Radial basis function neural networks applied to efficient QRST cancellation in atrial fibrillation

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ABSTRACT

The most extended noninvasive technique for medical diagnosis and analysis of atrial fibrillation (AF) relies on the surface electrocardiogram (ECG). In order to take optimal profit of the ECG in the study of AF, it is mandatory to separate the atrial activity (AA) from other cardioelectric signals. Traditionally, template matching and subtraction (TMS) has been the most widely used technique for single-lead ECGs, whereas multi-lead ECGs have been addressed through statistical signal processing techniques, like independent component analysis. In this contribution, a new QRST cancellation method based on a radial basis function (RBF) neural network is proposed. The system is able to provide efficient QRST cancellation and can be applied both to single and multi-lead ECG recordings. The learning algorithm used for training the RBF makes use of a special class of network, known as cosine RBF, by updating selected adjustable parameters to minimize the class-conditional variances at the outputs of the network. The experiments verify that RBFs trained by the proposed learning algorithm are capable of reducing the QRST complex dramatically, a property that is not shared by other methods and conventional feed-forward neural networks. Average Results (mean ± std) for the RBF method in cross-correlation (CC) between original and estimated AA are CC = 0.95 ± 0.038 being the mean square error (MSE) for the same signals, MSE = 0.311 ± 0.078. Regarding spectral parameters, the dominant amplitude (DA) and the mean power spectral (MP) were DA = 1.15 ± 0.18 and MP = 0.31 ± 0.07, respectively. In contrast, traditional TMS-based methods yielded, for the best case, CC = 0.864 ± 0.041, MSE = 0.577 ± 0.097, DA = 0.84 ± 0.25 and MP = 0.24 ± 0.07. The results prove that the RBF based method is able to obtain a remarkable reduction of ventricular activity and a very accurate preservation of the AA, thus providing high quality dissociation between atrial and ventricular activities in AF recordings.

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1. Introduction

Atrial fibrillation is a common arrhythmia with a prevalence of approximately 0.4–1.0% in the general population [1], which increases with age, thus being present in 5% of those older than 65, and 10% of those older than 70 [2]. AF is associated with an increased risk of stroke and mortality, as well as impaired exercise tolerance, fatigue, and heart failure [3]. The diagnosis of AF, as such, has been based mainly on visual inspection of the surface electrocardiogram (ECG) [4]. However, due to the atrial activity (AA) reduced amplitude, in contrast to the much higher amplitude of the ventricular activity (VA), that is, the QRS and the T wave, the cancellation of VA is mandatory to study AF from surface recordings [5]. This step would facilitate the extraction of AA under the best conditions, which is crucial to study the electrophysiological processes underlying AF, such as refractory periods, autonomic response, and drug effects [4]. The extraction of the AA during AF requires nonlinear signal processing, since spectra of atrial and ventricular activities overlap and, in consequence, they cannot be separated by simple linear filtering [6]. Different approaches are generally used to perform this task: source separation algorithms and template matching and subtraction (TMS) or its variants. Source separation algorithms try to find uncorrelated components within a multi-lead ECG using principal component analysis (PCA) [7,8], or to find independent components in a linear mixture using independent component analysis (ICA) [6]. The algorithms based on PCA or ICA exploit the property that the atrial and ventricular activities are originated from different and uncoupled bioelectric sources. PCA has been employed to monitor the effects of drugs [7] and assess the effects of linear left atrial ablation [9]. Instantaneous and convolutive ICA has been applied in order to obtain ECG signals devoid of VA...
involvement [6,10,11]. On the other hand, there are systems based on neural networks that have also been used with limited success [12]. Finally, a method based on least mean squares (LMS) has been recently introduced to enhance fibrillatory waves [13], however, this method was presented to deal with AF invasive recordings.

The methods based on standard or improved TMS assume that, for the same patient, ventricular complexes generally exhibit a limited number of forms. An average template of these distinct complexes is then used to subtract the VA [5]. This method relies on the assumption that the average beat can represent approximately each individual beat. However, QRST morphology is often subject to minor changes caused by respiration, patient movement, etc. and, therefore, QRST residua and noise are often present in the estimated AA or remainder ECG [14]. To solve this problem, different improvements have been presented, like the spatiotemporal QRST cancellation [15], where the average beats of adjacent leads are mathematically combined with the average beat of the analyzed lead. In this way, the method is able to suppress the electrical axis alterations and yield improved cancellation. A modification has been introduced by Lemay et al. [16] in which the QRS complex and the T wave are separately processed. However, the improvement introduced by these techniques is notably reduced when only single-lead ECG recordings are available. In this respect, to overcome the aforementioned shortcomings, an adaptive QRST cancellation method based on adaptive singular value cancellation has been applied to each single beat in [17].

In this paper, a new QRST cancellation method based on a radial basis function (RBF) neural network, is proposed. The system can be applied both to single and multi-lead ECG recordings. The proposed RBF network has been developed like a hierarchically layered structure. It starts with a small number of RBFs and then adds new RBFs if the approximation error is larger than some predetermined threshold and there is no existing RBF that can efficiently represent the current input. The adaptation strategy for the weight matrix of the RBF network is developed using the Lyapunov approach. Different types of RBFs can be employed by the proposed self-organizing RBF network. The implementations using Gaussian and Cosine RBF are compared with the widely applied TMS technique for QRST cancellation. Through the proposed system, ECG recordings with one and 12 leads have been used. However, once the network is trained, it can be used for any number of leads.

The remainder of this paper is organized as follows. The materials are introduced in the second section. The RBF network is described in the third place. In the fourth section, the comparison between the different methods discussed in the Introduction section and RBF is presented. Finally, discussion and conclusions are found in the ending sections.

2. Materials

In the present study both real and synthesized recordings were used. Real recordings were taken from the AF database available in PhysioBank [18] and simulated signals were created from real AA and VA as will be later described. Simulated signals were needed in order to assess a robust comparison between estimated and original AA. For real signals, 50 recordings from different databases (MIT-BIH Atrial Fibrillation Database, Long-Term AF Database, MIT-BIH Arrhythmia Database, AF Termination Challenge Database, ...) were selected from PhysioBank with different types of QRST morphologies. If the signals were noisy themselves, then the noise was previously reduced [19].

The set of synthesized signals was composed of 50 recordings with 12 leads and 50 recordings with two leads, that were created following the methodology described in [17] by combining real AA and VA. The parts used to synthesize the signals were taken from recordings different from the set composing the real signals database. The AA was obtained from the smooth concatenation of successive TQ segments extracted from real AF recordings, as described in [17,20]. The VA was extracted from normal sinus rhythm ECG recordings after P-wave cancellation as described in [8]. In addition, with the aim of creating simulated AF signals as similar to real AF recordings as possible, some variations in the signal were introduced. The QRS complex amplitude and width and the RR intervals were randomly varied following the specific characteristics of real AF recordings. More concretely, the QRST complex amplitude was randomly reduced or enlarged between 0% and 20% of its original size. On the other hand, the QRS width was randomly varied between 340 and 420 ms making use of different upsampling and downsampling factors. Finally, the TQ intervals were randomly reduced or enlarged in order to obtain a 40% of variability in the RR intervals with respect to their mean value. To reduce a TQ interval, consecutive samples were removed. In contrast, in order to expand the TQ interval, a linear combination of the three preceding TQ intervals was introduced in the middle of the considered interval [17]. As aforesaid, signals with two and 12 leads were synthesized, in such a way that the method is able to work in the two types of scenarios.

All the signals were classified in three groups. The first group comprised 40% of the signals randomly selected, which were employed for network training. The second group (25%) helped to validate the proper RBF-based system operation. Finally, the third group (35%) was used to compare the ANN proposed system with other previously proposed methods deeply established in the scientific community.

3. Methods

3.1. Artificial Neural Network

Artificial Neural Networks (ANN) are a type of non-linear processing systems ideally suited to a wide range of tasks, especially those in which there is no existing algorithm for task completion. An ANN consists of a collection of highly interconnected processing elements that transform a set of inputs into a set of desired outputs [21]. The result of the transformation is determined by the characteristics of the elements and the weights associated with the interconnections among them. By modifying the connections between the nodes, the network is able to adapt to the desired outputs. The goal is to choose the weights of the network that achieve a desired input/output relationship, known as training the network [22].

3.2. Proposed system

RBF neural networks are function approximation models that can be trained by means of examples to implement a desired input–output mapping [21,23]. In fact, RBF models are closely related to function approximation models used to perform interpolation [24]. Under certain mild conditions, RBF neural networks are capable of approximating arbitrarily well any function [25]. The performance of a RBF depends on the number and centers of the radial basis functions, their shapes, and the method used for learning the input–output mapping. The centers of the RBF are often determined by the k-means clustering algorithm [26].

The proposed RBF network in the present work has a hierarchically layered structure. It starts with a small number of RBFs and then adds new RBFs if the approximation error is larger than some predetermined threshold and there is no existing RBF that
can efficiently represent the current input. Moreover, some of the existing RBFs can be removed if the approximation error is small and other conditions still are satisfied [27]. The adaptation strategy for the weight matrix of the RBF network is developed using the Lyapunov approach. The proposed approximation strategy guarantees uniform ultimate boundedness of the approximation error, which is proved using the second Lyapunov method. Furthermore, is also capable of achieving uniform asymptotic stability of the approximation error if the RBF network can capture the dynamics of uncertainties perfectly. Different types of RBFs have been employed to develop the proposed QRST complex cancellation. The implementations using Gaussian RBF (GRBF) and raised-cosine RBF (RCRBF) are discussed and compared. Although the GRBF network possesses the property of universal approximation, the network’s training and output evaluation are still time consuming. This is because the GRBF has the unbounded support and thus each RBF has non-zero output over the whole input space. On the other hand, the RCRBF is proposed because of its compact support [28].

The property of compact support enables much faster network training and output evaluation as the complexity of the network and the dimensionality of the input space increase [29].

The proposed RBF network structure is shown in Fig. 1, which consists of one input layer, one output layer, and one hidden layer. For the given input \( x = [x_1, \ldots, x_n]^T \), the overall response at the \( k \)th output neuron is:

\[
y_k = \sum_{j=1}^M w_{kj} \phi \left( \frac{|x_i - c_{kj}|}{\sigma_{kj}} \right)
\]

where \( w_{kj} \) is the weight from the \( k \)th hidden neuron to the \( j \)th output neuron. In the following, the notation \( \tilde{y}_k(x) = \tilde{y}_k(x; c_{kj}, \sigma_{kj}) \) will be used, which refers to the RBF located at the \( k \)th hidden neuron. The vector \( c_{kj} = [c_{kj}, \ldots, c_{kj}] \) is the center of \( \tilde{y}_k(x) \) and the parameter \( \sigma_{kj} \), \( i = 1, \ldots, n \), is the radius or the width of \( \tilde{y}_k(x) \) in the \( i \)th coordinate. Finally, \( \phi : [0, \infty) \rightarrow \mathbb{R}^+ \) is the activation function, which characterizes the shape of the RBF, where \( \mathbb{R}^+ \) is the set of non-negative real numbers. Usually, the activation function \( \phi \) is constructed so that it is radially symmetric. The largest value of \( \phi \) is obtained when \( x_i = c_{kj} \), whereas the value of \( \phi \) vanishes or becomes very small when \( |x_i - c_{kj}| \) becomes large. Let \( w_k = [w_{k1}, \ldots, w_{kn}]^T \) be the weight vector for the \( k \)th output neuron. Then the expression for the response of the \( k \)th output neuron can be rewritten as:

\[
y_k = W_k \tilde{y}_k(x) \quad \text{and} \quad W_k = [w_{k1}, \ldots, w_{kn}]^T
\]

Regarding the two aforesaid different implementations of RBFs, the GRBF is characterized by the following activation function:

\[
\tilde{y}_k(x) = \exp \left( -\frac{(x-c)^2}{2 \sigma^2} \right)
\]

whereas the RCRBF is a compact support RBF where the one dimension (1-D) RCRBF is defined as

\[
\tilde{y}_{1\sigma}(x) = \begin{cases} 
\frac{1}{2} \left( 1 + \cos \left( \frac{\pi (x - c)}{\sigma} \right) \right) & \text{if } |x-c| \leq \sigma \\
0 & \text{if } |x-c| > \sigma
\end{cases}
\]

The support of this function is the compact set \([c-\sigma, c+\sigma]\), which has length twice of its radius \( \sigma \). In the \( n \)-dimensional space, the RCRBF centered at point \( c = [c_1, \ldots, c_n] \) with the radius \( \sigma = [\sigma_1, \ldots, \sigma_n] \) has been also represented as the product of 1-D RCRBFs. Finally, the nonlinear continuous-time dynamical systems considered in this paper are modeled by the following equation:

\[
\tilde{y}_i = f \left( w_{i0} + \sum_{j=1}^M w_{ij} g_j(|x - y_j|^2) \right) \quad 1 < i < p
\]

where \( f(x) \) is a non-decreasing, continuous and differentiable function. The our model in Eq. (4) describes a radial basis function neural network with inputs from \( \mathbb{R}^n \), \( c \) radial basis functions, and \( p \) output units if \( g_j(x^2) = \phi_j(x) \) and \( \phi(x) \) are radial basis functions. In such a case, the response of the radial basis function neural network to the input vector \( x_k \) is

\[
\tilde{y}_{ik} = \sum_{j=1}^c w_{kj} h_{jk} \quad 1 < i < p
\]

where \( h_{0k} = 1, v_k \), and \( h_{jk} \) represents the response of the radial basis function located at the \( j \)th prototype \( y_j \) to the input vector \( x_k \), that is, \( h_{jk} = g_j(|x_k - y_j|^2) \), \( 1 < j < c \). So far, the description of cancellation QRST system has been defined. In the next section, the RBF training will be explained and demonstrated.

### 3.2.1. Training of the radial basis function neural network

The proposed RBF neural network has been trained to map \( x_k \in \mathbb{R}^n \) into \( y_k = [y_{k1}, y_{k2}, \ldots, y_{km}]^T \in \mathbb{R}^m \), where the vector pairs \( (x_k, y_k) \) represent the training set with \( k \) from 1 and \( M \). If \( x_k \in \mathbb{R}^n \) is the input to a radial basis function neural network, its response is

\[
y_k = \sum_{j=1}^M w_{kj} \phi_j(|x_k - y_j|^2), \quad \tilde{y}_{ik} = f(w_{ik} h_{ik}) \quad 1 < i < p
\]

is the actual response of the \( i \)th output unit \( x_k \), with

\[
h_k = [h_{0k}, h_{1k}, \ldots, h_{ck}]^T, \quad h_{0k} = 1, 1 \leq k \leq M
\]

where \( g_j(x) \) is the linear generator function and \( w_i = [w_{i0}, w_{i1}, \ldots, w_{ic}]^T \) the weights vector [31].

Training of a RBF neural network by gradient descent requires that the active regions of the available RBFs cover completely the input space. The definition of the active region indicates that the likelihood of complete coverage of the input space by active regions of radial basis functions improves considerably as the values of \( q_j \) increase. Thus, the training process can be facilitated by updating the reference distances \( q_j \) based on the requirements of the desired input–output mapping [21,22]. It is expected that updating \( q_j \) during the learning process would allow the implementation of a desired input–output mapping by cosine RBF neural networks with a relatively small number of RBFs. This is
due to the fact that the reduction of the number of RBFs can be compensated during learning by increasing the values of \( a_j \), which is expected to expand the active regions of the corresponding RBFs. Reducing the number of RBFs is expected to improve the generalization ability of the corresponding cosine RBF neural network [26,32,33]. Two types of training algorithms have been chosen, as shown below.

**Batch Learning Algorithms** The proposed RBF neural network has been trained by batch learning algorithms [31], which have been developed using gradient descent to minimize the error \( E \) following the sequel expression

\[
E = \frac{1}{2} \sum_{k=1}^{M} \sum_{i=1}^{p} (y_{ik} - \tilde{y}_{ik})^2 \tag{8}
\]

This network has been trained by a gradient descent algorithm in a sequence of adaptation cycles [34], where an adaptation cycle involves the update of all adjustable parameters of the network. An adaptation cycle begins by increasing each weight vector \( w_i \), \( 1 \leq i \leq p \), by the amount \( \Delta w_i = -\alpha \nabla w_i E \) as [32,33]

\[
w_i + \Delta w_i = w_i + \alpha \sum_{k=1}^{M} a_{ik} \tilde{h}_k
\tag{9}
\]

where \( \alpha \) is the learning rate and \( a_{ik} \) is the output unit error, given as

\[
a_{ik}^{0} = f(\tilde{y}_{ik}/y_{ik})\tag{10}
\]

Following the update of these weight vectors, each prototype \( v_j, 1 \leq j \leq c \), is increased by an amount \( \Delta v_j = -\alpha \nabla v_j E \) as [31]

\[
v_j + \Delta v_j = v_j + \alpha \sum_{k=1}^{M} a_{ik}^{0}(x_k - v_j) \tag{11}
\]

where \( \alpha \) is the learning rate and \( a_{ik}^{0} \) is the hidden unit error, defined as

\[
a_{ik}^{h} = \frac{2}{m-1}(\tilde{h}_k)^{m-1}(x_k - v_j)^2 \sum_{i=1}^{p} a_{ik}^{0} w_{ij} \tag{12}
\]

If \( g_{i,0} = 1 + \delta_j x \), then \( g_{i,0}^{0}(x_k - v_j)^2 = \delta_j \), the hidden unit error corresponding to the RBF neural network has been obtained from (12) for \( m = 3 \) and \( \delta_j = 1/a_j^{0} \) as

\[
a_{ik}^{h} = \left( \frac{h_k}{h_j} \right)^{\frac{p}{m-1}} \sum_{i=1}^{p} a_{ik}^{0} w_{ij} \tag{13}
\]

Training of RBF neural networks also involves updates of the reference distances \( a_j \), \( 1 \leq j \leq c \), which has been increased by an amount \( \Delta a_j = -\eta \varepsilon_j/a_j \) as

\[
a_j + \Delta a_j = a_j + \left( \frac{\eta}{a_j} \right) \sum_{h,k} h_{jk}(1-h_{jk}) a_{ik}^{0} \tag{14}
\]

where \( \eta \) is the learning rate and \( a_{ik}^{0} \) is defined in (13).

**Sequential Learning Algorithms** The proposed RBF neural network has been also trained “on-line” by sequential learning algorithms, which have been developed using gradient descent to minimize \( E \) with the expression

\[
E_k = \frac{1}{2} \sum_{i=1}^{p} (y_{ik} - \tilde{y}_{ik})^2 \tag{15}
\]

for \( k = 1, 2, \ldots, M \). After an example \( (x_k, y_k), 1 \leq k \leq M \), is presented to the RBF neural network, the new estimate \( w_i \), \( 1 \leq i \leq p \), is obtained by increasing its current estimate \( w_{ik} \) by an amount \( \Delta w_{ik} = -\alpha \nabla w_i E_k \) as

\[
w_i = w_{ik} + \Delta w_i = w_{ik} + \alpha \sum_{k=1}^{M} a_{ik}^{0} \tilde{h}_k
\tag{16}
\]

where \( \alpha \) is the learning rate and \( a_{ik}^{0} \) is the output unit error defined in (10). Following the update of all the weight vectors \( w_i \), \( 1 \leq i \leq p \), the new estimate \( v_{jk} \) of each prototype \( v_j, 1 \leq j \leq c \), has been obtained by increasing its current estimate \( v_{jk} \) by the amount \( \Delta v_{jk} = -\alpha \nabla v_i E_k \) as

\[
v_{jk} = v_{jk} + \Delta v_{jk} = v_{jk} + \alpha \sum_{k=1}^{M} a_{ik}^{0}(x_k - v_{jk}) \tag{17}
\]

where \( \alpha \) is the learning rate and \( a_{ik}^{0} \) is the hidden unit error defined in (13). The hidden unit error for RBF neural networks is given in (13). Finally, the new estimate \( a_{jk} \) of each reference distance \( a_j, 1 \leq j \leq c \), has been obtained by increasing its current estimate \( a_{jk} \) by the amount \( \Delta a_{jk} = -\eta \varepsilon_{jk}/a_{jk} \) as

\[
a_j = a_{jk} + \Delta a_{jk} \tag{18}
\]

thereafter, an adaptation cycle is completed in this case after the sequential presentation, to the RBF neural network, of all the examples included in the training set.

### 3.2.2. Experimental results

The performance of RCRBF neural networks was evaluated and compared with that of Feedforward Neural Networks (FFNN) with sigmoid hidden units and conventional RBF neural networks with GRBF. Conventional radial basis function neural networks were trained by a hybrid learning scheme similar to that proposed by Moody and Darken [35]. The centers of the RBF were determined according to an unsupervised procedure relying on the k-means algorithm. The widths of the GRBF were computed according to the nearest heuristic prototype [36]. The centers of the RBF were fixed during the supervised learning process. Radial basis function neural networks were trained by a fully supervised procedure based on gradient descent [32,33]. This procedure involved the update of the output weights and the centers \( v_i \) of the radial basis functions. The learning rate \( \eta \) used to update the reference distances \( a_j \) was one order of magnitude lower than the learning rate used to update the output weights and the prototypes.

The results of these experiments are summarized in Table 1, which shows the number of adaptation cycles required for training the neural networks mentioned above, and the percentage of errors produced on average by the trained neural networks on the training and testing sets. According to Table 1, RCRBF networks produced the smallest percentage of errors on both training and testing sets among all the RBF models tested in the experiments. The performance differences among RBF models became more significant as the number of RBFs decreased.

<table>
<thead>
<tr>
<th>Feedforward NNs</th>
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<tr>
<td>( n_0 )</td>
<td>20</td>
<td>25</td>
<td>30</td>
<td>35</td>
</tr>
<tr>
<td>( N )</td>
<td>204.5</td>
<td>217.9</td>
<td>221.6</td>
<td>224.8</td>
</tr>
<tr>
<td>( E_{ts}(n_0) )</td>
<td>12.34(1.28)</td>
<td>11.02(1.17)</td>
<td>10.24(1.35)</td>
<td>10.02(1.09)</td>
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<tr>
<td>( E_{ts}(n_0) )</td>
<td>13.23(1.32)</td>
<td>12.34(1.23)</td>
<td>11.56(1.26)</td>
<td>11.17(1.12)</td>
</tr>
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<th>Gaussian RBF NNs</th>
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<tr>
<td>( c )</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
</tr>
<tr>
<td>( N )</td>
<td>357.7</td>
<td>368.9</td>
<td>371.6</td>
<td>384.8</td>
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<tr>
<td>( E_{ts}(n_0) )</td>
<td>11.02(1.16)</td>
<td>10.67(1.21)</td>
<td>10.23(1.07)</td>
<td>9.65(1.02)</td>
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<tr>
<td>( E_{ts}(n_0) )</td>
<td>11.54(1.19)</td>
<td>11.02(1.24)</td>
<td>11.67(1.11)</td>
<td>10.12(1.09)</td>
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<td>204.5</td>
<td>217.9</td>
<td>221.6</td>
<td>224.8</td>
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<tr>
<td>( E_{ts}(n_0) )</td>
<td>2.89(0.72)</td>
<td>2.12(0.56)</td>
<td>2.03(0.51)</td>
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<td>( E_{ts}(n_0) )</td>
<td>2.97(0.81)</td>
<td>2.23(0.58)</td>
<td>2.10(0.52)</td>
<td>1.96(0.50)</td>
</tr>
</tbody>
</table>

The number in parentheses represent the standard deviation.

### Table 1

Average number \( N \) of adaptation cycles required for training various RBF with \( c \) radial basis functions and FFNNs with \( n \) hidden units to yield RQST cancellation and percentage of average error produced on the training set \( (E_{ts}) \) and the testing set \( (E_{ts}) \).
In applications where the goal is to create a system that generalizes well in unseen examples, the problem of over-training has emerged. This arises in over-complex or over-specified systems when the capacity of the network significantly exceeds the needed free parameters. In order to avoid this problem, cross-validation technique to check the presence of over-training and select optimal parameters in order to minimize the generalization error has been used [21]. Fig. 2 shows the cross-correlation study for RCRBF, which has been selected because of its better adaptation to the problem. In this figure an optimum training point is obtained.

### 3.3. Performance assessment

The proposed method was thoroughly tested and compared with the most widely used method for QRST cancellation, i.e., the TMS technique, using the quantitative measures of performance that will be next described. The TMS algorithm was implemented following the description in [5]. In summary, the method was based on the cancellation of each QRST complex through the subtraction of an average QRST template computed over the recording under analysis [37].

Regarding synthesized recordings, the AA estimation performance was computed by comparing the estimated and the original atrial activities in terms of the cross-correlation (CC) [8] and mean square error (MSE) [16]. The cross-correlation measures the similarities between two signals. CC becomes 1 in the case of perfect matching and 0 in the case of unrelated signals, therefore it can be defined as

\[
CC = \frac{C_{AA \hat{A}A}}{\sigma_{AA} \sigma_{\hat{A}A}}
\]

where \(C_{AA \hat{A}A}\) is the covariance between the original and the estimated AA, being \(\sigma_{AA}\) and \(\sigma_{\hat{A}A}\) their standard deviations, respectively.

On the other hand, the normalized mean square error is defined as

\[
\text{MSE} = \sqrt{\frac{\sum (AA - \hat{AA})^2}{\sum (AA)^2}}
\]

(21)

where AA and \(\hat{AA}\) represent the same as in Eq. (20).

To evaluate the method's performance on real signals, it has to be remarked that the original AA on the ECG was obviously unknown. As a consequence, the only evidence for a successful ventricular complex cancellation was the absence of QRST residua. Unfortunately, there are no available parameters able to robustly quantify the existence of ventricular residua in the extracted AA. Indeed, each previous work where this aspect has been studied presents different parameters that have been proposed by their authors. Therefore, performance on real signals was evaluated following a previous work in which the ECGs were divided into atrial and ventricular segments [38]. A ventricular segment was defined as a time interval of 150 ms with the R-peak (ventricular depolarization) at its center. This interval was wide enough to include all the QRS morphologies. On the other hand, the atrial segments were composed of the remaining parts between ventricular segments of the electrocardiogram [38].

Within the ventricular segments, performance was evaluated by estimating the ventricular depolarization reduction (VDR), i.e., the beat-by-beat reduction of the R-peak amplitude that the algorithm under evaluation is able to achieve. Therefore, VDR was a vector of values defined as

\[
\text{VDR(dB)} = 10 \log \left( \frac{R_{\text{REC}}}{R_{\text{R}}^{\text{PK}}} \right)
\]

(22)

where \(R_{\text{REC}}\) is the R-peak amplitude of the original ECG, and \(R_{\text{R}}^{\text{PK}}\) is the residual R-peak amplitude of the electrocardiogram after ventricular activity reduction. High positive values of VDR will indicate good performance of the algorithm. Values close to zero are associated with poor performance and negative values indicate reduction errors because the peak is larger than before.

Regarding the atrial segments, the performance was evaluated by measuring the waveform degree of similarity (S). This index was defined as the cross correlation between the original and the estimated AA for each atrial segment. Hence, its definition follows Eq. (20) but the CC is computed only on the atrial segments and not in the whole ECG as before. Thereafter, similarity was a vector of values defined as

\[
S = \frac{C_{AA \hat{A}A}}{\sigma_{AA} \sigma_{\hat{A}A}}
\]

(23)

where \(C_{AA \hat{A}A}\) is the covariance of the two atrial segments under evaluation (original and estimated), and \(\sigma_{AA}\) and \(\sigma_{\hat{A}A}\) are their standard deviations, respectively. The similarity will provide information about how QRST cancellation algorithms preserve the atrial waveform in those intervals where the AA should remain unchanged. Both for ventricular and atrial segments, performance indices were computed on a beat-by-beat basis, thus providing detailed information of the evaluated methodologies [38].

To further compare the performance between methods, a set of several parameters in the spectral domain have also been used, as suggested by previous works [39,40]. Firstly, the dominant frequency (DF) was computed as the largest frequency peak of the AA spectrum within the 3–12 Hz frequency range. The AA power spectrum was computed using the Welch Periodogram with a Hamming window of 4096 points in length, 50% overlapping between adjacent sections and 8192-points Fast Fourier Transform. Besides the dominant amplitude (DA), defined as the amplitude of the dominant peak, was also computed prior to

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**Fig. 2.** Cross correlation graph obtained for the RCRBF. The small dashed line shows the evolution of the MSE for the training phase, in this phase, the signals of group 1 have been used. The large dashed line shows the evolution of the MSE for the validation phase, in this phase, the signals of group 2 have been used. The circle represents optimum point.
normalization of the spectral magnitude. Note that a higher value of the DA indicates greater power in the dominant peak. Finally, the mean (MP) and standard deviation (SP) in the normalized power spectral magnitude, from 3 to 12 Hz, were also compared. A lower mean and standard deviation of the spectral profile would indicate a lower noise floor, and a lower amplitude of any other spectral peaks that may be present other than the dominant peak. Therefore, this will imply improved spectral quality. When the dominant peak is increased in amplitude relative to a noise floor already near zero, it actually increases the mean spectral profile. Therefore, the higher mean spectral profile, like higher dominant amplitude, would indicate more power in the dominant frequency peak, i.e., greater power in the preserved atrial activity component [40].

4. Results

In this section, several recordings are considered to compare the performance between the proposed RBF approach and TMS. Firstly the method has been tested with signals of 12 leads and, later, with single lead recordings.

4.1. Twelve lead recordings

Both methodologies were first applied to the simulated AF recordings. The cross-correlation and the normalized mean square error between original and estimated AA signals, together with the ventricular residue and the similarity between atrial segments of the ECG and the estimated AA, were used to compare the performance of both methods. Table 2 summarizes the average obtained values of these parameters for the set of 12 leads ECGs. Given that the original AA in a real ECG is unknown, the CC coefficient and MSE were useless in this case. As a consequence, only VDR of the ventricular segments and S of the atrial segments were computed. Note that remarkable differences between RBF and TMS are reported for all the studied parameters and the analyzed recordings. Regarding spectral analysis, Table 3 shows the mean values for TMS and RBF. As can be observed, RBF achieves higher DA and MP than TMS, whereas similar DF and lower SP values are obtained. Note that MP is higher for the proposed system because the dominant peak is practically the only spectral peak. These results prove that the performance of the RBF-based method is higher than TMS because its AA extraction is more accurate.

As a graphical summary, Fig. 3 shows the estimated AA signals corresponding to a real AF recording with 12 leads when TMS and RBF methods are applied. As can be appreciated, the estimated AA through RBF matches the original AA with more fidelity than the TMS method. This fact agrees with the CC index and the MSE mean values presented in Table 2. In addition, it can be observed that the AA extracted by TMS presents QRST residua of larger amplitude, which is coherent with the computed VDR average value. In addition, another relevant observation is the absence of sudden transitions into the AA segments provided by RBF. This result justifies the higher similarity values obtained with RBF. In contrast, the AA obtained with TMS presents notable sudden transitions. On the other hand, Fig. 4 shows the AA spectrum obtained by TMS and RBF from lead V1 of the 12 lead ECG. As can be observed, DF is similar in both methods. Nevertheless, DA is higher in the case of the RBF method because RBF achieves an accurate extraction of atrial activity.

4.2. Single lead recordings

For the case of single-lead ECGs, the obtained results are summarized in Table 4. In the same way as with 12 leads, the AA obtained with RBF presents lower ventricular residue and higher similarity between atrial segments than those obtained with TMS. Moreover, the ventricular residue presents slight variations and the similarity increases considerably when the recording length is increased. Significant differences between RBF and TMS were also obtained for all the studied parameters. An example applied to a real single-lead recording is shown in Fig. 5, which presents the ECG of a patient in AF and the corresponding AA signals provided by TMS and RBF. This figure proves how the proposed method achieves a better AA extraction and a better QRST complex reduction maintaining the atrial signal characteristics. On the other hand, Table 5 summarizes the spectral analysis of TMS and RBF. As can be seen, RBF achieves better values than TMS (higher DA and lower SP values). These results demonstrate again that RBF is more accurate than TMS in AA extraction. An example of power spectral density is shown in Fig. 6 where RBF achieves a DA greater than TMS and with an accurately DF detection.

In view of the results, remark that the RBF-based method considers dynamics in the QRST waveform and, thereafter, a more accurate cancellation template is obtained. As a consequence, it behaves more robustly in those ECGs with variable QRST morphologies. In contrast, the AA estimated by other methods will be, unavoidably, more contaminated by QRST residua.

5. Discussion

In this study, an effective technique for QRST complex cancellation is proposed and applied to ECG recordings. The technique incorporates a RBF network architecture that can add or remove RBFs dynamically depending on the current approximation error.

Table 2

<table>
<thead>
<tr>
<th>Method</th>
<th>CC</th>
<th>MSE</th>
<th>VDR</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated</td>
<td>TMS</td>
<td>0.864 ± 0.041</td>
<td>0.577 ± 0.097</td>
<td>3.326 ± 0.561</td>
</tr>
<tr>
<td>AF recordings</td>
<td>RBF</td>
<td>0.962 ± 0.038</td>
<td>0.302 ± 0.071</td>
<td>6.595 ± 0.321</td>
</tr>
<tr>
<td>Real</td>
<td>TMS</td>
<td>–</td>
<td>–</td>
<td>3.298 ± 0.425</td>
</tr>
<tr>
<td>AF recordings</td>
<td>RBF</td>
<td>–</td>
<td>–</td>
<td>6.357 ± 0.334</td>
</tr>
</tbody>
</table>

Table 3

Average spectral results provided by the comparison between TMS and RBF obtained for synthesized and real AF recordings with 12 leads. Values indicate mean ± standard deviation.

<table>
<thead>
<tr>
<th>Method</th>
<th>DF</th>
<th>DA</th>
<th>MP</th>
<th>SP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated</td>
<td>TMS</td>
<td>86.2 ± 0.93</td>
<td>0.84 ± 0.25</td>
<td>0.24 ± 0.07</td>
</tr>
<tr>
<td>AF recordings</td>
<td>RBF</td>
<td>58.3 ± 0.81</td>
<td>1.18 ± 0.18</td>
<td>0.34 ± 0.06</td>
</tr>
<tr>
<td>Real</td>
<td>TMS</td>
<td>58.4 ± 0.97</td>
<td>0.81 ± 0.28</td>
<td>0.23 ± 0.10</td>
</tr>
<tr>
<td>AF recordings</td>
<td>RBF</td>
<td>58.2 ± 0.85</td>
<td>1.16 ± 0.20</td>
<td>0.32 ± 0.08</td>
</tr>
</tbody>
</table>
The structure of the RBF network varies over time in order to control the network complexity that is just sufficient for the approximation so that the computational efficiency is ensured. A generalization of the second method of Lyapunov to prove the guaranteed performance of the proposed RBF network has been used. Two different RBFs (GRBF and RCRBF) have been implemented and their performance summarized in Table 1. The network based in RCRBF achieved lower error both on the training set \((E_{tr})\) and the testing set \((E_{ts})\). Fig. 2 presents the cross correlation graph for the proposed network, in which the network achieves an optimum training point. This RBF has been mainly chosen because its adaptability to the non-linear and time-varying features of the QRST complex.

Non-invasive assessment of the atrial fibrillatory waves and particularly the fibrillation frequency is gaining acceptance as a tool for characterizing the arrhythmia in individual patients and
for assessing the impact of different treatment strategies [7,41–43]. However, analysis of the atrial fibrillatory waves directly from the ECG is very limited because the atrial signal is largely obscured by the ventricular activity. During last years, numerous algorithms have been proposed to extract the atrial fibrillation signal from the ECG [5–7,15,17]. The most extended and established is TMS [5]. This method assumes that, for the same patient, ventricular complexes generally exhibit a limited number of shapes. As a consequence, the average beat can represent approximately each individual beat. This average beat is then used to subtract the VA from each single beat. However, QRST morphology is often subject to minor changes caused by respiration, patient movement, etc, and, therefore, QRST residua and noise are often present in the estimated AA. Due to this, when there are alterations in the QRST morphology a complete reduction is not achieved. As a way to improve TMS, the average beats of adjacent leads could mathematically be combined with the average beat of the analyzed lead in order to suppress the electrical axis alterations and produce improved reduction. This is the core of the spatiotemporal cancellation method [15]. However, at least, one minute of signal is necessary for proper QRST cancellation, therefore in ambulatory ECG systems of short duration recording the method is not recommended. A modification of the aforesaid algorithm has been recently introduced by Lemay et al. [16] in which the QRS complex and the T wave are separately processed. However, the improvement introduced by these techniques is notably reduced when single-lead ECG recordings are only available. Other authors presented a strategy in which the temporal dependence of the AA was exploited [8], using principal component analysis, to estimate the AA from single-lead ECG recordings. The main features of ventricular and atrial activities are extracted, and several basis signals for each subspace are determined. Finally, the AA is reconstructed back exclusively from the basis signals that formed the atrial subspace. The algorithms based on PCA or ICA exploit the property that the atrial and ventricular activities originate from different and uncoupled bioelectric sources [6]. From the different techniques proposed to cancel out the QRST complex in ECGs with AF, TMS has been selected for comparison with the proposed RBF-based technique because is the most widely used technique in the literature. This study was validated using recordings from PhysioBank in order to give standardization and accessibility to the study. Tables 2–5 have shown how RBF gets the most appropriate values to the targets. Furthermore, Figs. 3–6 also demonstrate how the RBF-based system achieves a significant cancellation of the QRST complex in different scenarios both in time and frequency domains.

Taken together, these results demonstrate that the proposed approach can serve as a new framework to achieve efficient QRST cancellation. The RBF method has shown that accurate AA extraction can be provided both single and multi-lead ECG recordings. It has been found that the system works more efficiently if the ECG recording contain several leads. This is because a lot of leads carry more information about the signal that helps the neural network to get a better approximation of the function. Thus, the QRST complex reduction is more effective. Besides, the RBF works with multi-lead signals in a parallel fashion. Though traditional techniques can be used for QRST removal from the ECG in AF, over-cancellation can remove or distort relevant medical information. On the contrary, the results of this study suggest that clinical information can be preserved by selecting a flexible dynamic model for the ECG with an adaptive update of the RBF-based model parameters. Such information will provide optimal information about the electrophysiological phenomena surrounding AF, thus leading to improved diagnosis and treatments.

### 6. Conclusion

The present work introduced a RBF-based neural network applied to QRS-T cancellation in ECG recordings of AF. Throughout

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**Table 4**

Average results provided by the comparison between TMS and RBF obtained for simulated and real AF recordings with one lead. Values indicate mean ± standard deviation.

<table>
<thead>
<tr>
<th>Method</th>
<th>CC</th>
<th>MSE</th>
<th>VDR</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated AF recordings</td>
<td>TMS</td>
<td>0.855 ± 0.043</td>
<td>0.521 ± 0.098</td>
<td>3.269 ± 0.524</td>
</tr>
<tr>
<td></td>
<td>RBF</td>
<td>0.958 ± 0.038</td>
<td>0.311 ± 0.078</td>
<td>6.561 ± 0.321</td>
</tr>
<tr>
<td>Real AF recordings</td>
<td>TMS</td>
<td>–</td>
<td>–</td>
<td>3.195 ± 0.567</td>
</tr>
<tr>
<td></td>
<td>RBF</td>
<td>–</td>
<td>–</td>
<td>6.435 ± 0.316</td>
</tr>
</tbody>
</table>

**Table 5**

Average spectral results provided by the comparison between TMS and RBF obtained for simulated and real AF recordings with one lead. Values indicate mean ± standard deviation.

<table>
<thead>
<tr>
<th>Method</th>
<th>DF</th>
<th>DA</th>
<th>MP</th>
<th>SP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated AF recordings</td>
<td>TMS</td>
<td>5.85 ± 0.91</td>
<td>0.83 ± 0.24</td>
<td>0.23 ± 0.06</td>
</tr>
<tr>
<td></td>
<td>RBF</td>
<td>5.84 ± 0.78</td>
<td>1.16 ± 0.17</td>
<td>0.32 ± 0.06</td>
</tr>
<tr>
<td>Real AF recordings</td>
<td>TMS</td>
<td>5.82 ± 0.93</td>
<td>0.80 ± 0.26</td>
<td>0.22 ± 0.09</td>
</tr>
<tr>
<td></td>
<td>RBF</td>
<td>5.83 ± 0.82</td>
<td>1.15 ± 0.18</td>
<td>0.31 ± 0.07</td>
</tr>
</tbody>
</table>
all the stages, the RBF has been adapted through the Lyapunov approach, which has been improved in order to achieve optimal results. This has allowed to get a very accurate VA representation, thus providing high quality AA extraction. The proposed method can be applied to a wide number of ECG recording situations, such as long-term Holter monitoring and short channel ambulatory ECGs. In all these practical cases, the RBF-based system has provided outperforming results, both in QRST reduction and low atrial activity distortion, thus facilitating a more proper clinical analysis for medical professional use.

**Conflict of interest statement**

None declared.

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