Stable discrete-time impedances for haptic systems with vibration modes and delay

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Abstract

The stability of haptic rendering is affected by many factors that limit the range of impedances that can be applied to virtual objects. This paper addresses the effect of the vibration modes of the haptic interface on the stability of the impedance control loop. It is well known that experimental stability boundaries present complex shapes, making it difficult to predict the final Z-width of the haptic system. This paper shows how the vibration modes of the mechanical interface highly affect the size of the Z-width, causing a sudden reduction in the critical virtual stiffness K if the virtual damping B is increased beyond a certain value. The inclusion of the most significant vibration modes in the theoretical model of the haptic system—together with the viscous damping, time delay and sampling rate—makes it possible to obtain the stable impedances associated with the haptic device. A PHANToM Premium 1.0 haptic interface was used as a test bed to validate the study. Although results have been tested only on this device, this paper proposes a methodology for obtaining the Z-width that can be generalized for any other haptic system.

1 Introduction

Haptic interactions with virtual objects require a mechatronic interface to provide the user with the contact forces. In this work we focus on impedance-type kinesthetic interfaces, which are easily backdrivable and typically use a fast servo loop with DC motors to output a force vector as a function of the tool position. The kinematic design of the interface imposes some functional limitations (e.g., the reachable workspace, the type of grasping, etc.), and its mechanical properties can restrict, among other things, the impedance of the virtual objects. For example, virtual environments cannot feel stiffer than the interface itself because both impedances are in series. Thus, the mechanical stiffness of the haptic device limits the virtual stiffness that can be rendered. For relatively small (e.g., desktop-type) metallic mechanisms, this mechanical stiffness is usually sufficiently high such that the maximum virtual stiffness is imposed by the critical value of the control loop. However, for relatively large devices or those made of nonmetallic materials, the previous assumption may not be true, and even if high—and stable—virtual stiffness coefficients are implemented, the user might feel the objects to be as soft as the device itself.

Regarding the stability of the control loop, some mechanical properties of the device and other parameters of the control loop impose severe limitations. Sampling rate, viscous damping [1, 2], time delay [3], Coulomb friction, sensor quantization [4, 5] and interface flexibility [6] are some of the well-known phenomena that restrict the range of stable values for virtual stiffness. The combination of all these factors in multi-degree-of-freedom haptic interfaces makes it difficult to address the stability analysis in a generic way. In addition, the complex shapes that some experimental stability boundaries exhibit [7, 8, 9]—even for a single-joint movement—make it clear that multiple effects escape our full understanding, leaving room for further research on this topic.
To cope with this persistent challenge, different solutions have been explored. To compare the performance of different strategies and interfaces, the concept of Z-width has been used in the case of haptic interactions with a virtual impedance that consists of a spring $K$ and a damping $B$. The region containing the stable values of these parameters is called the Z-width [7]. A number of control strategies have been developed with the aim of increasing the Z-width of haptic systems [8, 10, 11]. Another possible approach is to modify the impedance feedback by introducing a control algorithm to dissipate energy and ensure passivity. This strategy also ensures the stability of the entire haptic loop, and it has been used successfully in haptic applications and teleoperated systems [12, 13, 14, 15].

This paper analyzes the influence of internal vibration modes of the mechanical interface on the stability of haptic rendering. The complexity of the study requires that some starting simplifications be established. Thus, the analysis covers movements along only one degree of freedom (DoF). The theoretical and experimental limits are found for a specific haptic interface in widespread use: the PHANToM Premium 1.0 [16]. The characterization method of the system dynamics will be described in detail in order to show the effects of each mechanical parameter in the frequency response of the system and provide an easy method for obtaining them for any interface. System description and modeling follow previous work [17, 9] but include some new elements to characterize the delay within the haptic loop, which can be a fraction of the sampling period. The main contribution of this paper is the study of the causes of the complex shape of the Z-width of the haptic system. In particular, the analysis shows how the vibration modes of the mechanical interface can be one of the most restricting elements in the size of the Z-width, causing a sudden reduction in the critical virtual stiffness $K$ if the virtual damping $B$ is increased beyond a certain value. Many factors concurrently affect the stability. Therefore, the root locus, a classical control tool, can be used to inspect the influence of different factors on the position of the closed-loop poles of the system. These graphics allow one to identify the poles that become unstable and the vibration modes associated with each branch of the plot. In addition, this method also shows the influence of time delay on the stability of the system, offering an explanation for the resulting Z-width of the haptic interaction, which can change greatly.

This paper is organized as follows. Section 2 describes the haptic interaction from the control point of view in order to identify the characteristic equation of the system after enumerating the main parameters of the haptic loop. Section 3 proposes a characterization method for experimentally identifying all the parameters that are involved in the system. The basis for the characterization process is a frequency response analysis, which is divided into two parts: an initial manual fitting and a subsequent automatic iterative method. Using the resulting system model, the stability analysis is described in Section 4. In this section, the theoretical results are also corroborated experimentally. The most relevant findings regarding the influence of the vibration modes on the Z-width of the haptic system are discussed in Section 5, and some final conclusions are drawn in Section 6.

2 Haptic interaction description

From the control point of view, haptic interaction can be described as a discrete-time closed-loop system (Fig. 1). The mechanical interface has a transfer function $G(s)$, and the impedance of the virtual environment is a viscoelastic force law with stiffness $K$ and damping $B$. To achieve good transparency, the dynamics of the interface should be negligible compared with the impedance of the virtual object. When this criterion is satisfied, the user of such a device will feel only the contact with the virtual environment. This constraint leads to relatively light mechanical solutions for the interface, and it means that the analysis of the vibration modes of the haptic device is quite important during the design phase.

The impedance of the virtual environment is usually implemented in discrete-time microprocessors that require sample and hold operations. As in many haptic applications, and due to the high
sensitivity of the sense of touch, the sampling period $T$ is set to 1 ms in the present analysis [18, 19]. Assuming the use of high-resolution encoders, the effect of sensor quantization is negligible. The zero-order hold models the digital-to-analog conversion. Depending on the complexity of the virtual environment, the collision detection algorithms could take several milliseconds, and therefore, the virtual force $F_e$ is available only after a few sampling periods. This effect is represented in this paper (Fig. 1) by a discrete-time delay of $d$ sampling periods, $z^{-d}$, in which $d$ is assumed to be a small constant value ($d = 0, 1, 2, 3...$), and ideally $d = 0$.

Certain physical elements (such as electronics, amplifiers and actuators) introduce additional time delay within the haptic loop. All these physical delays are usually shorter than one sampling period $T$—e.g., up to hundreds of microseconds [5]—and they are included in transfer function $G(s)$. Thus, in this paper the delay caused by the hardware setup is modeled within $G(s)$ while the delay introduced by the computations of the virtual environment is represented by $d$.

By changing impedance parameters $K$ and $B$, it is possible to simulate different virtual contacts. To provide realistic simulation of hard contacts, virtual stiffness $K$ should be set as large as possible. For this reason, it is important to know how the rest of the parameters of the haptic loop affect the stability limits for parameters $K$ and $B$, that is, the $Z$-width of the haptic system.

### 2.1 Mechanical interface model

The mechanical interface is involved in the haptic interaction loop by means of transfer function $G(s)$. This dynamic model combines the mechanical characteristics of the device along the DoF under analysis and the overall physical time delay $\tau$ present in the loop, but it does not include the discrete-time delay $d$ described previously. A simple mass-damper model (Fig. 2-a) is commonly used to describe the mechanical characteristics of the device along one DoF:

$$
G(s) = \frac{1}{ms^2 + bs}
$$

(1)

This model has mass $m$ and viscous damping $b$, and it assumes that the device is perfectly rigid. The inclusion of time delay $\tau$ is defined as follows:

$$
G(s) = \frac{e^{-\tau s}}{ms^2 + bs}
$$

(2)

From now on, these three parameters ($m$, $b$ and $\tau$) will be referred to as the parameters of the rigid model. To create a more general model for the haptic interface with $n$ vibration modes, the transfer function is multiplied by $n$ second-order transfer functions (one per vibration mode):

$$
G(s) = \frac{e^{-\tau s}}{ms^2 + bs} \prod_{i=1}^{n} \frac{\omega_i^2}{\nu_i^2 (s^2 + 2\zeta_i \nu_i s + \nu_i^2)}
$$

(3)

Each vibration mode $i$ consists of two underdamped complex poles ($\zeta_i < 1$) located at natural frequency $\omega_i$, and two underdamped complex zeros ($\xi_i < 1$) found at natural frequency $\nu_i$. The resulting model is a $(2 + 2n)$-th order type-I transfer function including $4n + 3$ parameters.
Inspired by previous works [20, 14, 21], Fig. 2-b shows a mechanical model with $n$ vibration modes and $4n + 2$ parameters. This model is equivalent to transfer function (3)—the only missing parameter is time delay $\tau$. The mass is divided into $n + 1$ masses, which are connected by $n$ links. The overall mass $m$ is equal to the sum of the masses, $m_1 + m_2 + \ldots + m_{n+1}$, and the characteristics of the spring-damper links are related to the natural frequencies and damping coefficients of the modes.

### 2.2 Characteristic equation of the system

The characteristic equation of the system is identified to analyze the stability of the haptic interaction. Using the block diagram shown in Fig. 1, the sampled output position signal is:

$$X(z) = \frac{Z[F(s)G(s)]}{1 + z^{-d} \left( K + B \frac{1 - z^{-1}}{T} \right) Z \left[ \frac{1 - e^{-Ts}}{s} G(s) \right]}$$

Since $Z[F(s)G(s)] \neq Z[F(s)]Z[G(s)]$, it is not possible to isolate the closed-loop transfer function for the system. However, the characteristic equation can be identified by taking the denominator of the sampled position signal [22]:

$$1 + z^{-d} \left( K + B \frac{1 - z^{-1}}{T} \right) Z \left[ \frac{1 - e^{-Ts}}{s} G(s) \right] = 0$$

The haptic interaction is stable if and only if all the roots of the characteristic equation—the poles of the system—are located inside the unit circle. Finding the analytical roots of expression (5) with the general model (3) is impossible. However, it is possible to perform the stability analysis for a particular system model $G(s)$. The following section details a method for characterizing the system.

### 3 System characterization method

This section shows how to empirically find the transfer function $G(s)$ for a particular haptic interface. The aim is to obtain the physical parameters included in the continuous-time transfer function (3). For the stability analysis, the characteristic equation (5) requires the discrete-time transfer function of the system, sometimes called pulse transfer function [23], which is the $Z$-transform of the continuous-time transfer function $G(s)$ multiplied by the expression of the zero-order hold:

$$G(z) = Z \left[ \frac{1 - e^{-Ts}}{s} G(s) \right]$$

This transfer function can be characterized by using the open-loop frequency response of the system (Fig. 3). Only one experiment is necessary if a proper input force $F(z)$—covering a wide range of frequencies—is commanded (e.g., a white-noise signal or a swept sine).
The input force $F(z)$ is a discrete-time signal computed by a microprocessor every sampling period $T$, while the zero-order hold represents the digital-to-analog converter. The measured response $X(z)$ is also sampled by a data-acquisition board. As a consequence, the experiment provides the discrete-time transfer function of the system:

$$\frac{X(z)}{F(z)} = Z\left[\frac{1 - e^{-Ts}}{s} G(s)\right] = G(z)$$

A blind fitting process—that is, without using the expected transfer function (3) with physical meaning—could be performed to find a discrete-time transfer function by matching the frequency response of the system. The resulting transfer function $G(z)$ can be used for the stability analysis, but the connection with the physical parameters of the system may be lost. As an alternative, this section proposes a fitting process that uses the $4n+3$ physical parameters of model (3) translated into the discrete-time domain.

This approach should pay special attention to time delay $\tau$. As previously stated, $\tau$ is typically shorter than one sampling period ($\tau < T$), and therefore, it is not possible to compute the pulse transfer function of $G(s)$ (7) by looking at $Z$-transform tables. To be rigorous, the modified $Z$-transform should be used [23]. However, most commonly used mathematical software packages, e.g., MATLAB, do not compute modified $Z$-transforms (the delay must be an exact multiple of the sampling period). To cope with this problem, time delay can be approximated by means of a simple pole with time constant $\tau$ [24]:

$$e^{-\tau s} \approx \frac{1}{1 + \tau s}$$

Therefore, the model for the mechanical characteristics of the system is:

$$G(s) \approx \frac{1}{ms^2 + bs} \frac{1}{1 + \tau s} \prod_{i=1}^{n} \frac{\omega_i^2(s^2 + 2\zeta_i\nu_is + \nu_i^2)}{\nu_i^2(s^2 + 2\zeta_i\omega_is + \omega_i^2)}$$

This approximation is valid only at low frequencies, that is, below the cut-off frequency of the simple pole ($\omega < \frac{1}{2\tau}$ rad/s). This fact does not compromise the characterization method because the Nyquist frequency falls within this frequency range. Thus, the approximation is good for all the visible frequencies of the pulse transfer function $G(z)$ ($\omega < \frac{\pi}{T}$ rad/s). Note that this is also true because the sampling rate is set to 1 kHz. Faster sampling rates (e.g., over 10 kHz) may require a different approach. In these cases, even a very short time delay $\tau$ could be longer than the sampling period ($\tau > T$), and thus the physical delay should be divided into two parts: $\tau = \delta T + \tau'$. The number of whole sampling periods $\delta$ could be added to digital-time delay $d$, and the last fraction of time $\tau'$ could be approximated by a simple pole. Nevertheless, approximation (8) is correct for most haptic systems with a sampling rate of 1 kHz.

The following subsections show the characterization process for a specific haptic interface. Section 3.1 presents the device and its frequency response. The fitting method for the physical parameters is divided into two parts: an initial manual fitting to obtain approximate values for most of the parameters (Section 3.2), and a subsequent iterative process (which has been here programmed in MATLAB but could be implemented in any mathematical or programming software) to obtain the best solution based on a least-square method (Section 3.3). This automatic process takes the parameters that have been estimated manually as the initial guess for the solution.
3.1 Experimental setup and frequency response

The first joint of the PHANToM Premium 1.0 haptic interface (φ-axis in Fig. 4) is used to show how $G(s)$ (9) can be characterized. Since the analysis only involves one DoF, the rest of the joints have been mechanically locked in a particular configuration. This kinematic configuration (Fig. 4) is the standard position for initializing the incremental encoders of the device to zero, and thus it is known as the zero configuration [25]. The end-effector of the interface (stylus or gimbal) has been removed to avoid the presence of moving parts during the experiments.

The system is controlled by a dSPACE DS1104 data-acquisition board running at a sampling frequency of 1 kHz. The motor’s drive is a Maxon 4-Q-DC Servoamplifier ADS 50/10 in current mode. The input signal $F(z)$ is a torque command in the motor that moves the base joint of the device, while the measured output $X(z)$ is the rotation of the encoder coupled to the motor. Both signals are mapped to the φ-axis in SI units. Therefore, the transfer function $G(z)$ contains rotational parameters, and it applies only to the zero configuration.

A white-noise input signal lasting 20 s is used to obtain the experimental frequency response of the system. This signal is limited to 500 Hz, the Nyquist frequency. Using the input and output signals from the experiment, we run an algorithm to estimate the experimental transfer function and use it for the further adjustment of the theoretical model and to plot the resulting Bode diagram (Fig. 5). In our case, MATLAB’s empirical transfer function estimation algorithm (tfestimate) is used. A hanning window 4096 points in length with a 50% overlap is selected for data processing. The length of the experiment, 20 s, allows the tfestimate algorithm to average the resulting Bode plot, leading to a smooth and reliable frequency response. A similar experimental frequency response was achieved by [25] with the same device and configuration.

3.2 Initial manual characterization

The theoretical frequency response of $G(z)$ is matched with the experimental one by using a semi-automatic iterative curve-fitting process based on the least-squares method [21]. It is not fully automatic because the number of vibration modes has to be selected manually before launching the algorithm, and the initial values for all the parameters have to be provided as well. The number of
parameters is so large that the fast convergence of the method strongly depends on the accuracy of these initial values. A visual inspection of the Bode diagram (Fig. 5) is used to set the initial values for the iterative method.

Regarding the number of vibration modes, Fig. 5 reveals four clear resonant peaks (local maxima in the gain plot) arising at frequencies close to 150, 200, 350 and 400 Hz. The same number of anti-resonant peaks (local minima in the gain plot) are located at approximately 70, 180, 310 and 380 Hz. In addition to this, a very small resonant mode can be detected around 100 Hz. The influence of this fifth mode is slightly noticeable only in the phase plot. Therefore, \( n = 5 \) is the choice for the number of vibration modes to be modeled in the fitting process:

\[
G(s) \approx \frac{1}{ms^2 + bs + 1} \prod_{i=1}^{5} \frac{\omega_i^2}{\nu_i^2} (s^2 + 2\xi_i s + \nu_i^2) \prod_{i=1}^{5} \frac{\omega_i^2}{\nu_i^2} (s^2 + 2\zeta_i \omega_i s + \omega_i^2) \quad (10)
\]

The number of modes selected also determines the number of parameters to be modeled. In expression (10), 23 parameters are involved. Three of these parameters (\( m, b \) and \( \tau \)) can be obtained by means of various lines and points manually drawn on the frequency response (points A, B and C in Fig. 6), and the estimation procedure is explained in the following paragraphs.

The inertia of the device is the predominant physical effect over a wide range of frequencies. This is visible in the gain diagram as a line with a slope of \(-40 \text{ dB/decade}\) (in this case from 2 Hz up to
Figure 6: Experimental frequency response and manual fitting points to estimate inertia (A), viscous
damping (B) and delay (C)

approximately 20 Hz). Within this range, the transfer function can also be approximated by

\[ G(s) \approx \frac{1}{ms^2}. \]  

(11)

The inertia value \( m \) can be easily estimated by taking any point of the gain diagram within the
decreasing range with slope \(-40\ \text{dB/decade}\) (e.g., point A in Fig. 6). For example, at 4 Hz (or
8\pi \text{ rad/s}) the gain is 0 dB, and therefore:

\[ 20 \log |G(s)|_{s=8\pi j} \approx 0 \text{ dB} \]  

(12)

\[ |G(s)|_{s=8\pi j} \approx \left| \frac{1}{ms^2} \right|_{s=8\pi j} = \frac{1}{64\pi^2 m} \approx 1 \text{ rad/N·m} \]  

(13)

From (13), the value for inertia \( m \) is estimated as \(1.58 \text{ g·m}^2\). Viscous damping \( b \) is more difficult
to estimate from the Bode diagram because its influence can be clearly seen in the Bode diagram
only at low frequencies—lower than the inertial part. For these very low frequencies, the transfer
function can also be approximated by

\[ G(s) \approx \frac{1}{ms^2 + bs}. \]  

(14)
The more visible effect of the viscous damping is a gradual transition from $-90^\circ$ to $-180^\circ$ in the phase diagram. The middle point of this transition, that is, the phase of $-135^\circ$, is located exactly at the position of the simple pole, $\frac{b}{m}$ rad/s. In our case, the middle point (B in Fig. 6) is manually placed at 1.5 Hz (or $3\pi$ rad/s), and therefore:

$$ \frac{b}{m} \approx 3\pi \text{ rad/s} \quad (15) $$

By using the previous estimation of the inertia and (15), viscous damping $b$ is 0.015 N-m-s/rad. Notice that the accuracy of the Bode diagram at low frequencies depends on the number of points selected for the hanning window (4096 points in this case). A more averaged and smoother Bode diagram could be obtained if fewer points were taken, but the first frequency of the plot could be over $\frac{b}{m}$ rad/s, making the influence of the viscous damping on the plot invisible.

The time delay $\tau$ is even more difficult to estimate manually. The effect of the delay can only be seen as an exponential decrease of phase at high frequencies [24]. However, this influence overlaps with the change of phases for the vibration modes. The anti-resonant peaks in the gain plot correspond to a phase increase of $180^\circ$, while the resonant peaks correspond to a phase decrease of $-180^\circ$. Without any kind of delay, the phase diagram of a sequence of zeros-poles-zeros-poles-etc., would be similar to a sinusoidal shape enclosed between $-180^\circ$ and $0^\circ$. These oscillations would be quite abrupt if the zeros or the poles had very small damping coefficients, and their amplitudes would also vary depending on the proximity to the adjacent zeros or poles. The two limits ($-180^\circ$ and $0^\circ$) would be reached if the adjacent zeros and poles were far enough away. With time delay, the oscillations and the limits fall exponentially.

A rough approximation for the exponential lower limit is manually drawn in Fig. 6. The exponential decrease for the lower limit with respect to the horizontal line of $-180^\circ$ is proportional to $-\omega \tau \text{ rad}$ [24]. Thus, in point C of Fig. 6:

$$ -\omega \tau \text{ rad} = -400\pi \tau \text{ rad} \approx -55^\circ \frac{\pi}{180^\circ} \text{ rad} \quad (16) $$

From (16), an estimation of the delay is $\tau \approx 0.76$ ms. However, a major correction has to be made, because the estimated delay does not only correspond to the physical delay of the system. A significant part also corresponds to the time discretization of the transfer function (the effect of the sampling and hold operations). This discretization delay is half the sampling period:

$$ G(z) = Z \left( \frac{1 - e^{-Ts}}{s} G(s) \right) \approx G(s) e^{-\frac{T}{2} s} \quad (17) $$

Therefore, the correct formulation for the phase lag due to the overall delay is:

$$ -\omega \left( \tau + \frac{T}{2} \right) \text{ rad} = -400\pi \left( \tau + \frac{T}{2} \right) \text{ rad} \approx -55^\circ \frac{\pi}{180^\circ} \text{ rad,} \quad (18) $$

and the estimation for the physical delay is now $\tau \approx 0.26$ ms, which is consistent with the order of magnitude of the few hundred microseconds usually given to the physical delay [26].

Note that manual estimation of the delay is not always possible. In the case under study (Fig. 6), it was possible to detect that between the poles of the third mode ($\omega_3$) and the zeros of the fourth mode ($\nu_4$) there is more than one octave (from 200 to 310 Hz), and therefore, the lower exponential limit is almost reached by the phase diagram. But it may not always be possible to perform this manual approach.

### 3.3 Iterative fitting method

The physical parameters estimated in the previous subsection are used as the initial values of an iterative fitting method. The only parameters that have not been estimated are the damping coeffi-
The fitting method compares the experimental frequency response (Fig. 5) with the Bode diagram of the theoretical model (10). An index of quality for the fitted model is the sum of squared residuals covering all the frequencies (from 0.7 Hz up to 500 Hz). The method finds the minimum of this index by evaluating the 23 parameters of the model around their initial values. The number of variables is so high that providing good initial values is crucial to obtaining a reasonable convergence time. In addition, with such a large number of parameters, this least-square method can easily converge to an incorrect solution (a local minimum far away from the best fit) if completely arbitrary values are given as initial values.

Note that the residuals can be computed in very different ways. One simple definition could be computing the difference of the gains of the frequency responses, but this approach does not provide useful values for parameters $b$ and especially $\tau$, whose influence is seen only in the phase diagram. Therefore, the performance index has to include not only the sum of the squares of the errors in gains, but also the sum of the squares of the errors in phases along all the frequencies. Since gains and phases have different units, the sum of their residual errors has to be multiplied by appropriate factors before being added, thus obtaining a performance index in which both diagrams have similar weight.

For the characterization method we programmed a function with 23 input parameters in order to compute the sum of squared residuals. Then we used an algorithm to find the minimum of this
Table 1: Parameters of the rigid part of the model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Initial value</th>
<th>Final value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inertia</td>
<td>( m )</td>
<td>1.58 g·m²</td>
<td>1.48 g·m²</td>
</tr>
<tr>
<td>Damping</td>
<td>( b )</td>
<td>0.015 N·m·s/rad</td>
<td>0.0128 N·m·s/rad</td>
</tr>
<tr>
<td>Time delay</td>
<td>( \tau )</td>
<td>0.26 ms</td>
<td>0.304 ms</td>
</tr>
</tbody>
</table>

Table 2: Parameters of the five vibration modes

<table>
<thead>
<tr>
<th>Mode</th>
<th>( \nu_1 ) (Hz)</th>
<th>( \xi_1 )</th>
<th>( \omega_1 ) (Hz)</th>
<th>( \zeta_1 )</th>
<th>( \nu_1 ) (Hz)</th>
<th>( \xi_1 )</th>
<th>( \omega_1 ) (Hz)</th>
<th>( \zeta_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>70</td>
<td>0.1</td>
<td>100</td>
<td>0.1</td>
<td>65.9</td>
<td>0.101</td>
<td>124.9</td>
<td>0.314</td>
</tr>
<tr>
<td>Mode 2</td>
<td>120</td>
<td>0.1</td>
<td>150</td>
<td>0.1</td>
<td>126.5</td>
<td>0.251</td>
<td>152.7</td>
<td>0.062</td>
</tr>
<tr>
<td>Mode 3</td>
<td>180</td>
<td>0.1</td>
<td>200</td>
<td>0.1</td>
<td>168.2</td>
<td>0.091</td>
<td>209.8</td>
<td>0.137</td>
</tr>
<tr>
<td>Mode 4</td>
<td>310</td>
<td>0.1</td>
<td>350</td>
<td>0.1</td>
<td>317.7</td>
<td>0.050</td>
<td>341.7</td>
<td>0.098</td>
</tr>
<tr>
<td>Mode 5</td>
<td>380</td>
<td>0.1</td>
<td>400</td>
<td>0.1</td>
<td>376.6</td>
<td>0.073</td>
<td>417.8</td>
<td>0.076</td>
</tr>
</tbody>
</table>

Scalar function of several variables starting at an initial estimate. The initial values of the iterative process were given as input parameters for this process. For the minimization method MATLAB’s `fmincon` command was used. The result of the fitting procedure is shown in Fig. 7. The parameters of the rigid part of model are presented in Table 1, while the parameters corresponding to the vibration modes are shown in Table 2. It can be seen in Fig. 7 that the model fits very well with the experimental data, and the final values for the parameters are quite close to those resulting from the manual characterization.

4 Stability analysis

Once the system dynamics \( G(s) \) is modeled, the stability analysis of the system can be performed, and the shape and size of the \( Z \)-width of the haptic interaction can be obtained. The analysis was performed both theoretically and experimentally in order to validate the results.

4.1 Theoretical stability boundaries

One way to analyze the stability of the system consists of numerically computing the roots of the characteristic equation (which are the poles of the system), evaluating their modulus and testing whether all of them are inside the unit circle [2, 27]. This procedure can be performed by substituting the estimated system dynamics \( G(s) \) within characteristic equation (5). Note that the characteristic equation contains three variables: the delay of the virtual environment \( d \) and the pair \((K, B)\), that is, the impedance of the virtual contact. Therefore, it is possible to check which impedances are stable—or not—for different delays \( d \), and the \( Z \)-width of the haptic system can be plotted by marking the stable impedances in a \((K, B)\)-plane.

Another approach [3] calculates the gain margin of the system from its open-loop transfer function (e.g., via the MATLAB’s `margin` command). Thus, the characteristic equation (5) is rearranged in such way that stiffness \( K \) is isolated as follows:

\[
1 + K F(z) = 0,
\]

where \( F(z) \) is a function of \( d \) and \( B \). Then, the critical stiffness is given by

\[
K_{CR} = \text{Gm}\{F(z)\},
\]
Figure 8: Stable (dots) and unstable impedances (crosses) found by calculating the position of the poles, and stability boundary found by computing the gain margin (line), for system model $G(s)$ with $d = 1$

where $\text{Gm}\{.\}$ refers to the gain margin of the transfer function within brackets. $F(z)$ is not properly the open-loop transfer function of the system, but it can be used in this method as if it were. $F(z)$ can be derived from characteristic equation (5), once it is rearranged as follows:

$$1 + K \frac{G(z)}{z^d + B \frac{1-z^{-1}}{T} G(z)} = 0,$$

(21)

and the critical stiffness is

$$K_{CR} = \text{Gm}\left\{ \frac{G(z)}{z^d + B \frac{1-z^{-1}}{T} G(z)} \right\}.$$

(22)

Defining appropriate value ranges for $d$ and $B$, the stability limit can be easily obtained. This method avoids checking all the poles of the characteristic equation and directly finds the boundary of the $Z$-width of the haptic system.

Fig. 8 shows the results of the two described methods: 1) checking the placements of all the poles of the system, and 2) computing the gain margin of the so-called open-loop transfer function $F(z)$. In both cases, for this example, the time delay was arbitrarily set to $d = 1$. Although the results are equal, the first method requires defining value ranges for $B$ and $K$, while the second one only for $B$. The first method can be improved by programming an algorithm to significantly reduce the number of tested impedances. However, from the point of view of computational cost, using the second method and calculating the gain margin is more effective.

The theoretical stability regions for $d = [1, 2, 4, 8]$, obtained computing the gain margin, are shown in Fig. 9. In this figure it is interesting to note two main issues regarding the shape and size of the $Z$-width, which will be discussed in Section 5 in more detail. The first is that the boundaries present a sharp bend, that is, a point of discontinuity in the slope. The cause of this change is the presence of vibration modes in the model [9], but even knowing the cause, other related questions remain unexplored (e.g., whether this point is always a maximum or the number and accuracy of the modes that should be modeled to observe this effect). The second issue is that the stability
regions enclosed by the boundaries do not become smaller with the delay. There is a wide range of impedances (for example, those with $B = 0.2 \text{ Nm s/rad}$) that are unstable for short delays ($d = 1$ and $d = 2$), but become stable for a longer delay ($d = 4$), and become unstable again for the longest delay that has been analyzed ($d = 8$). This is an unexpected result from the control point of view, which usually relates longer delays with more unstable systems.

Apart from the above, other phenomena that are already well known in the literature can be seen. For small values of virtual damping $B$, the first part of the boundary (up to the discontinuity in the slope) has a recognizable pattern: 1) the starting critical stiffness (for $B = 0$) is always smaller the longer the time delay $d$, 2) the starting slope is also smaller the longer the delay $d$, and 3) the slope tends to fall as $B$ is increased (this last effect is more pronounced the greater the delay). This pattern can be predicted by using a rigid model for the system [3]. Moreover, the stability criterion (23)—derived from the rigid model in [3]—can be used to estimate both the starting critical stiffness and the starting slope of the boundary:

$$K < \frac{b + B}{\tau + \tau + Td} = \frac{\sum \text{Damping}}{\sum \text{Delay}}$$

The critical impedance—that is, the critical stiffness $K_{CR}$ obtained with a particular damping $B$—is also associated with critical oscillation frequency $\omega_{CR}$. This is important information that the $Z$-width plot (Fig. 9) does not show. The critical frequency is provided by the margin command (this is the phase crossover frequency), but it can also be obtained with the method that checks whether the positions of the poles are within the unitary circle. In this last case, the first pair of poles that lay over the unitary circle make an angle $\theta$ with the origin and the real axis that, in radians, is proportional to the oscillation frequency and the sampling period: $\theta = \omega T$ [23].

Whatever method is used, Fig. 10 shows the critical oscillation frequencies of the system for the same time delays as in Fig. 9. It is clear that the point of discontinuity in the $Z$-width plot is associated with a finite jump in the critical oscillation frequency. This also means that the poles that are causing the instability are different on both sides of the discontinuity. Therefore, it is reasonable to assume that the poles causing instability in the first section of the $Z$-width (for smaller virtual damping coefficients) are associated with the rigid component of the model, while the poles causing
instability in the second section of the Z-width (for larger virtual damping coefficients) are associated with the part of $G(s)$ that models the vibration modes of the interface. Section 5 will discuss this hypothesis.

### 4.2 Experimental stability boundaries

The critical impedances were experimentally found by using the relay method \[28\] to verify the theoretical results of the previous subsection. This method consists of a relay feedback $F_r$ that makes the system oscillate around a reference position $X_r$ (Fig. 11). In steady state, the relay force is a square wave, the output position signal $X$ is similar to a sinusoidal wave, and they have opposite phases. The ultimate frequency $\omega_{CR}$ is the oscillation frequency of both signals. Instead of observing it in the time-domain plot, this frequency can be found by locating the maximum in the power spectral density function (both signals can be used for this).

To obtain the critical values for different damping coefficients and delays, the relay force is not directly commanded to the device as defined in \[28\]. The viscous force due to the virtual damping is added to the relay force, and then both of them are delayed (Fig. 11). This way, the relay method obtains empirically the critical gains $K_{CR}$ that were computed in (22).

Table 3 presents the oscillation frequencies and critical gains of the 44 experiments that were
Table 3: Experimental results

<table>
<thead>
<tr>
<th>d</th>
<th>B (N·m·s/rad)</th>
<th>( K_{CR} ) (N·m/rad)</th>
<th>( \omega_{CR} ) (Hz)</th>
<th>d</th>
<th>B (N·m·s/rad)</th>
<th>( K_{CR} ) (N·m/rad)</th>
<th>( \omega_{CR} ) (Hz)</th>
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<td>0.000</td>
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<td>0.144</td>
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Figure 12: Experimental stability limits for \( d = [1, 2, 4, 8] \). These results should be compared with the theoretical ones (Fig. 9).

carried out. The time delay of the impedance feedback was set to 1, 2, 4 and 8 ms, that is, \( d = [1, 2, 4, 8] \), while virtual damping \( B \) was increased to get a complete plot of the Z-width in the \( (K, B) \)-plane. The resulting stability limits are shown in Fig 12, and these experimental boundaries are quite similar to those obtained theoretically (Fig 9). The experimental ultimate frequencies also fit very well with the theoretical ones (Fig 10). These interesting results suggest that: 1) no
matter how complex the experimental stability regions appear to be, having a good model of the system makes it possible to obtain an acceptable prediction of the shape of these regions; and 2) it should be possible to find an explanation for the shape of the stability regions in terms of the physical parameters that were modeled, in particular, the vibration modes. The first statement would require many more tests on different devices in order to be reasonably accepted. This is not the aim of the present study and remains a suggested open issue. With respect to the second statement, the following section attempts to explain some possible causes for the shape and size of the stability regions that were obtained.

5 Discussion

This section presents a deeper analysis of the reasons behind the actual shape and size of the Z-width. Although Bode diagrams proved to be a powerful tool during the characterization process, root loci diagrams are used here for the discussion.

The root locus represents the evolution of the position of all the poles of a system in the complex plane (in this case, the Z-transform complex plane) when a design parameter changes its value from zero to infinity. The system under study has three different parameters that can change in the characteristic equation: virtual stiffness \( K \), virtual damping \( B \), and discrete-time delay \( d \). Among these parameters, only \( K \) and \( B \) vary continuously from zero to infinity to cover all possible impedances, while \( d \) takes only small discrete values—in previous sections, the stability regions were depicted for \( d = [1, 2, 4, 8] \). Therefore, two different root loci are needed for each delay \( d \), one root locus as a function of \( K \) and another one as a function of \( B \).

Fig. 13 shows the procedure for depicting the two root loci. The first root locus is obtained for \( B = 0 \) as a function of \( K \) (vertical arrow in Fig. 13). This diagram identifies the poles that first lie outside the unitary circle (at \( K = K_0 \)) when the virtual stiffness increases in absence of virtual damping. This is the case for the system with elastic virtual contacts, instead of viscoelastic interaction, which is a more general situation. The second root locus is depicted as a function of \( B \) for a particular stiffness coefficient \( K = K_1 \) (horizontal arrow in Fig. 13).

To investigate the possible reasons why the stable region for \( d = 2 \) is smaller than for \( d = 4 \), stiffness \( K_1 \) is set to 20 N·m/rad (see Fig. 13). Next, the key elements of these diagrams are introduced with the two root loci for delay \( d = 2 \). Afterwards, both root loci are depicted for \( d = 4 \) in order to compare them and derive some conclusions.
5.1 System root locus as a function of $K$

In general, for any delay $d$, the characteristic equation of the system where $B = 0$ can be written as follows:

$$1 + K \frac{G(z)}{z^d} = 0$$  \hspace{1cm} (24)

Therefore, in the first place we calculate and plot the root locus of a fictitious open-loop transfer function defined as follows:

$$F_0(z) = \frac{G(z)}{z^d}$$  \hspace{1cm} (25)

It is clear that $F_0(z)$ and system model $G(z)$ have the same zeros. However, $F_0(z)$ contains not only the poles of system model $G(z)$ but also as many poles in the origin of the $Z$-transform complex plane ($z = 0$) as the number of discrete-time delays $d$.

Fig. 14 shows the root locus of $F_0$, that is, without virtual damping ($B = 0$) and with discrete-time delay $d = 2$. Some key elements of system model $G(z)$ and the open-loop transfer function $F_0(z)$ have been identified in this diagram. The poles and the zeros associated with the vibration modes are called using their natural frequencies ($\omega_i$ for the poles and $\nu_i$ for the zeros). The poles associated with the rigid part of the model are identified as “rigid poles”. The two poles at $z = 0$ (overlapping, so only one “×” is visible) are due to the discrete-time delay $d$ and they are named “delay $d$ poles”. Very close to those is another real pole due to the physical delay $\tau$, which is named “delay $\tau$ pole”. The other two unidentified real zeros come from the time discretization. Of these two zeros, the one closer to $z = 0$ arises only if physical delay $\tau$ is approximated by a pole. Otherwise, only the zero outside the unit circle appears.

It is interesting to note that all the open-loop poles and zeros associated with the vibration modes are complex-conjugate; the rest of the poles and zeros are real. Also note that $F_0(z)$ and $G(z)$ are type-I transfer functions, and therefore they have a pole at $z = 1$ (the equivalent point of $s = 0$ in Laplace). Except this one, the rest of poles are within the unit circle (the two “rigid poles” are very close, but only one is on $z = 1$). Thus, this root locus always starts being stable for small virtual stiffness values. However, the existence of a zero outside the unit circle means that sooner or later the discrete-time system becomes unstable, regardless of the number of poles and the time delay. The number of asymptotes is $1 + d$. The branches starting at the complex-conjugate poles associated with the vibration modes do not necessarily end at their corresponding zeros.

Regarding the critical gain of this root locus, which is the critical stiffness in the case of simple elastic force feedback, there are three possible complex-conjugate solutions in which the branches cross over the unit circle. However, the closed-loop poles marked with + are the first ones that lie outside the unit circle, and therefore, set the critical gain $K_0$. We use then an algorithm which increases the gain value and computes the positions of the closed-loop poles for each given gain and checks that the poles are stable (their modulus are less than 1). With this method we obtain a critical gain of $K_0 = 4.89$ N·m/rad, which is coherent with the result in Fig. 13. In our case we make use of MATLAB’s $rlocfind$ command to calculate the position of the closed-loop poles for each gain.

The last interesting items shown in Fig. 14 are the closed-loop poles for gain $K_1 = 20$ N·m/rad, which are marked with dots. These closed-loop poles are the open-loop poles of the second root locus as a function of $B$, that is, the starting points of the branches of this second diagram.

5.2 System root locus as a function of $B$

In general, for any delay $d$, the characteristic equation of the system with $K = K_1$ as a function of $B$ can be written as follows:

$$1 + B \frac{(z - 1)G(z)}{Tz[z^d + K_1G]} = 0$$  \hspace{1cm} (26)
Therefore, we can plot the second root locus using the following transfer function:

\[ F_1(z) = \frac{(z - 1)G(z)}{Tz^d + K_1G(z)} \]  

(27)

Fig. 15 shows the root locus of \( F_1 \) for the gain \( K_1 = 20 \text{ N} \cdot \text{m/rad} \) and discrete-time delay \( d = 2 \). Note that, for the selected gain, the open-loop poles of this diagram (marked with “\( \times \)” in Fig. 15) are the same closed-loop poles marked with dots in Fig. 14 and an extra pole at \( z = 0 \). The open-loop zeros of the root locus are the same open-loop zeros of the previous diagram (circles in both plots) plus an extra zero in \( z = 1 \).

As expected from Fig. 13, by increasing virtual damping \( B \) the system changes from unstable to stable and then to unstable again. These critical transitions are marked with plus signs in Fig. 15 (\( B_0 = 0.041 \text{ N} \cdot \text{m-s/rad} \) for the unstable to stable transition and \( B_1 = 0.117 \text{ N} \cdot \text{m-s/rad} \) for the stable to unstable transition). There are other branches that cross over the unit circle, but the corresponding virtual damping is larger than \( B_1 \), so the system just remains unstable. The critical closed-loop poles for \( B_0 \) and \( B_1 \) are located at \( z_0 = 0.9935 \pm 0.114j \) and \( z_1 = 0.5672 \pm 0.8235j \), respectively. In polar form, these positions are \( z_0 = 1\angle \pm 0.1143 \text{ rad} \) and \( z_1 = 1\angle \pm 0.9776 \text{ rad} \). The arguments of these complex numbers can be directly related to the critical oscillation frequency: \( \theta = \omega_{CR}T \) \[23\]. These resulting oscillation frequencies, 18.18 and 154 Hz, respectively, are consistent with the previous results shown in Fig. 10.

The root locus depicted in Fig. 15 confirms the suggested explanation for the behavior obtained in Fig. 10: the finite jump in the critical oscillation frequencies is due to the existence of different
critical poles on the unit circle. The diagram also confirms that the first part of the stability boundary (for small virtual damping coefficients) is related to the rigid part of the dynamics of the system, while the second part of the stability boundary (the brusque truncation for larger virtual damping coefficients) is related to the vibration modes of the haptic mechanism.

During the fitting process for the dynamics of the system, by using its frequency response, it is hard to predict which poles—or vibration modes—may limit the stability of the system. When analyzing the root loci, it is reasonable to expect that the poles closer to the unit circle (in this case the poles associated with $\omega_2$) are the best candidates for making the system unstable. Thus, to enlarge the $Z$-width of the haptic system under analysis, the designer could try to dampen the second vibration mode, moving its corresponding poles away from the unit circle. However, these kinds of recommendations are difficult to accomplish. Moreover, the stability analysis presented in this work does not provide an easy way to predict the order of magnitude of the critical virtual damping $B_1$ that is associated with the vibration modes. The inclusion of virtual damping is useful for dissipating energy from the system, but its damping coefficient cannot be set arbitrarily large.

5.3 Influence of the discrete-time delay

This subsection shows the two root loci of the system with a different discrete-time delay, $d = 4$, but using the same stiffness coefficient $K_1 = 20 \text{ N}\cdot\text{m}/\text{rad}$. The comparison of these root loci with the previous diagrams is useful for analyzing the influence of the discrete-time delay on stability.

Fig. 16 shows the root locus of $F_0$, that is, without virtual damping ($B = 0$) and with discrete-time delay $d = 4$. As a consequence of this delay $d$, four open-loop poles are located in $z = 0$ and the
The root locus has five asymptotes. As the discrete-time delay increases, the existence of new branches leads to an unpredictable reconfiguration of their paths. In other words, the branches begin and end in different open-loop poles and zeros from one delay to another. For example, for $d = 2$ in Fig. 14, the poles associated with $\omega_2$ take two of the three asymptotes. But now, in Fig. 16 for $d = 4$, the rigid poles and those associated with $\omega_3$ take four of the five asymptotes.

The critical poles depicted with “+” symbols in Fig. 16 correspond to a gain of $K_0 = 2.79 \text{ N-m/rad}$. For $K_1 = 20 \text{ N-m/rad}$, only two poles are located outside the unit circle. These positions can be identified as open-loop poles in the second root locus, which is depicted in Fig. 17.

This second root locus presents a new reconfiguration of the branches with respect the first one (Fig. 16). The branches associated with the rigid poles do not approach any asymptote, but end at the zeros associated with $\nu_1$, as happened for $d = 2$. These branches cross over the unit circle, defining the unstable to stable transition marked with plus signs for virtual damping $B_0 = 0.0914 \text{ N-m-s/rad}$. The branches associated with the poles of the second vibration mode $\omega_2$ again approach a pair of asymptotes. However, their departure from the unit circle does not impose the transition from stable to unstable. The critical poles associated with the branches of the third vibration mode $\omega_3$ arise sooner for a virtual damping $B_1 = 0.2569 \text{ N-m-s/rad}$. And the critical oscillation frequency of these poles, frequency 250.12 Hz, is nearly half the Nyquist frequency. This is also consistent with Fig. 10.

The analysis of the root loci allows a better understanding of the influence of the vibration modes and delay on the stability of the system. On the one hand, poorly damped vibration modes introduce poles near the unit circle, which can make the system become unstable as virtual damping increases. On the other hand, the number of poles—and therefore the number of asymptotes—
increases directly with the discrete-time delay. Both phenomena together can set the stability boundaries to unexpected positions (Fig. 9). In fact, in the system under analysis, it cannot be said that longer delays imply a reduction of the region of stability. However, this curious effect is not erratic or unpredictable. This work also shows that the stability boundaries can be predicted after obtaining a good model of the system.

6 Conclusions

The quality of virtual contact using a haptic device greatly depends on the maximum stiffness achievable by the device. Normally, this limitation is imposed by the control loop where, among other factors, the vibration modes of the device and time delays have a great influence. Increasing virtual damping up to a critical value helps to achieve a higher stable stiffness value. Therefore, finding this critical limit ends up being of high importance in order to achieve more rigid and realistic virtual contact.

A good characterization of the system, one that takes these vibration modes into account, becomes necessary in order to create an accurate theoretical model of the device. The present research used a linear model with five vibration modes. A thorough explanation of the process employed to create this model from experimental data was provided. Once this precise model of the interface was obtained, the $Z$-width of the system—the region of stability—could be successfully estimated.

This paper also extends the results and conclusions from the literature and provides a deeper
discussion about the influence of the vibration modes and the time delay on the Z-width of a haptic system and the reasons behind these complex shapes. The theoretical and experimental results confirm that the vibration modes of the mechanical interface and the time delay significantly influence the Z-width of the system, and impose an upper limit on the virtual damping coefficient that can be implemented.

Even though only the first joint of a PHANToM Premium 1.0 haptic interface has been characterized in this paper, the process described here can be used for other configuration, joints, and interfaces. The detailed description of the process may help researchers obtain satisfactory models for any other mechatronic system.

References


