RADIATION PATTERNS OF A DIPOLE ANTENNA ARRAY ON BIANISOTROPIC SUBSTRATES WITH A SOFT- AND HARD-SURFACE: THE CLARIFICATION OF THE CONTINUOUS MAGNETIC GROUP OF SYMMETRY

W.-Y. Yin and L. W. Li

Department of Electrical Engineering
National University of Singapore
Singapore 119260

Abstract—A comparative investigation is performed for examining the radiation characteristics of a dipole antenna array on multilayered bianisotropic substrate with a grounded soft and hard surface possessing certain direction of the conductivity. Eight continuous magnetic groups of symmetries for describing the constitutive features of such bianisotropic structure are considered, respectively. Numerical examples are presented to show the effects resulted from the changing of magnetic groups on the co- and cross-polarized far-field normalized patterns of linear and square dipole antenna arrays. This work provides a basis for understanding the possible differences among various kinds of bianisotropic media.

1. INTRODUCTION

The interaction of electromagnetic waves with layered bianisotropic structures has been investigated by some researchers in the past few years. Based on different constitutive relations of the bianisotropic media, these studies have been motivated both by interest in developing new techniques for solving radiation and propagation problems and by particular engineering applications [1–8]. In these studies, the analytical approach presented by Tsalamengas, i.e., the technique of the generalized spectral-domain exponential matrix (GSDEM) should be mentioned, as is very efficient and powerful to deal with most multilayered bianisotropic linear media, and hence it has been adopted in
Figure 1. Dipole antenna array on a multilayered bianisotropic structure with a SHS or a perfectly conducting plane.

our previous work [9–13].

On the other hand, a novel method for describing the four independent constitutive tensors of bianisotropic media by using the theory of the continuous groups of symmetry has been developed more recently by Dmitriev [14, 15], and the constitutive characteristics of “Kamenskii’s media” has been suggested [16–18]. Physically, according to the magnetic group theory, a whole and clear picture of the constitutive features of many kinds of bianisotropic media can be obtained and understood.

In this study, the authors pay attention to the radiation characteristics of a dipole antenna array on various multilayered bianisotropic structures possessing some magnetic groups of the symmetry, such as $D_{\infty}(C_{\infty}), C_{\infty v}(C_{\infty}), C_{\infty h}(C_{\infty}), D_{\infty h}(C_{\infty v}), D_{2d}(C_{2v}), D_{2d}, C_{s}, C_{2v}(C_{s})$. In the next section, the technique of GSDEM is employed for the mathematical treatment, however, the grounded plane of the bianisotropic substrate is assumed to be a retrodirective surface known as soft- and hard-surface (SHS) [19–21]. This novel artificially corrugated surface has numerous practical applications in microwave engineering, of which the unique property is that it has the same boundary condition for both the electric and magnetic fields.
2. THEORY

Fig. 1(a) shows the cross section of a grounded $N$-layer bianisotropic substrate structure. The ground surface may be a SHS or a perfectly conducting plane. Usually, the region $z > 0$ is a free space $(\varepsilon_0, \mu_0)$.

Each layer of this bianisotropic structure is characterized by the relative permittivity tensor $[\varepsilon^{(i)}(\omega)]$, the relative permeability tensor $[\mu^{(i)}(\omega)]$, and the relative magnetoelectric tensors $[\xi^{(i)}(\omega)]$ and $[\eta^{(i)}(\omega)]$, respectively. The generalized constitutive equations are stated as (the time dependence $e^{j\omega t}$ is assumed):

$$
\begin{bmatrix}
\bar{D}^{(i)}(\omega) \\
\bar{B}^{(i)}(\omega)
\end{bmatrix} =
\begin{bmatrix}
\varepsilon_0 [\varepsilon^{(i)}(\omega)] & \sqrt{\mu_0 \varepsilon_0} [\xi_e^{(i)}(\omega)] \\
\sqrt{\mu_0 \varepsilon_0} [\xi_m^{(i)}(\omega)] & \mu_0 [\mu^{(i)}(\omega)]
\end{bmatrix}
\begin{bmatrix}
\bar{E}^{(i)}(\omega) \\
\bar{H}^{(i)}(\omega)
\end{bmatrix},
$$

$i = 1, \ldots, N$ (1a, b)

where

$$
\begin{bmatrix}
\varepsilon^{(i)} \\
\mu^{(i)} \\
\xi^{(i)} \\
\eta^{(i)}
\end{bmatrix} =
\begin{bmatrix}
c_{xx}^{(i)} & c_{xy}^{(i)} & c_{xz}^{(i)} \\
c_{yx}^{(i)} & c_{yy}^{(i)} & c_{yz}^{(i)} \\
c_{zx}^{(i)} & c_{zy}^{(i)} & c_{zz}^{(i)}
\end{bmatrix},
$$

$c = \varepsilon$, $\mu$, $\xi_e$, $\xi_m$ (1c)

and $[c^{(i)}]$ can be simplified for certain magnetic group of the symmetry, for instance,

$$
1^o(D_\infty(C_\infty)) : [\varepsilon^{(i)}] =
\begin{bmatrix}
\varepsilon_{xx}^{(i)} & \varepsilon_{xy}^{(i)} & 0 \\
-\varepsilon_{yx}^{(i)} & \varepsilon_{xx}^{(i)} & 0 \\
0 & 0 & \varepsilon_{zz}^{(i)}
\end{bmatrix},
$$

$$
[\mu^{(i)}] =
\begin{bmatrix}
\mu_{xx}^{(i)} & \mu_{xy}^{(i)} & 0 \\
-\mu_{yx}^{(i)} & \mu_{xx}^{(i)} & 0 \\
0 & 0 & \mu_{zz}^{(i)}
\end{bmatrix},
$$

$$
[\xi^{(i)}] =
\begin{bmatrix}
\xi_{xx}^{(i)} & \xi_{xy}^{(i)} & 0 \\
-\xi_{yx}^{(i)} & \xi_{xx}^{(i)} & 0 \\
0 & 0 & \xi_{zz}^{(i)}
\end{bmatrix},
$$

$$
[\eta^{(i)}] = -[\xi^{(i)}];
$$

(2a)
2° \( (C_{\infty v}(C_{\infty})) \): \[
\begin{bmatrix}
\varepsilon^{(i)} \\
\mu^{(i)} \\
\xi^{(i)}
\end{bmatrix}
\] are the same as 1°, except that \[
\eta^{(i)} = \xi^{(i)} ;
\] (2b)

3° \( (C_{\infty h}(C_{\infty})) \): \[
\begin{bmatrix}
\varepsilon^{(i)} \\
\mu^{(i)} \\
\xi^{(i)}
\end{bmatrix}
\] are the same as 2°, except that \[
\eta^{(i)}_{xy} = -\eta^{(i)}_{yz} = -\xi^{(i)}_{xy} ;
\] (2c)

4° \( (C_{s}) \):
\[
\begin{bmatrix}
\varepsilon^{(i)} \\
\mu^{(i)} \\
\xi^{(i)} \\
\eta^{(i)}
\end{bmatrix} =
\begin{bmatrix}
\varepsilon^{(i)}_{xx} & 0 & 0 & 0 \\
0 & \varepsilon^{(i)}_{yy} & \varepsilon^{(i)}_{yz} & 0 \\
0 & \varepsilon^{(i)}_{yz} & \varepsilon^{(i)}_{zz} & 0 \\
0 & \xi^{(i)}_{xy} & \xi^{(i)}_{yz} & \xi^{(i)}_{zz}
\end{bmatrix} ,
\] (2d)

5° \( (C_{2v}(C_{s})) \):
\[
\begin{bmatrix}
\varepsilon^{(i)} \\
\mu^{(i)} \\
\xi^{(i)} \\
\eta^{(i)}
\end{bmatrix} =
\begin{bmatrix}
\varepsilon^{(i)}_{xx} & 0 & 0 & 0 \\
0 & \varepsilon^{(i)}_{yy} & 0 & 0 \\
0 & 0 & \varepsilon^{(i)}_{zz} & 0 \\
0 & \xi^{(i)}_{xy} & \xi^{(i)}_{yz} & \xi^{(i)}_{zz}
\end{bmatrix} ,
\] (2e)
\[6^\circ(D_{2d}) : \quad \text{The parameters are the same as } 4^\circ, \text{ except that} \]
\[
\varepsilon_{xx}^{(i)} = \varepsilon_{yy}^{(i)} = 0, \quad \mu_{xx}^{(i)} = \mu_{yy}^{(i)} = 0, \quad \xi_{yx}^{(i)} = \xi_{xy}^{(i)} = 0; \quad (2f)
\]

\[7^\circ(D_{2d}(C_{2\nu})) : \quad \text{The parameters are the same as } 6^\circ, \text{ except that} \]
\[
\eta_{zy}^{(i)} = -\xi_{yz}^{(i)}, \quad \eta_{yx}^{(i)} = -\xi_{xy}^{(i)}; \quad (2g)
\]

\[8^\circ(D_{ooh}(C_{oo\nu})) : \quad \text{The parameters are the same as } 7^\circ, \text{ except that} \]
\[
\xi_{xy}^{(i)} = -\xi_{yx}^{(i)}; \quad (2h)
\]

It is understood that, strictly speaking, all the elements in (2) should be a function of the operating angular frequency that is suppressed here. According to the technique of GSDEM, the transverse field components in the spectral-domain are, as in Fig. 1(a), governed by

\[
\frac{d}{dz} \begin{bmatrix} \tilde{f}^{(i)} \\ \end{bmatrix}_{4 \times 1} = \begin{bmatrix} P^{(i)} \\ \end{bmatrix}_{4 \times 4} \begin{bmatrix} \tilde{f}^{(i)} \\ \end{bmatrix}_{4 \times 1} \quad (3)
\]

where \[\begin{bmatrix} \tilde{f}^{(i)} \\ \end{bmatrix}_{4 \times 1} = \begin{bmatrix} \tilde{E}_{x}^{(i)} \\ \tilde{E}_{y}^{(i)} \\ \tilde{H}_{x}^{(i)} \\ \tilde{H}_{y}^{(i)} \\ \end{bmatrix}^{T}, \text{ the superscript } T \text{ stands for the transpose of matrix, and the expressions of the elements } P_{11}^{(i)} - P_{44}^{(i)} \text{ can be referred to [12]. In the } z > 0 \text{ region, the fields are determined by [1]} \]

\[
\begin{bmatrix} \tilde{f}^{(i)} \\ \end{bmatrix}^{(0)} = \begin{bmatrix} [L] \\ [M] \end{bmatrix} \begin{bmatrix} \beta_{1} \\ \beta_{2} \end{bmatrix} + \sum_{p=1}^{q} \tilde{f}_{d}^{(p)} \right|_{z=0} \quad (4a)
\]

and

\[
\begin{bmatrix} \tilde{f}_{d}^{(p)} \\ p_{x}^{(p)} \\ p_{y}^{(p)} \end{bmatrix} \right|_{z=0} = \frac{e^{i [k_{x} x^{(p)} + k_{y} y^{(p)}]}}{8\pi^{2}\gamma_{0}} \begin{bmatrix} j\gamma_{0} (k_{x}^{2} - k_{0}^{2}) / k_{0} \\ j\gamma_{0} k_{x} k_{y} / k_{0} \\ j\gamma_{0} / \gamma_{0} \end{bmatrix} \quad (4b)
where \( x^{(p)} \) and \( y^{(p)} \) are the coordinates of the \( p \)-th dipole antenna at \( z = 0 \). For a SHS at \( z = -d = -\sum_{l=1}^{N} D^{(l)} \), the boundary conditions become

\[
\bar{v} \cdot \bar{E}^{(N)}|_{z=-d} = 0, \quad \text{and} \quad \bar{v} \cdot \bar{H}^{(N)}|_{z=-d} = 0 \tag{5}
\]

where \( \bar{v} = \nu_x \bar{e}_x + \nu_y \bar{e}_y \) is the direction of conductivity on the SHS, and \( \nu_x^2 + \nu_y^2 = 1 \). So, we further have

\[
\begin{bmatrix}
\beta_1 \\
\beta_2
\end{bmatrix} = \begin{bmatrix}
D_{11} & D_{12} \\
D_{21} & D_{22}
\end{bmatrix}^{-1} \begin{bmatrix}
D_{01} \\
D_{02}
\end{bmatrix} \tag{6a}
\]

and

\[
D_{21} = C_{21} L_{11} + C_{22} L_{21} + C_{23} M_{11} + C_{24} M_{21},
\]

\[
D_{22} = C_{21} L_{12} + C_{22} L_{22} + C_{23} M_{12} + C_{24} M_{22},
\]

\[
D_{01} = -C_{11} \sum_{p=1}^{q} \bar{E}^{(p)}_{yd}|_{z=0} - C_{12} \sum_{p=1}^{q} \bar{E}^{(p)}_{yd}|_{z=0}
\]

\[
- C_{13} \sum_{p=1}^{q} \bar{H}^{(p)}_{yd}|_{z=0} - C_{14} \sum_{p=1}^{q} \bar{H}^{(p)}_{yd}|_{z=0},
\]

\[
D_{02} = -C_{21} \sum_{p=1}^{q} \bar{E}^{(p)}_{zd}|_{z=0} - C_{22} \sum_{p=1}^{q} \bar{E}^{(p)}_{yd}|_{z=0}
\]

\[
- C_{23} \sum_{p=1}^{q} \bar{H}^{(p)}_{zd}|_{z=0} - C_{24} \sum_{p=1}^{q} \bar{H}^{(p)}_{yd}|_{z=0},
\]

\[
C_{11} = \nu_x T_{11}(-d) + \nu_y T_{21}(-d), \quad C_{12} = \nu_x T_{12}(-d) + \nu_y T_{22}(-d),
\]

\[
C_{13} = \nu_x T_{13}(-d) + \nu_y T_{23}(-d), \quad C_{14} = \nu_x T_{14}(-d) + \nu_y T_{24}(-d),
\]

\[
C_{21} = \nu_x T_{31}(-d) + \nu_y T_{41}(-d), \quad C_{22} = \nu_x T_{32}(-d) + \nu_y T_{42}(-d),
\]

\[
C_{23} = \nu_x T_{33}(-d) + \nu_y T_{43}(-d), \quad C_{24} = \nu_x T_{34}(-d) + \nu_y T_{44}(-d) \tag{6b}
\]

where \( T_{mn}(-d)(m, n = 1, \ldots, 4) \) are the elements of transmission matrix \( [T(-d)] \), which relate the tangential electromagnetic fields at \( z = -d \) to another at \( z = 0 \). By combining (6a) with (4a) and by using the stationary phase asymptotic integration technique, the radiated field in the far-zone of the above dipole antenna array can be evaluated, i.e.,

\[
E_\theta = j \frac{2\pi \psi_1}{R \sin \theta} e^{-jk_0 R}, \quad E_\varphi = -j \frac{2\pi \psi_2}{R \tan \theta} e^{-jk_0 R} \tag{7a, b}
\]
where

\[ \tilde{\psi}_1 = k_x \tilde{E}_x^{(0)}(k_x, k_y, 0) + k_y \tilde{E}_y^{(0)}(k_x, k_y, 0) \]  \hspace{1cm} (7c)

\[ \tilde{\psi}_2 = k_y \tilde{E}_x^{(0)}(k_x, k_y, 0) - k_x \tilde{E}_y^{(0)}(k_x, k_y, 0) \]  \hspace{1cm} (7d)

and \( k_0 = \omega \sqrt{\mu_0 \varepsilon_0} \).

3. NUMERICAL RESULTS AND DISCUSSION

Following the GSDEM technique briefly described as above, we have developed some computer codes for investigating the combined effects of constitutive parameters corresponding to different magnetic groups of the symmetry on the far-field patterns of various dipole antenna arrays. Herein the losses of the multilayered bianisotropic substrate are not taken into account. As the measured datum of bianisotropic media with certain magnetic group of symmetries are not available in the literature now, the authors have to assume within a realizable physical range the constitutive parameters of bianisotropic media used for calculation.

At first, our codes have been checked by considering the radiation of an \( x \)-directional dipole antenna in the ordinary nonreciprocal ferrite substrate. It is clearly shown in Fig. (2a) that an excellent agreement between our results and those [23] has been achieved. In the following numerical samples, the normalization means that each set of \( E_{\theta, \varphi} \) is normalized by its own maximum (i.e., \( \frac{|E_{\theta, \varphi}|}{\max(|E_{\theta, \varphi}| \text{ or } |E_{\theta, \varphi}|)} \)).

In Fig. 2, the first layer of the substrate is assumed to be ferrite biased by an internal DC field \( (M_s^{(1)} \mu_0 = 0.16T, \omega^{(1)} / \omega^{(1)} = 0.2) \), while the second layer is isotropic with \( [\varepsilon^{(2)}] = 12.9 \) and \( [\mu^{(2)}] = 1.0 \). From above it is clear that the relative levels of the cross-polarized component \( E_{\theta} \) with respect to the \( x \)-direction dipole antenna and the co-polarized component \( E_{\varphi} \) can be adjusted effectively only by choosing the direction factors of the conductivity on the SHS. For instance, for curve (b) \( E_{\theta} \) is a significant component, conversely, for curve (d) \( E_{\varphi} \) becomes dominant. When \( \nu_x = \nu_y = \frac{\sqrt{2}}{2} \), \( E_{\theta} \) is comparable to \( E_{\varphi} \) in magnitude (curve(c)). Practically, it would be very difficult to change the pattern sharp by varying the direction factors of conductivity on the SHS. However, information such as shown by curves (b), (c) and (d) can be used to help design an antenna that will have optimum pattern sharp. On the other hand, pattern reshaping can also be achieved only by varying the bias field intensity.
Figure 2. Normalized radiation patterns of a dipole antenna in the x-direction mounted on a two-layer nonreciprocal ferrite substrate with a grounded PEC and a SHS. \( f = 17.25 \text{ GHz}, \ D^{(1)} = 1.524 \times 10^{-3} \text{ m}, \ D^{(2)} = 2.286 \times 10^{-3} \text{ m}, \) and \( [\varepsilon^{(2)}] = 12.97. \)

(a) PEC: \( [\varepsilon^{(1)}] = 15.17, \ \mu_{yy}^{(1)} = 1.0, \ \mu_{xx}^{(1)} = \mu_{zz}^{(1)} = \mu_{y}^{(1)} = 0.0, \ [\mu^{(2)}] = 1.0, \ \mu_{xx}^{(2)} = \mu_{zz}^{(2)} = 1.0 + \frac{\omega_0^{(1)} \omega_m^{(1)}}{\omega_0^{(1)} + \omega_0^{(2)} - \omega^2}, \)

\[ \mu_{xx}^{(1)} = -\mu_{xx}^{(1)} = -\frac{j\omega \omega_m^{(1)}}{\omega_0^{(1)} - \omega^2}, \ \frac{\omega_0^{(1)}}{\omega_m^{(1)}} = 0.2, \ M_s^{(1)} \mu_0 = 0.16T, \]

\[ \omega_m^{(1)} = 2.21 \times 10^5 M_s^{(1)}, \ \left[ \xi^{(i)} \right] = \left[ \eta^{(i)} \right] = 0.07, \ i = 1, 2. \]

(b) SHS: The parameters are the same as (a), except that \( \nu_x = 1.0, \ \nu_y = 0.0. \)

(c) \( \nu_x = \nu_y = \frac{\sqrt{2}}{2}. \)

(d) \( \nu_x = 0.0, \ \nu_y = 1.0. \)

\(|E_\varphi|: \text{solid line}; \ |E_\theta|: \text{dotted line.} \)

Fig. 3 shows the radiation pattern of an x-direction dipole antenna array \( (q = 9) \) on a two-layer bianisotropic substrate with a grounded PEC and a SHS, respectively, while the second layer of substrate is also isotropic.

In Fig. 3, the locations of nine dipole antennas along the x-axis \( (p_x = 1, p_y = 0) \) are determined by \( 1 \left( -\frac{1}{2}, 0 \right), 2 \left( -\frac{3}{8}, 0 \right), 3 \left( -\frac{1}{4}, 0 \right), 4 \left( \frac{1}{8}, 0 \right), 5 \left( 0, 0 \right), 6 \left( \frac{1}{4}, 0 \right), 7 \left( \frac{3}{4}, 0 \right), 8 \left( \frac{3}{8}, 0 \right), 9 \left( \frac{1}{2}, 0 \right), \) and \( \lambda \) is the operating wavelength. Here three kinds of the magnetic groups are considered simultaneously. It is demonstrated in Figs. 3(a) and (b)
Figure 3. Normalized radiation patterns for an x-direction dipole antenna array on a two-layer bianisotropic substrate with a grounded PEC and a SHS. \( f = 10 \text{ GHz}, \ D^{(1)} = 1.524 \times 10^{-3} \text{ m}, \ D^{(2)} = 2.286 \times 10^{-3} \text{ m}, \ \varepsilon^{(2)} = 12.97, \ \mu^{(2)} = 1.0, \)
\[
\begin{align*}
[\varepsilon^{(1)}] &= \begin{bmatrix} 5.0 & j0.5 & 0.0 \\
-j0.5 & 5.0 & 0.0 \\
0.0 & 0.0 & 6.0 \end{bmatrix}, \quad [\mu^{(1)}] = \begin{bmatrix} \mu_{xx}^{(1)} \\
\mu_{yy}^{(1)} \end{bmatrix}, \quad \frac{\omega_0^{(1)} \omega_m^{(1)}}{\omega_0^{(1)} - \omega^2}, \\
\omega_0^{(1)}/\omega_m^{(1)} &= 0.2, \quad \mu_{xy}^{(1)} = -\mu_{yx}^{(1)} = -\frac{j\omega_0 \omega_m^{(1)}}{\omega_0^{(1)} - \omega^2}, \quad M_s^{(1)} \mu_0 = 0.16T, \\
\omega_m^{(1)} &= 2.21 \times 10^5 M_s^{(1)},
\end{align*}
\]

(a) PEC: \([\xi^{(1)}] = -[\eta^{(1)}] = [C], \)
\[
[C] = \begin{bmatrix} j0.5 & 0.3 & 0.0 \\
-0.3 & j0.5 & 0.0 \\
0.0 & 0.0 & j0.6 \end{bmatrix}, \quad (D_{\infty}(C_{\infty}));
\]
\[
[\xi^{(1)}] = [\eta^{(1)}] = [C](C_{\infty})(C_{\infty}) \quad \text{(square dot)};
\]
\[
[\xi^{(1)}] - [C], [\eta^{(1)}] = \begin{bmatrix} j0.5 & -0.3 & 0.0 \\
0.3 & j0.5 & 0.0 \\
0.0 & 0.0 & j0.6 \end{bmatrix}, \quad (C_{2\infty})(C_{\infty}) \quad \text{(circular dot}).
\]

(b) SHS (\(\nu_x = 1.0, \ \nu_y = 0.0\)): the parameters are the same as (a).
(c) SHS (\(\nu_x = 0.0, \ \nu_y = 1.0\)): the parameters are the same as (b). 
|\(E_\varphi|\) : solid line; |\(E_\theta|\) : dotted line.
that by changing the forms of the magnetoelectric tensors \([\xi^{(1)}]\) and \([\eta^{(1)}]\) from \(D_\infty(C_\infty) \rightarrow C_{\infty\nu}(C_\infty) \rightarrow C_{\infty h}(C_\infty)\), the normalized pattern \(E_\varphi\) is nearly the same for the grounded PEC as well as the SHS case \(|E_\varphi| > |E_\theta|\). The resulted major change is in the shape and magnitude of the cross-polarized field \(E_\varphi\). In Fig. 3(c), since \(\nu_x = 0.0, \nu_y = 1.0\), the cross-polarized component \(E_\varphi\) becomes dominant \(|E_\varphi| < |E_\theta|\). Relatively, the effect of changing the magnetic groups on \(E_\theta\) is obvious in Figs. 3(a) and (b), while in Fig. 3(c) the effect on \(E_\varphi\) is clear.

Fig. 4 depicts the radiation pattern of an \(x\)-direction dipole antenna array \((q = 9)\) along the \(y\)-axis on a two-layer bianisotropic substrate with a grounded SHS, and the constitutive parameters of the first substrate layer are the same as in Fig. 3 \((D_\infty(C_\infty))\).

In Fig. 4, the location of nine dipole antennas along the \(y\)-axis \((p_x = 1, p_y = 0)\) are just determined by 1 \((0, -\frac{\lambda}{2})\), 2 \((0, -\frac{3\lambda}{8})\), 3 \((0, -\frac{\lambda}{4})\), 4 \((0, -\frac{\lambda}{8})\), 5 \((0, 0)\), 6 \((0, \frac{\lambda}{8})\), 7 \((0, \frac{3\lambda}{8})\), 8 \((0, \frac{\lambda}{4})\), 9 \((0, \frac{\lambda}{2})\), and also three kinds of magnetic groups are introduced in the second layer of substrate \((D_{2d}, D_{\infty h}(C_{\infty\nu}), D_{2d}(C_{2\nu}))\), respectively, but the second layer is magnetically isotropic \(([\mu^{(2)}] = 1.0)\). From Fig. 4, it is found that for these three kinds of magnetic groups, the normalized pattern shapes are nearly the same and they are symmetrical about the direction \(\theta = 0^\circ\).

Furthermore, Fig. 5 shows the radiation pattern of an \(x\)-direction dipole antenna square array \((q = 9)\) on a two-layer bianisotropic substrate with a grounded SHS, and the constitutive parameters of the first layer of substrate are the same as in Fig. 3 \((D_\infty(C_\infty))\).

In Fig. 5, the second layer of substrate is supposed to be a ferrite biased by an internal DC field in the direction \((\theta_0 = 90^\circ, \varphi_0)\). So, the relative permeability tensor \([\mu^{(2)}]\) is calculated by [23]:

\[
\begin{bmatrix}
\mu + (1 - \mu) \cos^2 \varphi_0 & (1 - \mu) \cos \varphi_0(2) \sin \varphi_0(2) & j\kappa \sin \varphi_0(2) \\
(1 - \mu) \cos \varphi_0(2) \sin \varphi_0(2) & \mu + (1 - \mu) \sin \varphi_0(2) & -j\kappa \cos \varphi_0(2) \\
-j\kappa \sin \varphi_0(2) & j\kappa \cos \varphi_0(2) & \mu
\end{bmatrix}
\]

(8a)

and \(\mu = 1 + \frac{\omega_0(2)\omega_m(2)}{\omega_0(2)^2 - \omega^2}, \kappa = \frac{\omega_m(2)}{\omega_0(2)^2 - \omega^2}, \) and \(\omega_m(2) = 2.21 \times 10^5 M_s(2)\).

The locations of the nine dipole antennas in the square array are given by: 1 \((\frac{\lambda}{4}, \frac{\lambda}{4})\), 2 \((\frac{3}{4}, 0)\), 3 \((\frac{1}{4}, -\frac{\lambda}{4})\), 4 \((0, \frac{\lambda}{4})\), 5 \((0, 0)\), 6 \((0, -\frac{\lambda}{4})\), 7 \((\frac{3}{4}, -\frac{\lambda}{4})\), 8 \((\frac{1}{4}, 0)\), and 9 \((-\frac{\lambda}{4}, \frac{\lambda}{4})\). Increasing the bias field intensity \(\omega_0(2)/\omega_m(2)\)
Figure 4. Normalized radiation patterns for an \(x\)-direction dipole antenna array along the \(y\)-axis on a two-layer bianisotropic substrate with a grounded SHS. \(f = 10\, \text{GHz}, \ D^{(1)} = 1.524 \times 10^{-3}\, \text{m},\ D^{(2)} = 2.286 \times 10^{-3}\, \text{m},\ \varepsilon^{(2)}_{xx} = \varepsilon^{(2)}_{yy} = 9.4, \ \varepsilon^{(2)}_{zz} = 11.6, \ [\mu^{(2)}] = 1.0,\)

(a) SHS
\[
\begin{align*}
(\nu_x=1.0, \ \nu_y=0.0) : & \quad \begin{bmatrix} \xi^{(2)} \\ \eta^{(2)} \end{bmatrix} = [C], \quad [C] = \begin{bmatrix} 0.0 & 0.6 & 0.0 \\ 0.6 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix} (D_{2d}); \\
\begin{bmatrix} \xi^{(2)} \\ \eta^{(2)} \end{bmatrix} = - \begin{bmatrix} \xi^{(2)} \\ \eta^{(2)} \end{bmatrix} = \begin{bmatrix} 0.0 & 0.6 & 0.0 \\ -0.6 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \end{bmatrix} (D_{\infty h}(C_{\infty v})) \text{ (square dot)}; \\
\begin{bmatrix} \xi^{(2)} \\ \eta^{(2)} \end{bmatrix} = - \begin{bmatrix} \xi^{(2)} \\ \eta^{(2)} \end{bmatrix} = [C](D_{2d}(C_{2v})) \text{ (circular dot)}. 
\end{align*}
\]

(b) SHS (\(\nu_x = \nu_y = \frac{\sqrt{2}}{2}\)): the parameters are the same as (a).

(c) SHS (\(\nu_x = 0.0, \ \nu_y = 1.0\)): the parameters are the same as (b).

\(|E_{\varphi}|\): solid line; \(|E_{\theta}|\): dotted line.
Figure 5. Normalized radiation patterns for an $x$-direction dipole antenna square array on a two-layered bianisotropic substrate with a grounded SHS. $f = 10 \text{ GHz}$, $D^{(1)} = 1.524 \times 10^{-3} \text{ m}$, $D^{(2)} = 2.286 \times 10^{-3} \text{ m}$, $\varepsilon^{(2)} = 12.6$, $M_s^{(2)} \mu_0 = 0.275$,

(a) $p_x = 1$, $p_y = 0$, $\nu_x = 1.0$, $\nu_y = 0.0$, $\varphi_0^{(2)} = 60^\circ$, $\omega_0^{(2)}/\omega_m^{(2)} = 0.2$ (square dot); 1.0 (circular dot);

(b) The parameters are the same as (a), except that $\nu_x = \nu_y = \frac{\sqrt{2}}{2}$;

(c) The parameters are the same as (b), except that $\nu_x = 0.0$, $\nu_y = 1.0$.

$|E_\varphi|$: solid line; $|E_\theta|$: dotted line.

from 0.2 to 1.0, the main beam in Fig. 5(a) nearly scans from $\theta_m = 1^\circ$ to $14^\circ$, and correspondingly, the cross-polarized field is increased. Here, $\theta_m$ is the main beam direction defined as the beam due to the co-polarized component $E_\varphi$ [23]. From Fig. 5(b) or (c) it is clear that pattern reshaping can still be realized effectively only by changing the internal bias field, i.e., its magnitude or direction. Various numerical experiments prove that,
Figure 6. Normalized radiation patterns for a dipole antenna square array on a three-layer bianisotropic substrate with a grounded SHS. $f = 10$ GHz, $p_x = p_y = 1$, $\nu_x = 1.0$, $\nu_y = 0.0$, $D^{(1)} = 0.5 \times 10^{-3}$ m, $D^{(2)} = 1.524 \times 10^{-3}$ m, $D^{(3)} = 2.286 \times 10^{-3}$ m, $\varepsilon_{xx}^{(2)} = \varepsilon_{yy}^{(2)} = 9.4$, $\varepsilon_{zz}^{(2)} = 11.6$, $[\varepsilon^{(3)}] = 12.9$, $M_s^{(3)} \mu_0 = 0.275$, $\omega_0^{(3)} / \omega_m^{(3)} = 0.2$, $[\mu^{(2)}] = 1.0$, $[\xi^{(2)}] = [\eta^{(2)}] = [C]$, 

(a) $[C] = \begin{bmatrix} 0.6 & 0.0 & 0.0 \\ 0.0 & 0.6 & 0.0 \\ 0.0 & 0.0 & 0.8 \end{bmatrix} \quad (D_{\infty})$ (square dot); $[\xi^{(2)}] = [\eta^{(2)}] = [C](D_{\infty}h(D_{\infty}))$ (circular dot).

(b) $\varphi_0^{(2)} = 0^\circ$, $M_s^{(2)} \mu_0 = 0.275$, $\omega_0^{(2)} / \omega_m^{(2)} = 0.2$, $[\xi^{(2)}] = [C]$, 

$[C] = \begin{bmatrix} 0.0 & 0.5 & 0.6 \\ 0.5 & 0.0 & 0.0 \\ 0.6 & 0.0 & 0.0 \end{bmatrix}$, $[\eta^{(2)}] = \begin{bmatrix} 0.0 & -0.5 & 0.6 \\ -0.5 & 0.0 & 0.0 \\ 0.6 & 0.0 & 0.0 \end{bmatrix} (C_{2\nu}(C_s))$ (square dot); $[\xi^{(2)}] = [\eta^{(2)}] = [C](C_s)$ (circular dot).

\[
E_{\theta,\varphi} \left( \mp 360^\circ + \varphi, \mp 180^\circ + \varphi_0^{(2)} \pm \theta_0^{(2)} \right) = E_{\theta,\varphi} \left( \varphi, \varphi_0^{(2)} \mp \theta_0^{(2)} \right) \quad (9a)
\]
\[
\theta_m \left( \mp 360^\circ + \varphi, \mp 180^\circ + \varphi_0^{(2)} \pm \theta_0^{(2)} \right) = \theta_m \left( \varphi, \varphi_0^{(2)} \mp \theta_0^{(2)} \right) \quad (9b)
\]

and (9) is also true if we inverse the orientation of all the dipole antennas simultaneously.

Finally, Fig. 6 depicts the radiation pattern of a dipole antenna square array ($q = 9$) on a three-layer substrate, and also the first layer of substrate processes the magnetic group $D_{\infty}(C_{\infty})$ as shown in Fig. 3(a), while the third layer is a ferrite with the internal bias DC...
field $\varphi_0^{(3)} = 30^\circ$, $\theta_0^{(3)} = 90^\circ$.

In Fig. 6, the locations of nine dipole antennas are the same as in Fig. 5 but $p_x = p_y = 1$. For case (a) the second layer of the substrate is a uniaxial bianisotropic medium with the magnetic groups $D_\infty$ and $D_{\infty h}(D_\infty)$, respectively. For case (b) the second layer possesses the magnetic groups $C_{2\nu}(C_s)$ and $C_s$ but $\mu_{yy}^{(2)} = \mu_{zz}^{(2)}$ and $\varepsilon_{yz}^{(2)} = \varepsilon_{zy}^{(2)} = 0.0$, respectively. As in Fig. 3, here also exists very little difference in the normalized pattern $E_\varphi$ for either $\{D_\infty, D_{\infty h}(D_\infty)\}$ or $\{C_{2\nu}(C_s), C_s\}$, for instance, for case (a) we find $E_\varphi(D_\infty) = E_\varphi(D_{\infty h}(D_\infty))$. The main effect resulted from the changing of the magnetic groups is on the shape and magnitude of $E_\theta$.

4. CONCLUSION

In this study, theoretical investigations have been carried out on the comparative effects of the magnetic groups of symmetry in bianisotropic substrates with a grounded soft and hard surface on the radiation patterns of some dipole antenna arrays. The mathematical treatment is based on the generalized exponential matrix technique in the spectral domain. It is believed that our study can provide much insight into the diverse electromagnetic properties of various bianisotropic composites and their potential applications in antenna engineering. Naturally, following the procedure adopted above, the characteristics of a large number of different magnetic groups in bianisotropic superstrate-substrate structures can be explored and understood physically.

ACKNOWLEDGMENT

This work was supported with funds by the National University of Singapore’s sponsored project RP3981676. W. Y. Yin also appreciates greatly the Alexander von Humboldt Research Foundation for its support during his research attachment in Germany.

REFERENCES


Wenyan Yin was born in 1961 in Jiangsu Province, P.R. China. He received M.S. and Ph.D. degrees in Electrical Engineering from Xi’an University (XU) in 1989 and Xi’an Jiaotong University (XJU) in 1994, respectively. Since 1993, he has worked in the Department of Electronic Engineering of Northwestern Polytechnic University (NPU) as an Associate Professor. He has been granted the research fellowship by the Alexander von Humboldt-Stiftung of Germany from 1997 to 1998 and he has done research work in the Department of Electrical Engineering of Duisburg University. Now he is doing research work in the Department of Electrical Engineering at the National University of Singapore as a Research Fellow. His research interests are in the areas of the electromagnetic characteristics of bi(an)isotropic...
materials and their applications, microwave and millimeter-wave theory and techniques, microstrip antennas and wave propagation in mobile communication, etc. He has been a senior member of Chinese Institute of Electronics, and a member of American Association of the Advancement of Science.

Le-Wei Li received the degrees of B.Sc. in Physics, M.Eng.Sc. and Ph.D. in Electrical Engineering from Xuzhou Normal University, Xuzhou, China, in 1984, China Research Institute of radiowave Propagation (CRIRP), Xinxiang, China, in 1987 and Monash University, Melbourne, Australia, in 1992, respectively. In 1992, he worked at La Trobe University (jointly with Monash University), Melbourne, Australia as a Research Fellow. Since 1992, He has been with the Department of Electrical Engineering at the National University of Singapore firstly as a Research Scientist, then as a Lecturer, and currently as a Senior Lecturer. His current research interests include electromagnetic theory, radio wave propagation and scattering in various media, microwave propagation and scattering in tropical environment, and analysis and design of antennas. Dr. Li received several scientific achievement awards from various Chinese institutions among which the most recent one is the National Science and Technology Advancement Award with medal rewarded by National Science and Technology Committee, China in 1996. He has been invited to serve editorial board of Journal of Electromagnetic Waves and Application and to become a member of The Electromagnetics Academy since 1998. He is currently a senior member of the IEEE.