Calibration for mobile robots with an invariant Jacobian

Joaquim A. Batlle, Josep M. Font-Llagunes, Ana Barjau *
Department of Mechanical Engineering, Universitat Politècnica de Catalunya, Diagonal 647, 08028 Barcelona, Spain

ABSTRACT

The kinematics of some mobile robots is described through a strictly invariant Jacobian matrix \( J \). This is the case for robots with three degrees of freedom and suitable omnidirectional wheels, and that for robots with conventional wheels and differential kinematics. This article proposes a calibration technique for the matrix \( J \) of such robots. It is based on four accurate configuration measurements associated with particular nominal motions where the generalized velocities maintain a constant proportional relationship. As a consequence, the nominal trajectories are arcs of circumference and may be part of the actual trajectories of the robot. An application example is presented.

1. Introduction

In order to achieve an autonomous operation, a mobile robot has to be able to estimate its pose (position and orientation) during its motion within the working environment. This operation, known in robotics as localization or positioning, is one of the fundamental problems in mobile robot navigation.

The process of robot localization under dynamic conditions (when the robot moves) relies highly on odometry. In this process, the encoder measurements are generally fused with data coming from other sensors in order to get the optimal pose estimation at any time. To avoid growth in the error as the distance traveled increases, at least one of those sensors has to be exteroceptive, i.e., must detect known features of the robot’s working environment.

In mobile robotics, the Extended Kalman Filter (EKF) \([1–3]\) is the most widely accepted tool to perform that data fusion. In a very general way, such a fusion can be represented as

\[
\text{[Odometry information]} \rightarrow \text{[EKF]} \leftarrow \text{[Exteroceptive information]} \downarrow \text{Optimal pose estimation.}
\]

The smaller the errors associated with the odometry and the measurements, the better the performance of the fusion algorithm.

Odometry errors can be classified into systematic and non-systematic (or random) errors \([4]\). The first are mainly associated with the idealized kinematical model of the mobile robot, and are particularly worrying because they accumulate along the robot’s trajectory. Nevertheless, they can be corrected through a suitable model calibration.

Conversely, non-systematic errors appear due to unpredictable causes such as random fluctuations on the relationship between the wheels’ angular velocity due to the control dynamics, and the robot’s interaction with features of the environment such as irregularities of the floor surface or casual wheel slippage. It is not possible to predict an upper bound for non-systematic errors; however, they are not usually as important as their systematic counterpart if the robot moves on a smooth and clean indoor surface \([5]\). In the work done by Borenstein \([6–8]\) it is shown that non-systematic errors can be reduced by orders of magnitude if redundant encoder data is used.

Several works concerning odometry calibration exist. Borenstein and Feng \([5,7]\) present an off-line calibration technique called the UMBmark test. It measures and corrects the systematic errors of a mobile robot with a differential drive. The calibration trajectory is a 4 m × 4 m square path followed in both the clockwise and counterclockwise directions. The method uses the final pose errors to correct the theoretical kinematical model of the robot.

The works by Larsen et al., Martinelli, Martinelli and Siegwart and Martinelli et al. \([9–12]\) propose an algorithm for an on-line calibration of the odometry systematic error. They use an Augmented Kalman Filter (AKF) that estimates both the robot’s pose and the parameters characterizing the odometry error during robot navigation. The work \([12]\) integrates also a new filter (the Observable Filter, OF) where the state variables to be estimated are the parameters characterizing the non-systematic error. The method is developed for differential and synchronous drive robots, and experiments with a differential platform are presented.
Other interesting works take into account the propagation of odometry errors from a theoretical point of view. An early work [13] presents a method to obtain the non-systematic error covariance matrix by incorporating knowledge of the path followed between odometric updates. Chong and Kleeman [14] present a different approach to model non-systematic errors. They compute the covariance matrix in closed form – when following constant curvature trajectories – by analyzing the position and orientation error noise obtained by repeating the same trajectory several times. Finally, Kelly [15] proposes a general solution for linearized systematic and random error propagation in odometry for any trajectory or error model.

This paper deals with an off-line odometry systematic error calibration for robots with an invariant Jacobian matrix. The presented method is based on a minimum of four accurate pose measurements (more measurements can also be used, but the method is then over-determined). The only requirement is that the three trajectories between the measured poses be followed with generalized velocities maintaining mutually constant relationships (different for each trajectory).

This calibration can be performed several times in an iterative way to improve the accuracy. The aim of this work is to show that with just one calibration the errors are effectively reduced, thus validating the proposed method. In a practical application, the iteration is advisable in order to obtain a more accurate Jacobian matrix. Once the off-line calibration has been done, further accuracy can be achieved if constrained trajectories leading to a linear algebraic odometry are used [16] and on-line methods (as those mentioned before) are implemented.

The presented method is more flexible than the UMBmark method [5,7] because the calibration trajectories may be actual trajectories of the mobile robot. Moreover, the method by Borenstein and Feng is intended for differential robots, whereas our method deals with all kinds of mobile robots with an invariant Jacobian. Certainly, robots with a differential drive belong to this group, but robots with omnidirectional wheels and three degrees of freedom (DOFs) [17,18] - like the one used in the experiments - are also suitable for calibration with this approach.

The paper is structured as follows. In Section 2, the analytical approach to calibrate 3-DOF mobile robots with an invariant Jacobian is presented. Section 3 gives a geometrical interpretation of this method, and Section 4 presents the experimental equipment to prove the good performance of the calibration method. The vehicle used is a 3-DOF mobile robot with omnidirectional wheels, whose design guarantees the invariance of the robot's Jacobian. This section also reports the pose errors obtained before and after calibration. Section 5 refers to calibration of differential drive mobile robots, and finally Section 6 summarizes the main points of this paper.

2. Analytical approach for 3-DOF mobile robots with an invariant Jacobian

For a robot having the three DOFs of rigid body plane motion (Fig. 1) and an invariant Jacobian matrix $[J]$, the rotation velocities $\{\dot{\theta}\} = \{\dot{\theta}_1, \dot{\theta}_2, \ldots, \dot{\theta}_N\}$ of its $N$ omnidirectional wheels, usually controlled by means of $N$ motors, are related to the generalized velocities of the robot frame $\{u\} = \{v_x, v_y, \psi\}$ (longitudinal and transversal velocity of a point, and rotation, respectively) through

$$\{\dot{\theta}\} = [J] \{u\}. \quad (1)$$

Due to geometrical errors in the model, the Jacobian $[J]$ is not exactly known. Only an approximation of it, denoted by $\tilde{J}$, is available. The use of $\tilde{J}$ to introduce the nominal velocities $\{u\}_{\text{nom}}^T = \{v_{\text{nom}}, \psi_{\text{nom}}\}$ leads to the actual velocities $\{u\}^T = \{v_x, v_y, \psi\}$:

$$\{u\} = [\tilde{J}]^{-1} \{u\}_{\text{nom}} \equiv [G] \{u\}_{\text{nom}}, \quad \text{with } [G] \text{ invariant.} \quad (2)$$

The robot’s pose is defined through the position of one of its points and its orientation $\psi$. If the Cartesian coordinates $(x, y)$ of point $O$ are used,

$$\begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\psi}(t) \end{pmatrix} = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} [G] \{u\}_{\text{nom}}. \quad (3)$$

This equation cannot be integrated analytically in principle. However, if the calibration trajectories are chosen properly, such integration can be easily done. This is the case for calibration trajectories with nominal velocities $\{u\}_{\text{nom}}(t)$ that maintain a constant relationship between themselves:

$$\{u\}_{\text{nom}}(t) = \lambda(t) \{u\}_{\text{nom},0}, \quad \text{with } \{u\}_{\text{nom},0} \text{ constant}, \quad (4)$$

where $\lambda(t)$ is a time evolution that may include starting and stopping transients; see Fig. 2. The corresponding actual velocities are

$$\{u(t)\} = \lambda(t) [G] \{u\}_{\text{nom},0}. \quad (5)$$

Time integration of the equation for $\dot{\psi}(t)$,

$$\dot{\psi}(t) = \lambda(t) \begin{bmatrix} g_{31} & g_{32} & g_{33} \end{bmatrix} \{u\}_{\text{nom},0} \rightarrow$$

$$\frac{d\psi}{dr} \lambda^{-1}(t) = \begin{bmatrix} g_{31} & g_{32} & g_{33} \end{bmatrix} \{u\}_{\text{nom},0}, \quad (6)$$
where \([g_{31} \ g_{32} \ g_{33}]\) is the third row of \([G]\), can be done assuming \(\psi(t = 0) = 0\) without lack of generality:

\[
\psi_e \equiv \int_0^t \dot{\psi}(t) \, dt = \eta(t_e) \begin{bmatrix} g_{31} & g_{32} & g_{33} \end{bmatrix} [u_{\text{nom},0}]
\]

\[
= \eta(t_e) \lambda^{-1}(t) \frac{d\psi}{dt},
\]

where \(\eta(t_e) \equiv \int_0^{t_e} \lambda(t) \, dt\).

In order to perform the time integration of the two first rows in Eq. (5), it can be taken into account from Eq. (6) and Eq. (7) that \(\psi_e \, dt = (\eta(t_e) \lambda^{-1}(t) \frac{d\psi}{dt}) \, dt\), and accordingly,

\[
\psi_e \frac{\dot{x}(t)}{\dot{y}(t)} \, dt = \eta(t_e) \begin{bmatrix} \cos \psi_e \, d\psi - \sin \psi_e \, d\psi \\ \sin \psi_e \, d\psi \end{bmatrix}
\]

\[
\times \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \end{bmatrix} [u_{\text{nom},0}].
\]

Integration of Eq. (8), assuming \(x(t = 0) = y(t = 0) = 0\), leads to

\[
\psi_e \begin{bmatrix} x_e \\ y_e \end{bmatrix} = \eta(t_e) \begin{bmatrix} \sin \psi_e - (1 - \cos \psi_e) \\ 1 - \cos \psi_e \sin \psi_e \end{bmatrix}
\]

\[
\times \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \end{bmatrix} [u_{\text{nom},0}].
\]

Pre-multiplying Eq. (9) by the inverse matrix

\[
\begin{bmatrix} \sin \psi_e - (1 - \cos \psi_e) \\ 1 - \cos \psi_e \sin \psi_e \end{bmatrix}^{-1} = \frac{1}{2(1 - \cos \psi_e)} \begin{bmatrix} \sin \psi_e & -1 - \cos \psi_e \\ 1 - \cos \psi_e & \sin \psi_e \end{bmatrix},
\]

one obtains

\[
\begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} = \frac{\psi_e}{2} \begin{bmatrix} \sin \psi_e - x_e + y_e \\ -x_e + \cos \psi_e \end{bmatrix}
\]

\[
\frac{\sigma_1}{\sigma_2} = \eta(t_e) [G][u_{\text{nom},0}] = [G] \begin{bmatrix} \sigma_1^{\text{nom}} \\ \sigma_2^{\text{nom}} \\ \psi_1^{\text{nom}} \end{bmatrix},
\]

where \((\sigma_1^{\text{nom}}, \sigma_2^{\text{nom}}, \psi_1^{\text{nom}})\) are the time integrals of the nominal generalized velocities (though only \(\psi_1^{\text{nom}}\) has the meaning of a coordinate increment).

The \([G]\) matrix is readily obtained from three different nominal motions and the measurement of the corresponding initial and final poses,

\[
[M] = \begin{bmatrix} \sigma_1 & \sigma_2 & \sigma_3 \\ \sigma_1 & \sigma_2 & \sigma_3 \\ \psi_1 & \psi_2 & \psi_3 \end{bmatrix} = [G] \begin{bmatrix} \sigma_1^{\text{nom},1} & \sigma_2^{\text{nom},1} & \psi_1^{\text{nom},1} \\ \sigma_1^{\text{nom},2} & \sigma_2^{\text{nom},2} & \psi_1^{\text{nom},2} \\ \sigma_1^{\text{nom},3} & \sigma_2^{\text{nom},3} & \psi_1^{\text{nom},3} \end{bmatrix} = [G][N],
\]

and the actual Jacobian matrix \([J]\) from it,

\[
[G] = [M][N]^{-1} \Rightarrow [J] = [J] [G]^{-1} = [J] [N][M]^{-1}.
\]

The matrix \([M]\) is obtained from the differences \(x_e, y_e, \psi_e\) between the initial and the final configuration measurements for each trajectory. Of course, the control system used to correct the errors along the path must not be activated while doing the calibration.

The values of \(\sigma_1\) and \(\psi_1\) and their interpretation are readily obtained by time integration of Eq. (5) and comparison to Eq. (11),

\[
\int_0^{t_e} [u(t)] \, dt = \eta(t_e) [G][u_{\text{nom},0}] \Rightarrow \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} = \int_0^{t_e} [v_{\text{ill}}] \, dt.
\]

Thus \(\sigma_1\) and \(\sigma_2\) are the time integrals of the actual longitudinal and transversal velocities \(v_{\text{ill}}\) and \(v_{\text{ir}}\), respectively.

### 3. Geometrical interpretation

Both the nominal and the actual trajectories are circular because in both cases the velocity \(V(O)\) of point \(O\) maintains a constant orientation relative to the mobile robot frame, and the relationship between its module \(v\) and \(\psi\) is also constant. This ratio defines a constant radius of curvature \(R = v / \psi\). Condition \(\psi_e \neq \pm n\pi\) rad requires that the calibration trajectories are not closed circumferences.

The circular geometry of the actual trajectory allows an easy determination of the time integration values \(\sigma_1\) and \(\sigma_2\) of the actual velocities \(v_{\text{ill}}\) and \(v_{\text{ir}}\) from geometrical considerations. From Fig. 3,

\[
\begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} = \int_0^{t_e} \begin{bmatrix} v \sin \gamma \\ v \cos \gamma \end{bmatrix} \, dt = \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} = \int_0^{t_e} [v_{\text{ill}}] \, dt.
\]

where \(\gamma\) is the constant angle between the actual velocity of point \(O\) and the mobile robot’s longitudinal axis, and \(\sigma\) is the actual path \(\psi_e, r\) followed along the actual circular arc of angle \(\psi_e\) and radius \(R\),

\[
R = \frac{1}{2} \sqrt{v_{\text{ill}}^2 + v_{\text{ir}}^2} \left(\sin(\psi_e/2)\right)^{-1}.
\]

The angle \(\gamma\) can be expressed from measurements as

\[
\gamma = \arctan \frac{y_e - \psi_e}{x_e}.
\]
4. Application case. The “Spherik” mobile robot

4.1. Experimental equipment

The calibration has been applied to a 3-DOF mobile robot with omnidirectional wheels [17,18] based on freely rotating spherical rollers (“Spherik” mobile robot; see Fig. 4).

The robot is equipped with a scanning laser sensor with center $P$ (Fig. 5) that detects different catadioptric landmarks placed on the workspace. The sensor emits a rotating laser beam that sweeps the environment and reflects back whenever a landmark is encountered. The angle of the reflected beam relative to the robot’s longitudinal axis is then measured by an incremental encoder. From the position of three landmarks and their corresponding angles it is possible to determine the robot’s pose through triangulation [19–25]. In the experiments, the sensor center $P$ coincides with point 0 used to define the robot’s generalized velocities.

The laser sensor used is the model LS6 from Guidance Control Systems. The scanner rotates at 8 Hz and delivers an angular accuracy in the measurements of 0.1 mrad leading to position and orientation errors below 1 mm and 1 mrad, respectively [19,26].

For the “Spherik” robot, the theoretical Jacobian, which is the Jacobian estimate $\tilde{J}$, is

$$
\tilde{J} = \frac{1}{r} \begin{bmatrix}
0 & -1 & -L \\
\cos \alpha & \sin \alpha & -d \cos \beta \\
-\cos \alpha & \sin \alpha & -d \cos \beta
\end{bmatrix},
$$

where $r$, $L$, $d$, $\alpha$ and $\beta$ are the robot’s geometrical parameters defined in Fig. 5 and given in Table 1.

4.2. Calibration trajectories. Correction matrix $[G]$

Four different nominal trajectories (a, b, c, d) have been used (one more than the number strictly required). The first two are straight with nominal robot displacements of 2 m, the third trajectory is a rotation of $\pi$ rad about the sensor center $P = 0$, and the fourth is a circular arc of $\pi/2$ rad and radius 1 m (with robot rotation of $\pi/2$ rad also); see Fig. 6(a).

The four trajectories were followed consecutively; the robot was at rest for 5 s at the end of each trajectory in order to measure its configuration accurately. As shown in Fig. 6(b), the reference frame is redefined at the beginning of each trajectory in such a way that the x-axis is aligned with the robot’s longitudinal axis and the y-axis is at 90° counterclockwise.

Two sets of three trajectories have been used to determine two $[G]$ matrices: trajectories a–b–c ($[G_{abc}]$) and trajectories a–b–d ($[G_{abd}]$). A third $[G_{abcd}]$ matrix has been obtained using all the trajectories, thus leading to an overdetermined linear system. Similarly to Eq. (12),

$$
[M'] = [G_{abcd}][N'] \rightarrow [N']^{T}[G_{abcd}]^{T} = [M']^{T},
$$

where $[M']$ and $[N']$ are 3 × 4 dimensional matrices, and the elements of $[G_{abcd}]$ are the unknowns of the linear system. Matrices $[M']$ and $[N']$ have an extra column corresponding to the fourth trajectory. The optimal solution of the overdetermined linear system in Eq. (18) can be obtained by minimizing the weighted squared error by means of the weighted pseudoinverse matrix [27]:

$$
[G_{abcd}]^{T} = \left([N'][W][N']^{T})^{-1}[N'][W][M']\right)^{T},
$$

where $[W]$ is the 4 × 4 dimensional weighting matrix used to calculate the squared error [27]. In this problem, the physical units used – meters and radians – allowed us to give the same weight to all elements, and thus, use the unit weighting matrix $[W] = [I]$ as in the Moore–Penrose pseudoinverse [28].

4.3. Trajectories using the calibrated Jacobians

The same four nominal trajectories used for calibration are followed anew by the robot with the corresponding calibrated Jacobians $\tilde{J}_{abc}$, $\tilde{J}_{abd}$ and $\tilde{J}_{abcd}$, obtained from the approximate Jacobian $\tilde{J}$ and the corresponding $[G]$ matrix using Eq. (13). Table 2 shows the associated position and orientation errors ($e_{x} = \|x_{nom} - x \|$ and $e_{y} = \|y_{nom} - y \|$ respectively), the mean error values and the relative error reduction (in percentage terms) relative to the initial mean error values before calibration.

The position and orientation errors are reduced significantly in all cases. The Jacobian obtained by applying the Moore–Penrose pseudoinverse is the one yielding the best results. This is not surprising as all the information about the four trajectories is taken into account in the calibration process.

The remaining error is due to the non-systematic odometry error, and to the residual error in the corrected Jacobian. The latter may come from a lack of a strict constant relationship between the three generalized velocities due to the control dynamics, ground irregularities, or wheel sliding.
5. Jacobian calibration for mobile robots with differential kinematics

A mobile robot with differential drive has two DOFs that can be described by means of the generalized velocities \( \{u\}^T = \{v, \dot{\psi}\} \). Fig. 7. The driving wheel velocities \( \{\dot{\theta}\}^T = \{\dot{\theta}_1, \dot{\theta}_2\} \) are related to them by means of an invariant Jacobian matrix \( \{J\} \), according to Eq. (1). Again, due to geometrical errors, \( \{J\} \) is not exactly known, and the theoretical Jacobian \( \{\tilde{J}\} \),

\[
\frac{1}{r} \left[ \begin{array}{c} L \\ -L \end{array} \right], \tag{20}
\]

is only an approximation of the actual Jacobian \( \{J\} \). As has been shown in Section 2, if the nominal velocities \( \{u_{\text{nom}}(t)\}^T = \{v_{\text{nom}}(t), \dot{\psi}_{\text{nom}}(t)\} \) maintain a constant relationship between themselves, as defined in Eq. (4), the actual velocities are given by Eq. (5) and both the nominal and the actual trajectories are circular. For the differential drive, the time integration of Eq. (5) leads to

\[
\begin{bmatrix}
\int_0^{t_e} v(t) \, dt \\
\int_0^{t_e} \psi(t) \, dt
\end{bmatrix} = \eta \left( \{u_{\text{nom}}(t)\} \right) \{u_{\text{nom,0}}\} \equiv \{G\} \begin{bmatrix}
\sigma_{\text{nom,0}} \\
\psi_{\text{nom,0}}
\end{bmatrix}, \tag{21}
\]

where \( \sigma_{\text{nom}} \) and \( \psi_{\text{nom}} \) are the time integrals of the nominal velocities; \( \psi_e \) is directly obtained from the final pose measurement and

\[
\dot{\psi}_e = \frac{x_e}{\sin \psi_e} - \frac{y_e}{1 - \cos \psi_e}, \tag{22}
\]

As in the case of 3-DOF mobile robots, considered in Section 2, the end angle \( \psi_e \) must be \( \psi_e \neq \pm n2\pi \) rad. From two independent trajectories,

\[
[M] = \begin{bmatrix}
\sigma_1 & \sigma_2 \\
\psi_{\text{nom,1}} & \psi_{\text{nom,2}}
\end{bmatrix} = \{G\} \begin{bmatrix}
\sigma_{\text{nom,1}} & \sigma_{\text{nom,2}} \\
\psi_{\text{nom,1}} & \psi_{\text{nom,2}}
\end{bmatrix} \equiv \{G\}[N]. \tag{23}
\]
From Eqs. (23) and (5) the actual or calibrated Jacobian $|J|$ is obtained as in Eq. (13).

6. Conclusions

The presented method allows the correction of systematic odometry errors for any kind of mobile robot having an invariant Jacobian matrix. The actual Jacobian matrix is determined exclusively from discrete pose measurements, instead of velocity ones. The calibration motions correspond to nominal generalized velocities maintaining a mutual constant relationship and, in the absence of non-systematic errors, the resulting calibration trajectories are circular, with radius varying from zero to infinity.

The method has been implemented and tested on a 3-DOF mobile robot with omnidirectional wheels. With just four calibration trajectories, the odometry errors have shown a reduction of between 38% and 83%.

References


Joaquim A. Batlle was born in Barcelona in 1943. He was awarded his M.S. in Mechanical Engineering in 1968 and his Ph.D. in 1975. Upon graduation he joined the Universitat Politècnica de Catalunya, where he is Professor in Mechanical Engineering and head of the research line on Mechanics and Acoustics. He is a member of the Royal Academy of Sciences and Arts of Barcelona (1991), and also a member of the American Society of Mechanical Engineering (1996) and the Acoustical Society of America (1990). His research work is mainly related to the mechanics of mobile robots, to percussive dynamics and to the acoustics of waveguides and windwinds.

Josep M. Font-Llagunes received his M.E. degrees in Mechanical Engineering (2002) and Biomedical Engineering (2004), both from the Universitat Politècnica de Catalunya (UPC). In 2007, he received his Doctor of Engineering degree from the University of Girona. Since 2004 he has been an assistant professor of Mechanics at UPC, where he has been involved in numerous research projects of the Reference Center for Advanced Production Techniques (CeRTAP) of the Autonomous Government of Catalonia. He held a Mobility Scholarship from UPC to pursue post-doctoral research at the Centre for Intelligent Machines (McGill University, Canada) during the year 2007–2008. His research interests cover mobile robotics, dynamics of multibody systems and biomechanics.

Ana Barjau obtained her diploma in Theoretical Physics in 1980, and her Ph.D. in 1987. Since then, she has been a professor in the Department of Mechanical Engineering of the Universitat Politècnica de Catalunya (UPC). Since 2004, she is the academic coordinator at UPC of a European master of Mechanical Engineering. Her research interests include theoretical mechanics, mechanics of robots and acoustics of axissymmetrical waveguides, with special application to musical instruments.