Super-Twisting Sensorless Control of Linear Induction Motors

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Abstract—In this work, a sensorless control scheme is presented for linear induction motors. The secondary fluxes are algebraically calculated by first determining the primary fluxes, then, a super-twisting observer for secondary fluxes is designed in order to retrieve the back-EMF components by means of the equivalent control method. Based on these components, the linear velocity is determined and used in a linear velocity and load force observer, where the estimated variables along with primary current and voltage measurements are used to control the linear induction motor for the tracking of a reference linear velocity signal and a square secondary flux modulus, all by means of a super-twisting controller. Simulations show that the proposed observer based controller scheme performs well when tracking a time varying linear velocity signal.

Keywords: Linear induction motors, Sensorless control, Sliding mode control

I. INTRODUCTION

Nowadays, linear induction motors (LIMs) are now widely used in many industrial applications including transportation, conveyor systems, actuators, material handling, pumping of liquid metal, and sliding door closers, etc., with satisfactory performance. The most obvious advantage of linear motor is that it has no gears and requires no mechanical rotary-to-linear converters. The linear electric motors can be classified into the following: D.C. motors, induction motors, synchronous motors and stepping motors, etc. Among these, the LIM has many advantages such as simple structure replacement of the gear between motor and motion devices, reduction of mechanical losses and the size of motion devices, silence, high starting thrust force, and easy maintenance, repairing and replacement. On the other hand, linear induction motors is becoming a common test bench in the automatic control theory framework due to the fact that represents a coupled MIMO nonlinear system, resulting in a challenging control problem. Starting from the pioneering work of Blaschke [1], field oriented control has been a classical control technique for the electric motors. Active research area include fuzzy control [2], adaptive backstepping [3], sliding mode [4], artificial neural networks [5], among others. Since cost and space reduction in the final setup for LIM are essential aspects, sensorless control schemes are gaining popularity among control engineers. Moreover, the avoidance of a mechanical sensor reduces the noise problem commonly present in measurements. Sensorless control of LIM drives is now receiving wide attention. The main reason is that the speed sensor spoils the ruggedness and simplicity of LIMs. In a hostile environment, speed sensors cannot even be mounted. However, due to the high order and nonlinearity of the dynamics of a LIM, estimate the states of speed and secondary flux with only primary currents and voltages becomes a challenging problem. The sensorless control of LIM is an active research area, and common observer designs are based on high frequency signal injection method as presented in [6], a fuzzy observer as proposed in [7], a sliding mode based observer as in [8]. Regardless of the technique used to design the observer, all of these works are characterized by complex and long procedures, that can demand heavy computational burden.

Therefore, in this work, one is compelled with a simple sensorless control design for linear induction motors, based on the super-twisting algorithm [9]. In this effort, the rotor fluxes are algebraically calculated by first determining the stator fluxes, then, with a rotor flux observer design, one determines the equivalent values for the injected signals. In this case, the injected signals are identical to the back-EMF. Then, by simple algebraic manipulation of such signals one can easily determine the rotor velocity. In order to force the rotor velocity and the rotor flux modulus to track desired references signals, a super-twisting controller is proposed. This technique has been replacing the classical sliding mode control mainly for its chattering elimination property and by retaining all the other properties of classical sliding mode as robustness and order reduction.

The remaining of this works is organized as follows: In Section 2, the mathematical model of LIM is presented. In Section 3, the control problem to be tackled is formulated, and the observer and controller are designed. Simulation results are presented in Section 4. Finally some comments conclude the work.

II. LINEAR INDUCTION MOTOR MODEL

In the following, the mathematical model of the LIM will be presented as a nonlinear affine system. Under the assumptions of equal mutual inductance and linear magnetic circuit. The LIM equations in the stator reference frame \((\alpha,\beta)\) are given
by [10]
\[
\begin{align*}
\frac{dv}{dt} &= \frac{\eta_0}{L_m} (\psi_\alpha i_\beta - \psi_\beta i_\alpha) - \frac{D}{L_m} v - \frac{F_l}{L_m}, \\
\frac{d\psi_\alpha}{dt} &= -\eta_1 \psi_\alpha - \eta_2 v \psi_\beta + \eta_1 L_m i_\alpha, \\
\frac{d\psi_\beta}{dt} &= -\eta_1 \psi_\beta + \eta_2 v \psi_\alpha + \eta_1 L_m i_\beta, \\
\frac{di_\alpha}{dt} &= -\eta_4 i_\alpha + \eta_1 \eta_5 \psi_\alpha + \eta_2 \eta_5 \psi_\beta + \frac{u_\alpha}{\eta_6}, \\
\frac{di_\beta}{dt} &= -\eta_4 i_\beta + \eta_1 \eta_5 \psi_\beta - \eta_2 \eta_5 \psi_\alpha + \frac{u_\beta}{\eta_6},
\end{align*}
\]
where \( v \) is the mover linear velocity, \( u_\alpha, u_\beta \) are the primary voltage components, \( i_\alpha, i_\beta \) are the primary current components, \( \psi_\alpha, \psi_\beta \) are the secondary flux components, and \( F_l \) is the load force disturbance, with \( D \) as the viscous friction and iron-loss coefficient, \( L_m \) is the magnetizing inductance, and \( M \) is the total mass of the moving element, moreover,
\[
\begin{align*}
\eta_0 &= \frac{3L_m \eta_2}{2L_r}, & \eta_1 &= \frac{1}{T_r}, & \eta_2 &= \frac{\eta_p \pi}{h}, & \eta_3 &= \eta_2 \eta_5, \\
\eta_4 &= \frac{R_s}{\sigma L_s} + \frac{1 - \sigma}{\sigma T_r}, & \eta_5 &= \frac{L_m}{\sigma L_s r}, & \eta_6 &= \sigma L_s, \\
T_r &= \frac{L_r}{R_s}, & \sigma &= 1 - \frac{L_m^2}{L_s},
\end{align*}
\]
with \( L_r \) and \( L_s \) as the secondary and primary inductances respectively, \( R_s \) and \( R_s \) as the secondary and primary resistances respectively, \( n_p \) is the number of pole pairs and \( h \) is the pole pitch.

III. Super-twisting Sensorless Controller for LIM

The control problem is to force the linear velocity \( v \) and the square of the secondary flux modulus \( \psi_m = \psi_m^2 + \psi_m^2 \) to track some desired references \( v_r \) and \( \psi_m^2 \), respectively, ensuring at the same time load force rejection. This control problem will be solved in this section along with the design of an observer for the estimation of unmeasured variables as secondary fluxes, mover linear velocity and load force.

A. Controller design

In order to solve the posed control problem using the super-twisting sliding mode approach, we first derive the expression of the tracking error dynamics \( z_1 = v - v_r \), \( z_2 = \psi_m - \psi_m^2 \) which are the outputs to be forced to zero. Making use of (1), the output error dynamics can be written as follows:
\[
\begin{align*}
\dot{z}_1 &= \frac{\eta_0}{L_m} (\psi_\alpha i_\beta - \psi_\beta i_\alpha) - \frac{D}{L_m} v - \frac{F_l}{L_m} + \epsilon_v, \\
\dot{z}_2 &= -2\eta_1 \psi_m + 2\eta_1 L_m (\psi_\alpha i_\alpha - \psi_\beta i_\beta) - \psi_m^2 + \epsilon_m.
\end{align*}
\]
Now, defining the vector \( \zeta_1 = (\zeta_{1,1}, \zeta_{2,1})^T = (z_1, z_2)^T \) its dynamic results as
\[
\dot{\zeta}_1 = f_1(\cdot) + G(\psi_\alpha, \psi_\beta) I \tag{2}
\]
where \( I = (i_\alpha, i_\beta)^T, f_1(\cdot) = (f_{1,1}(\cdot), f_{2,1}(\cdot))^T \)
\[
G(\psi_\alpha, \psi_\beta) = \begin{pmatrix}
-\eta_0 \psi_\beta / L_m & \eta_0 \psi_\alpha / L_m \\
L_m \eta_1 \psi_\alpha & L_m \eta_1 \psi_\beta
\end{pmatrix},
\]
with \( f_{1,1}(\cdot) = -Dv/L_m - F_l / L_m - \epsilon_v, f_{2,1}(\cdot) = -\eta_1 \psi_m - \psi_m^2 \).

For the stabilization of equation (2) one proposes a desired dynamic as
\[
\dot{\zeta}_1 = f(\cdot) + G(\psi_\alpha, \psi_\beta) I = K_1 \zeta_1 \tag{3}
\]
where \( K_1 = \text{diag}(k_{1,1}, k_{2,2}) \). From (3) one can calculate the current vector as a reference signal, i.e., \( I_r = G^{-1}(\cdot)(K_1 \zeta_1 - f_{1}(\cdot)) \) with \( I_r = (i_{\alpha, r}, i_{\beta, r})^T \). Due to the fact that \( \text{det}(G) = -\eta_0 \eta_1 L_m \psi_m \), one can note that \( G^{-1} \) exist under the obvious assumption that \( \psi_m \neq 0 \). The reference current signal is such that if circulating through the motor stator windings, the output tracking errors \( z_1 \) and \( z_2 \) will decay asymptotically to zero. In order to force the stator currents to be equal to the stator reference signals, one defines the variable \( \zeta_2 = (\zeta_{2,1}, \zeta_{2,2})^T = I - I_r \), where its dynamic along with the ones in (2) results as follows:
\[
\begin{align*}
\dot{\zeta}_1 &= K_1 \zeta_1 + G(\psi_\alpha, \psi_\beta) \zeta_2, \\
\dot{\zeta}_2 &= f_2(\cdot) + U / \eta_0 - \dot{I}_r \tag{4}
\end{align*}
\]
with \( f_2(\cdot) = (f_{2,1}(\cdot), f_{2,2}(\cdot))^T \), \( U = (u_\alpha, u_\beta)^T \) and \( f(\cdot) = -\eta_1 i_\alpha + \eta_1 \eta_5 \psi_\alpha + \eta_2 \eta_5 \psi_\beta + \frac{u_\alpha}{\eta_6} \). Considering \( \zeta_{2,1} \) and \( \zeta_{2,2} \) as sliding functions, one proposes the following super-twisting controller:
\[
\begin{align*}
u_\alpha &= u_{\alpha, 1} - k_b |\zeta_{2,1}|^{1/2} \text{sign}(\zeta_{2,1}), \\
u_\alpha &= -k_{b, 1} \text{sign}(\zeta_{2,1}), \\
u_\beta &= u_{\beta, 1} - k_b |\zeta_{2,2}|^{1/2} \text{sign}(\zeta_{2,2}), \\
u_\beta &= -k_{b, 1} \text{sign}(\zeta_{2,2}).
\end{align*}
\tag{5}
\]

Let us differentiate the sliding functions twice:
\[
\begin{align*}
\ddot{\zeta}_{2,1} &= f_{2,1}(\cdot) + \frac{u_\alpha}{\eta_6} - \frac{d i_{\alpha, r}}{dt}, \\
\ddot{\zeta}_{2,2} &= f_{2,2}(\cdot) + \frac{u_\beta}{\eta_6} - \frac{d i_{\beta, r}}{dt}, \\
\ddot{\zeta}_{2,1} &= \dot{f}_{2,1}(\cdot) + \frac{u_\alpha}{\eta_6} - \frac{d^2 i_{\alpha, r}}{dt^2}, \\
\ddot{\zeta}_{2,2} &= \dot{f}_{2,2}(\cdot) + \frac{u_\beta}{\eta_6} - \frac{d^2 i_{\beta, r}}{dt^2},
\end{align*}
\]
and then to assume that there are positive constants \( \Phi_\alpha \) and \( \Phi_\beta \) such that within the region \( |s| < s_0 \) the following inequalities hold \( \forall t, x \in X, u_\alpha, u_\beta \in U \):
\[
\begin{align*}
|\dot{f}_{2,1}(\cdot) - \frac{d^2 i_{\alpha, r}}{dt^2}| & \leq \Phi_\alpha, \\
|\dot{f}_{2,2}(\cdot) - \frac{d^2 i_{\beta, r}}{dt^2}| & \leq \Phi_\beta, \tag{6}
\end{align*}
\]
with \( U = \{u : |u| \leq U_m\} \). The corresponding sufficient conditions for the finite time convergence to the sliding manifolds are
\[
k_{a, 1} > \Phi_\alpha / \Gamma_m, \quad k_a^2 \geq \frac{4 \Phi_\alpha}{\Gamma_m \Gamma_m (k_{a, 1} + \Phi_\alpha)},
\]
bounded in a bounded domain. The first step is to calculate the primary fluxes based on conditions (6) and (7) are satisfied. In particular they state that the second time derivative of the sliding variable \( \dot{s} \), evaluated with fixed values of the control \( u \), is uniformly bounded in a bounded domain.

B. Observer design

In this subsection, we consider and fix the drawbacks of the proposed control law (5), due to the fact of unmeasurable states (mover linear velocity and secondary flux components). The first step is to calculate the primary fluxes based on primary current and voltage measurements

\[
\psi_{\alpha,s} = \int (v_\alpha - R_s i_\alpha) dt \\
\psi_{\beta,s} = \int (v_\beta - R_s i_\beta) dt
\]  

(8)

where \( \psi_{\alpha,s}, \psi_{\beta,s} \) are primary fluxes. In order to eliminate the dc offset produced by integrators one can follow the work [12]. Once the primary fluxes are retrieved, these can be used along with primary currents for the calculation of the secondary fluxes with the following well known relations

\[
\psi_{\alpha,c} = \frac{L_r}{L_m} (\psi_{\alpha,s} - \sigma L_s i_{\alpha,s}) \\
\psi_{\beta,c} = \frac{L_r}{L_m} (\psi_{\beta,s} - \sigma L_s i_{\beta,s})
\]  

(9)

where \( \psi_{\alpha,c} \) and \( \psi_{\beta,c} \) denotes calculated secondary fluxes. Then, we design a sliding mode observer for secondary fluxes based in (1), in order to extract the rotor velocity:

\[
\frac{d\hat{\psi}_\alpha}{dt} = -\eta_1 \hat{\psi}_\alpha + L_m n_1 i_\alpha + \rho_\alpha \\
\frac{d\hat{\psi}_\beta}{dt} = -\eta_1 \hat{\psi}_\beta + L_m n_1 i_\beta + \rho_\beta
\]  

(10)

where \( \hat{\psi}_\alpha \) and \( \hat{\psi}_\beta \) are secondary fluxes estimates and \( \rho_\alpha \) e \( \rho_\beta \) are the injected inputs to the observer. One defines the estimation errors \( \hat{\psi}_\alpha = \psi_{\alpha,c} - \hat{\psi}_\alpha, \hat{\psi}_\beta = \psi_{\beta,c} - \hat{\psi}_\beta \), whose dynamic can be expressed as

\[
\frac{d\hat{\psi}_\alpha}{dt} = -\eta_1 \hat{\psi}_\alpha - \eta_2 \hat{\psi}_\beta - \rho_\alpha \\
\frac{d\hat{\psi}_\beta}{dt} = -\eta_1 \hat{\psi}_\beta + \eta_2 \hat{\psi}_\alpha - \rho_\beta
\]  

(11)

and making use of the calculated rotor fluxes (9) we select the injection observer signals as follows:

\[
\rho_\alpha = b_{\alpha,1} \sqrt{\hat{\psi}_\alpha} |\text{sign}(\hat{\psi}_\alpha)| - \rho_{\alpha,1} \\
\dot{\rho}_{\alpha,1} = -b_{\alpha,2} \text{sign}(\hat{\psi}_\alpha) \\
\rho_\beta = b_{\beta,1} \sqrt{\hat{\psi}_\beta} |\text{sign}(\hat{\psi}_\beta)| - \rho_{\beta,1} \\
\dot{\rho}_{\beta,1} = -b_{\beta,2} \text{sign}(\hat{\psi}_\beta)
\]  

(12)

where \( b_{\alpha,1}, b_{\alpha,2}, b_{\beta,1}, b_{\beta,2} \) are constant design parameters. Again, one can assume that the following conditions holds:

\[
\begin{align*}
-\eta_1 \hat{\psi}_\alpha - \eta_2 \frac{d\hat{\psi}_\alpha}{dt} &\leq \hat{\Phi}_\alpha, \\
-\eta_1 \hat{\psi}_\beta + \eta_2 \frac{d\hat{\psi}_\alpha}{dt} &\leq \hat{\Phi}_\beta,
\end{align*}
\]

\[0 < \Gamma_m \leq 1 \leq \Gamma_M.
\]

The corresponding sufficient conditions for the finite time convergence to the sliding manifolds are [11]

\[
k_{\alpha,1} > \frac{\hat{\Phi}_\alpha}{\Gamma_m}, \quad k_{\alpha,2}^2 > \frac{4\hat{\Phi}_\alpha}{\Gamma_m^2} \Gamma_M (b_{\alpha,1} + \hat{\Phi}_\alpha) \\
k_{\beta,1} > \frac{\hat{\Phi}_\beta}{\Gamma_m}, \quad k_{\beta,2}^2 > \frac{4\hat{\Phi}_\beta}{\Gamma_m^2} \Gamma_M (b_{\beta,1} + \hat{\Phi}_\beta)
\]

When the estimation errors \( (\hat{\psi}_\alpha, \hat{\psi}_\beta)^T \) tend to zero in finite time, the structure of \( \rho_\alpha \) and \( \rho_\beta \) in (12) can be revealed by means of the equivalent control method which is calculated from \( \psi_\alpha = 0, \psi_\beta = 0 \) as follows:

\[
\rho_\alpha,eq = -\eta_2 \psi_\alpha, \quad \rho_\beta,eq = \eta_2 \psi_\beta.
\]  

(13)

Now, one can easily determine the rotor velocity \( \omega_c \) as follows:

\[
v_c = \frac{\psi_\alpha \rho_\beta - \psi_\beta \rho_\alpha}{(\psi_\alpha^2 + \psi_\beta^2)^{1/2}} \eta_2.
\]  

(14)

For the load force estimation we consider that it is slowly varying, so one can assume it is constant, i.e., \( F_l = 0 \). This fact can be valid since the electric dynamic of the mover is faster than mechanical one. Therefore one proposes the following observer based on the linear velocity estimation and primary current measurements:

\[
\begin{align*}
\frac{d\hat{v}}{dt} &= \frac{n_0}{L_m} (\hat{\psi}_\alpha i_\beta - \hat{\psi}_\beta i_\alpha) - \frac{D}{L_m} \hat{v} - \frac{\hat{F}_l}{L_m} + l_1 (v_c - \hat{v}) \\
\frac{d\hat{F}_l}{dt} &= l_2 (v_c - \hat{v}).
\end{align*}
\]

(15)

Defining the estimation errors as \( e_v = v_c - \hat{v} \) and \( e_{F_l} = F_l - \hat{F}_l \) one can determine the estimation error dynamic as follows:

\[
\begin{pmatrix} e_v \\ e_{F_l} \end{pmatrix} = \begin{pmatrix} -\frac{D}{L_m} - l_1 - \frac{1}{l_2} \\ 0 \end{pmatrix} \begin{pmatrix} e_v \\ e_{F_l} \end{pmatrix} + \begin{pmatrix} l_1 \Delta \nu + \frac{n_0}{L_m} (\hat{\psi}_\alpha i_\beta - \hat{\psi}_\beta i_\alpha) \end{pmatrix},
\]

where \( \Delta \nu = v - v_c \). When the estimation errors for the secondary fluxes in (10) are zero, equation (15) reduces to

\[
\begin{pmatrix} e_v \\ e_{F_l} \end{pmatrix} = \begin{pmatrix} -\frac{D}{L_m} - l_1 - \frac{1}{l_2} \\ 0 \end{pmatrix} \begin{pmatrix} e_v \\ e_{F_l} \end{pmatrix}
\]

where \( l_1 \) and \( l_2 \) can easily be determined in order to yield

\[
\lim_{t \to \infty} e_v(t) = 0 \quad \text{and} \quad \lim_{t \to \infty} e_{F_l}(t) = 0.
\]
IV. SIMULATIONS

In this part we verify the performance of the proposed control scheme by means of numeric simulations. We consider a LIM with the following nominal parameters: $L_r = 0.02846\,\text{H}$, $L_s = 0.02846\,\text{H}$, $L_m = 0.02419\,\text{H}$, $R_r = 3.5215\Omega$, $R_s = 5.3685\Omega$, $n_p = 2$, $h = 0.027\,\text{m}$, $M = 2.78\,\text{Kg}$ and $D = 36.0455\,\text{kg/s}$. The controller gain parameters have been set to $k_{1,1} = -300 - 2000|v_r|/v_{r,p}$, $k_{2,2} = -100 - 2300|v_r|/v_{r,p}$, $k_\alpha = 25080$, $k_\beta = 25080$, $k_{\alpha,1} = 1000$ and $k_{\beta,1} = 1000$. For the secondary flux observer, we set $b_{\alpha,1} = 1550$, $b_{\alpha,2} = 10000$, and for the load force observer $l_1 = 1007$ and $l_2 = -55600$. The reference signal is $v_r = v_{r,p}\sin2t\,\text{m/s}$, with $v_{r,p} = 3\sin\alpha$ in order to verify the performance of the proposed control scheme at low velocities. A load force of $350\,\text{N}$ with decrements of $150\,\text{N}$ at $5\,\text{s}$ and $6\,\text{s}$ has been considered. Moreover, the initial conditions of the flux observer have been set different from zero in order to avoid $d\xi(t) = 0$.

Fig. 1 shows the simulation of the output tracking for the linear velocity where it can be appreciated the good performance of the controller. Also, Fig. 2 shows the output tracking performance but of the secondary square flux modulus, where the performance is acceptable. Fig. 3 and 4 show the secondary currents and voltages respectively. Fig. 5 shows the performance of the proposed observer or the linear velocity estimation, here, one can appreciate that the estimated linear velocity tracks the real linear velocity with a good accuracy, moreover, in Fig. 6 the estimate of the applied linear force is shown, where its performance is acceptable.

V. CONCLUSIONS

In this paper a sensorless control of linear induction motors have been designed. The super-twisting algorithm is applied for a secondary flux observer design based on calculated secondary fluxes. The equivalent injected signals are determined to be equal to the back-EMF, allowing the estimation of the rotor velocity by means of simple computations. Also, the control design has been carried out with the same sliding mode algorithm, yielding a great performance of the linear motor when tracking a low velocity along with a constant square secondary flux modulus. This fact has been verified by numeric
simulations. Some interesting issues as the robustness of the sensorless controller with respect to parameter variations and disturbances, and real-time implementation are currently under study.

REFERENCES