of \( R_{\text{mean,barrel}} \) is obtained with Eq. (10).

\[ R_{\text{mean,barrel}} = \text{mean}(R_{\text{barrel}}(i)). \quad (10) \]

- Criterion 2 cancels the overcut, the radius is computed with Eq. (11).

\[ R_{\text{ancel,overcut,barrel}} = \text{min}(R_{\text{barrel}}(i)). \quad (11) \]

- Criterion 3 balances the maximum and minimum values of the interferences. Its utilization requires a numerical computation.

- Criterion 4 balances the volume of overcut and undercut machining of the part. Its utilization requires a numerical computation.

### 3.3.2. Shaped-tool method

In order to obtain the shaped-tool profile, all the distance profiles computed previously are transferred onto a graph presented in Fig. 10 for the studied surface. This method is able to change a three-dimensional problem into a two-dimensional problem. Therefore, a shaped-tool profile is associated at all these points using a criterion. Now the use of the four presented criteria (cf. Section 3.3) will be detailed.

- Criterion 1, the mean of all \( d(t_1) \) (for all the \( i \) rules, noted \( d_i(t_1) \)) values is computed for each value of \( t_1 \).

\[ R_{\text{shaped-tool}}(t_1) = \text{mean}[d_i(t_1)] \forall i. \quad (12) \]

- Criterion 2, the minimum of all the \( d(t_1) \) values is computed for each value of \( t_1 \).

\[ R_{\text{shaped-tool}}(t_1) = \text{min}[d_i(t_1)] \forall i. \quad (13) \]

- Criterion 3 balances the maximum and minimum values of the interferences for each \( t_1 \) value.

\[ R_{\text{shaped-tool}}(t_1) = [\max[d_i(t_1)] + \min[d_i(t_1)]]/2. \quad (14) \]

### 3.4. Computation of generated interferences

#### 3.4.1. Numerical comparison

The choice of a single tool to machine a surface will create a deviation \( E \). This deviation is computed subtracting \( d(t_1) \) (obtained by Eq. (6)) from the value of the tool radius for value \( t_1 \), \( r_{\text{tool,CATS}}(t_1) \), to obtain Eq. (16). This computation is presented in Fig. 11. Using this interference computation, where \( E \) is positive, the tool generates overcut.

\[ E(t_1) = r_{\text{tool,CATS}}(t_1) - d(t_1) \quad (16) \]

Subsequently the comparison of the two methods (barrel tool and shaped-tool cf. Section 2) using the four criteria (cf. Section 3.3) is carried out with a nominal tool radius \( r = 8 \text{ mm} \). These methods will be compared on the surface mentioned earlier. This industrial surface was sampled at 101 points in \( u \) parameter direction and 21 points in \( v \) parameter direction. To compare the
efficiency of machining methods, 3 performance indicators are created. These indicators translate the requirements expressed by the mechanical, hydraulic and machining professions.

- Indicator 1 is the extreme value of interferences. This indicator is present in two columns in Table 1: maximum overcut and undercut. This indicator expresses the requirement of the machining expert.
- Indicator 2 is $\Sigma(\text{Vol})$. This is the summation of the signed volume of interferences. This indicator summarizes the interference volume distribution in undercut and overcut. This indicator does not express directly the requirement of an expert.
- Indicator 3 is $\Sigma(|\text{Vol}|)$. This is the summation of the absolute value of the interference volume. This indicator enables an accurate evaluation of the distance between the machined surface and the theoretical surface and expresses the requirements of the hydraulic expert.

The value obtained by Eq. (17), $\text{Vol}_{\text{overcut}}$, is the volume of generated overcut. This volume translates the requirements of the mechanical expert.

$$\text{Vol}_{\text{overcut}} = [\Sigma(\text{Vol}) + \Sigma(|\text{Vol}|)]/2.$$  \hspace{1cm} (17)

Table 1 presents the values of the 3 indicators for the two methods using the 4 optimization criteria and the cylindrical cutter. The same tool path is used for all test cases. Only the tool profile changes. Note in Table 1 that:

- All the optimized tools allow a reduction of the 3 performance indicators compared to the cylindrical cutter.
- For this surface and this positioning, criteria 1, 3 and 4 generate similar optimized tool profiles for the half-barrel tool and shaped-tool methods. These tools have similar values for the 3 indicators. Indicator 3, $\Sigma(|\text{Vol}|)$, illustrates a reduction by a factor of 3 in the interference volume generated by this optimized tool compared to that generated by a cylindrical cutter.
- Criterion 2 optimizes the tool profiles to cancel the overcut but it generates a lot of undercut, although the shaped-tool is still more competitive than the cylindrical cutter.
- Note that indicator 2, $\Sigma(\text{Vol})$, for the shaped-tool optimized by criterion 4 is not exactly equal to 0. This is caused by computation rounding error.

For a better illustration of the benefit of these optimized tool profiles, the comparison was subsequently detailed between the cylindrical cutter and the half-barrel tool optimized by criterion 1. Eq. (16) applied to the studied surface is shown in Fig. 12. This figure represents the nominal surface where the color depends on the generated interferences. Fig. 12 presents the computed interferences obtained with a half-barrel tool defined by:

- $r = 8 \text{ mm}$;
- $R_{\text{criterion}_1} = 11880 \text{ mm}$.

Fig. 12 illustrates that with a $R_{\text{criterion}_1}$ the interferences are distributed in overcut and undercut, because in some rules $R_{\text{criterion}_1}$ is greater than $R_{\text{criterion}_2}$, which causes the overcut. It is interesting to note in this figure that the interferences generated by this method on the directrix curve located on the blade shank are equal to zero, which is typical with the positioning used.

Fig. 13 presents both deviations computed, using Eq. (16), with half-barrel and cylindrical cutters on the industrial surface, represented in parametric space. In the following, the deviation caused by the half-barrel cutter is noted E1 and that due to the cylindrical cutter, E2. Remember that parameter $v$ varies along the rule and parameter $u$ varies along the directrix curve.

The analysis of this deviation shows that deviations E1 and E2 are equal to zero on the blade shank (for the parameter $v$ equal to 0). This is caused by the choice of the blade shank directrix curve for the tangency curve for the tool path computation. Deviation E2 is always positive, which signifies that the cylindrical tool, in software positioning, generates only overcut, as was seen in Section 3.1. Deviation E2 contrary to E1, is alternately positive and negative because the chosen barrel radii is the mean of all the computed radius at each point on tool path. Note, that the range of interferences is reduced by around 20% in this case.

3.4.2. Experimental comparison

To verify the theoretical computation of interferences, industrial surfaces machined as shown in Fig. 14. The part is composed of two NDRS and its associated blade shank. One of the surfaces is machined using a cylindrical cutter and the other is machined using a half-barrel tool optimized with criterion 1. The material used is a machinable polyurethane slab.

To carry out this machining, a half-barrel is made. The tool profile is controlled by measuring a set of tool radii at different distances from the tool tip. The half-barrel is defined by:

- $r = 8.007 \text{ mm}$;
- $R_{\text{criterion}_1} = 10052 \text{ mm}$.

The tool produced is not exactly the computed tool. The difference between this theoretical and real tools could appear large (the computed value of $R_{\text{criterion}_1}$ is 11880 mm whereas...
Table 1
Comparison of different tool shape optimization methods on the industrial surface.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>R_hbarrel</th>
<th>Indicator 1</th>
<th>Indicator 2</th>
<th>Indicator 3</th>
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<td>Shaped-tool</td>
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<td>Maximum value of the</td>
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<td>undercut (mm)</td>
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<td>Shaped-tool</td>
<td>11 880</td>
<td>N/A</td>
<td>0.060</td>
<td>0.085</td>
</tr>
<tr>
<td></td>
<td>4 037</td>
<td>N/A</td>
<td>0.000</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>13 140</td>
<td>N/A</td>
<td>0.073</td>
<td>0.070</td>
</tr>
<tr>
<td></td>
<td>12 782</td>
<td>N/A</td>
<td>0.070</td>
<td>0.085</td>
</tr>
<tr>
<td>Cylindrical</td>
<td>Infinity</td>
<td>0.195</td>
<td>0.000</td>
<td>828</td>
</tr>
</tbody>
</table>

Experimental tests confirm a benefit from this new method. The experimental results may appear less interesting than the theoretical ones, but this is caused by the measurement of only one part of the surface whereas the computed tool profile is optimized for the entire surface.

In conclusion, the CATS method seems to be a good way to reduce NDRS machining error.

4. Application to the machining of academic surfaces

Now, CATS method will be tested on two representative academic surfaces.

First surface, Surface#1 (represented on Fig. 18), is a ruled surface defined by the Eq. (1) (Box 1) and by the following directrix curves:

\[ C_0(u) = [40; 80 \cdot u - 40; 0] \]

\[ C_1(u) = [-13.557 \cdot u + 40; 77.365 \cdot u - 40; 54.286] \]

\[ [u, v] = [0, 1] \times [0, 1] \]

Surface#1 is a regular surface, with values of \( \Delta h \) and \( \Delta \alpha \) parameters almost equal to zero. Its two directrix curves are segments and the twist value of the surface is near to 10° for each rule. The machining of this surface is studied using software positioning with a cylindrical tool of 8 mm of radius. The maximum value of generated overcut is equal to 0.119 mm and undercut is equal to 0 mm. The use of a shaped-tool with the profile defined in Fig. 17, gives the maximum values of overcut equal to 0 and undercut equal to −0.020 mm.

Fig. 18 presents the machined workpiece using a shaped-tool whose profile is presented in Fig. 17 (measured shape). On this machined surface the maximum measured value of overcut is
Radius of the shaped-tool (mm)

8.05
8
7.95
7.9
7.85
7.8
0
Tip of tool

20 40 Tool axes (mm)
Theoretical
Measured shape

Fig. 17. Shaped-tool profile to machine Surface #1.

Fig. 18. Machined Surface #1 in an aluminum alloy block and its associated shaped-tool.

equal to 0.018 mm and undercut equal to −0.006 mm. This machining checks the benefit of the shaped-tool to an academic regular surface and a simple positioning. In this case, the CATS method almost cancels out the interference problem.

To continue to test the usefulness of the CATS method, a second academic surface is studied using two positioning. The considered surface, Surface#2, is a ruled surface defined by Eq. (1) and by the following directrix curves:

\[
C_0(u) = [23.014 \cdot u; 20.429; 0]
\]

\[
C_1(u) = [23.014 \cdot u; 20.2324 \cdot u^2; 33.995]
\]

\[u, v] = [0, 1] \times [0, 1].\]

This surface is frequently used in articles [8,11,12] as reference. Its twist varies continuously from 0° to 60° and the length of these rules varies continuously from 40 mm to 34 mm. The use of a cylindrical cutter, with 10 mm of radius, and a software positioning gives the maximum values of overcut equal to 5.047 mm and undercut equal to 0 mm. The use of a shaped-tool whose profile is optimized with CATS method (criterion 3 cf. Section 3.3.1) generates the maximum values of overcut equal to 2.523 mm and undercut equal to −2.523 mm. In this case, CATS method enables a distribution of the interferences in overcut and undercut but this usefulness is limited.

Now the machining of Surface #2 is studied using the positioning DPO defined by Liu [8] with a cylindrical cutter with 10 mm of radius. In paper [8] the maximum interferences computed by the authors are: overcut = 0.321 mm and undercut = 0.819 mm. Using our algorithm to compute interferences, the maximum overcut is equal to 0.333 mm and undercut is equal to −0.807 mm. This difference is caused by the hypothesis used in [8] and the numerical approximations. Fig. 19 presents the Surface#2, the optimal shaped-tool with criterion 3 (cf. Section 3.3.1) and the computed distances profiles for 3 points of the tool path. The use of this shaped-tool gives the maximum values of overcut equal to 0.412 mm and undercut equal to −0.404 mm. In this case, CATS method enables a reduction by around 30% of the interferences amplitude. Fig. 20 presents the Surface#2 and its associated DPO tool path, the tool path color depends on the generated interferences respectively to cylindrical cutter at the left side and shaped-tool (cf. Fig. 19) at the right side. In conclusion, for this case study, the use of a shaped-tool enables a reduction of the interferences amplitude and a better repartition of the interferences as shown in Fig. 20.

5. Influence of surface regularity on the advantages provided by the CATS method

The previous test evaluates the benefit of the CATS method for an industrial application and for 2 academic surfaces. However, industrial parts present a wide variety of surfaces, and it is interesting to evaluate the adaptation of this method to different surfaces. These studies focus on the influence of surface regularity on the half-barrel and shaped-tool methods, using criterion 1. Only the criterion 1 was studied because it is standard, and in order to simplify the presentation of the results.

5.1. Numerical experimental design

Surface regularity is quantified by \(\Delta h\) and \(\Delta \alpha\) (defined in Section 2). This study is performed on 9 surfaces, illustrated in Fig. 21. These surfaces are chosen to represent the variety of surfaces existing in industry. Parameters \(\Delta h\) and \(\Delta \alpha\) of these surfaces vary almost regularly on a grid, as shown in Table 3.

Numerical results of this study are presented in Table 3. The definitions of notations used in this table are:
- $\Delta h_s$ is the standardized value of $\Delta h$;
- $\Delta \alpha_s$ is the standardized value of $\Delta \alpha$;
- $\text{Ampl}$ is the amplitude value (mm) of the interferences generated by the cylindrical cutter;
- $\text{Vol}$ is the value (mm$^3$) of the indicator $\Sigma(|\text{Vol}|)$ for the cylindrical cutter;
- $G_{\text{ampl}}$ is the gain in percentage of interference amplitude, obtained by the optimized tools (half-barrel or shaped-tool) compared to the cylindrical cutter. It is computed with Eq. (18).

$$G_{\text{ampl}} = 100 \cdot \left( \frac{\text{Ampl} - \text{Ampl}_{\text{optimized tool}}}{\text{Ampl}} \right).$$

With $\text{Ampl}_{\text{optimized tool}}$ the amplitude value of the interferences generated by the optimized tool.

- $G_{\text{vol}}$ is the gain, in percentage of the indicator $\Sigma(|\text{Vol}|)$, obtained by the optimized tools compared to the cylindrical cutter. It is computed with Eq. (19).

$$G_{\text{vol}} = 100 \cdot \left( \frac{\text{Vol} - \text{Vol}_{\text{optimized tool}}}{\text{Vol}} \right).$$

With $\text{Vol}_{\text{optimized tool}}$ the value of the indicator $\Sigma(|\text{Vol}|)$ for the optimized tool.

### 5.2. Analysis of results

Firstly the analysis is carried out on the values of Table 3. Note in this table a significant reduction of interferences on using the CATS method. Furthermore as expected, the CATS method benefit is reduced by a significant variation of the twist surface. Secondly a more accurate analysis is carried out. To study the influence of parameters $\Delta h$ and $\Delta \alpha$, a mathematical model is associated with the gains computed previously. This model, presented in Eq. (20), is a basic linear model defined by four parameters: $a$, $b$, $c$ and $d$. This model is associated with the gains using the least square model.
Table 3
Influence of surface regularity on the CATS method.

<table>
<thead>
<tr>
<th>Surface</th>
<th>$\Delta h$</th>
<th>$\Delta h_1$</th>
<th>$\Delta \alpha$</th>
<th>$\Delta \alpha_2$</th>
<th>Cylindrical cutter</th>
<th>Half-barrel tool</th>
<th>Shaped-tool</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(mm)</td>
<td>(mm)</td>
<td>(%)</td>
<td>(%)</td>
<td>Ampl (mm)</td>
<td>Vol (mm$^3$)</td>
<td>$G_{\text{ampl}}$ (%)</td>
</tr>
<tr>
<td>1</td>
<td>0.057</td>
<td>-1.11</td>
<td>0.185</td>
<td>-1.23</td>
<td>0.41</td>
<td>663</td>
<td>62</td>
</tr>
<tr>
<td>2</td>
<td>0.072</td>
<td>-1.07</td>
<td>0.842</td>
<td>-0.13</td>
<td>0.67</td>
<td>677</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>0.089</td>
<td>-1.02</td>
<td>1.715</td>
<td>1.34</td>
<td>1.09</td>
<td>764</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0.334</td>
<td>-0.28</td>
<td>0.114</td>
<td>-1.35</td>
<td>0.28</td>
<td>395</td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>0.337</td>
<td>-0.27</td>
<td>0.660</td>
<td>-0.43</td>
<td>0.43</td>
<td>432</td>
<td>62</td>
</tr>
<tr>
<td>6</td>
<td>0.340</td>
<td>-0.26</td>
<td>1.639</td>
<td>1.21</td>
<td>0.81</td>
<td>539</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>0.865</td>
<td>1.32</td>
<td>0.489</td>
<td>-0.72</td>
<td>0.25</td>
<td>225</td>
<td>45</td>
</tr>
<tr>
<td>8</td>
<td>0.869</td>
<td>1.33</td>
<td>0.908</td>
<td>-0.02</td>
<td>0.35</td>
<td>260</td>
<td>88</td>
</tr>
<tr>
<td>9</td>
<td>0.877</td>
<td>1.36</td>
<td>1.703</td>
<td>1.32</td>
<td>0.60</td>
<td>352</td>
<td>15</td>
</tr>
</tbody>
</table>

Table 4
Parameters of mathematical models for the four gains.

<table>
<thead>
<tr>
<th>Surface</th>
<th>$G_{\text{ampl}}$</th>
<th>$G_{\text{vol}}$</th>
<th>$G_{\text{ampl}}$</th>
<th>$G_{\text{vol}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface A</td>
<td>34</td>
<td>58</td>
<td>47</td>
<td>72</td>
</tr>
<tr>
<td>Surface B</td>
<td>12</td>
<td>9</td>
<td>27</td>
<td>13</td>
</tr>
<tr>
<td>Surface A</td>
<td>-16</td>
<td>-29</td>
<td>-10</td>
<td>-12</td>
</tr>
<tr>
<td>Surface B</td>
<td>1</td>
<td>-1</td>
<td>11</td>
<td>9</td>
</tr>
</tbody>
</table>

Fig. 22. Gains for two industrial surfaces.

criterion. The values of parameters $a$, $b$, $c$ and $d$ are presented in Table 4 for the four gains.

Gain = $a + b \cdot \Delta h_1 + c \cdot \Delta \alpha + d \cdot \Delta h_2 \cdot \Delta \alpha_2$. \hspace{1cm} (20)

The analysis of Table 4 shows:

- Parameter $a$ of this model gives the mean gain for the 9 surfaces. These average gains of the CATS method are particularly interesting. Note also that the shaped-tool method perform better than the half-barrel tool and that the CATS method, on average, divides by two the volume of interferences.
- The value of parameter $d$ relative to $b$ and $c$ determines if a relation exists between $\Delta h_1$ and $\Delta \alpha$. For the half-barrel method this relation is insignificant, whereas in the case of shaped-tool it is not.
- The sign of $b$ and $c$ determines if these parameters increase or decrease the benefits of the CATS method. $c$ is negative for all the gains; thus, if $\Delta \alpha$ increases, the benefit of the CATS method decreases. What may seem paradoxical is that $b$ is positive for all the models and particularly for the shaped-tool amplitude model; i.e., the gain of the CATS method increases when the rule length varies. This can be explained in the case of the shaped-tool gain amplitude. We note that the biggest values of $\alpha$ appear for the longest rules. Based on this observation, when $\Delta h$ is large the number of long rules is reduced; therefore the shaped-tool radius is optimized on a lower number of long rules, whence this tool is better optimized, whereas a cylindrical cutter generates a great amplitude of interferences. This explains a large positive value of $b$ for $G_{\text{ampl}}$ of the shaped-tool.

To verify the accuracy of these mathematical models, they were applied to two industrial surfaces:

- The first surface (noted surface A in Fig. 22) is the extrados of the inductor presented in Fig. 2. This surface has $\Delta h = 0.830$ and $\Delta \alpha = 0.887$. Thus it has a $\Delta h_1 = 1.22$ and $\Delta \alpha_2 = -0.05$.
- The second surface (noted surface B in Fig. 22) is the extrados of an impeller. This surface has $\Delta h = 0.653$ and $\Delta \alpha = 2.001$. Thus it has $\Delta h_1 = 0.681$ and $\Delta \alpha_2 = 1.81$.

The results are presented in Fig. 22 (note that in this figure the real gains of the half-barrel method on surface B are represented and are equal to zero). This figure illustrates the deviation between the computed gain and the gain obtained with a mathematical model of the experimental design. Note that in this figure the deviation is greater for the amplitude model than the volume model. These deviations can be explained by:

- The chosen model (Eq. (20)) is basic. The presented model is only used to determine the influence of parameters $\Delta h$ and $\Delta \alpha$ on the gains.
6. Conclusion

This paper proposes a new approach to reducing the problem of interferences during the flank milling of NDRS: the CATS method. This new approach is obtained by modifying the tool profile. CATS is not a new strategy to compute tool paths, it is used after the CAM system, and therefore, it is not in competition with tool positionings on NDRS. CATS method can be used with all types of positioning, and thus with all CAM systems. This approach is able to improve the machining efficiency of tool paths.

The tool profile is optimized for a chosen positioning, a nominal surface and functional criteria. Three criteria are defined with respect to specific industrial requirements, expressed by mechanical, hydraulic and machining experts. This method offers a new degree of freedom which is able to sculpt the shape of interferences. The different steps of this CATS method are the following:

1. Computation of the tool path.
2. Calculation of the distance profiles between the tool axes and the surface at each point of the tool path.
3. Association of a tool profile with the distance profiles: two methods are proposed (half-barrel and shaped-tool).
4. Computation of generated interferences.

This article applies this new method to software positioning for the machining of an industrial surface. Three performance indicators are created to quantify the geometrical efficiency of the machining process, these indicators being the expression of professional requirements. It is demonstrated (by computation and experimentation) that the CATS method ensures a reduction of the interference problem for the positioning/surface pair.

Subsequently, the influence of the surface on the usefulness of this method for software positioning was studied. This demonstrates the interest of the CATS method for almost all surface types. In conclusion, CATS method is a new approach, which can increase the competitiveness of all trajectories. But the use of this method will be really interesting to correct a machining which creates regular interferences; e.g. the machining of a NDRS using a simple positioning.

Other interesting study, opened up by the presented paper, would be the general application of CATS method to the free-form surface machining using multi-pass machining. Indeed, a barrel cutter can machine a smooth free-form surface with fewer paths than point milling for a same scallop height.

References