End-to-End Delay Analysis of the IEEE 802.11e with MMPP Input Traffic

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Outline

• Purpose of the paper
• Transmission Probability in IEEE 802.11e EDCA
• End-to-end Delay
  □ MAC delay distribution
  □ Queuing delay analysis
• Numerical results
• Conclusion
A new mathematical model for the end-to-end delay analysis in IEEE 802.11e WLAN, with MMPP input traffic
We envision the Binary Exponential Backoff algorithm as a function of two coordinates \((x_i, y_i)\):

\[m_i \rightarrow \text{number of backoff stages} \]

\[m_i' \rightarrow \text{number of } CW \text{ sizes} \]

After stage \(m_i\) a packet is dropped.
Transmission Probability (cont.)

• \( P_i(x_i=j \mid Tx) \): a station is at a specific stage \( x_i \), while it is transmitting:

\[
P(x_i = j \mid Tx) = \frac{(1-p_i)p_i^{x_i}}{1-p_i^{m_i+1}}
\]

where \( p_i \) is the collision probability.

• \( P_i(Tx \mid x_i=j) \): a station transmits, given that the station is being at the backoff stage \( x_i=j \):

\[
P(Tx \mid s_i = j) = \frac{1}{1+E[BD_{x,i}]} 
\]

where \( E[BD_{x,i}] \) is the average value of the Backoff Duration at stage \( x_i \).

• In order to find the transmission probability \( \tau_i \) (backoff counter becomes zero), we use the Bayes’ theorem and we sum over the region of \( x_i \in [0,m_i] \).

\[
\tau_i = \frac{1}{\sum_{x=0}^{m_i} P(x_i = j \mid Tx)} \frac{\sum_{x=0}^{m_i} P(x_i = j \mid Tx) P(Tx \mid x_i = j)}{P(Tx \mid x_i = j)}
\]
The interruption of the backoff procedure of a station may occur by two different events:

- The collision of two or more stations, with probability:
  \[ p_i = \left(1 - (1 - \tau)^{n-1}\right) \prod_{z<i} (1 - \tau_z) + \sum_{z>i} \tau_i \prod_{k>z} (1 - \tau_k) \]

- The transmission of an exactly one station with probability:
  \[ p'_i = \binom{n-1}{1} \cdot \tau \cdot (1 - \tau)^{n-2} \prod_{z>i} (1 - \tau_z) \]

The transmission probability of a packet that belongs to any AC:

\[ \tau = \sum_{i=0}^{3} (1 - p_{s,i}(0)) \tau_i \prod_{z>i} (1 - \tau_z) \]

\[ p_{s,i}(0) \] is the probability that the ACi’s queue is empty.
The Probability Generating Function (PGF) of each state \((x, y, i)\):

\[
D_{x,y,i}^{total}(z) = \frac{(1 - p_i) \cdot z^\sigma}{1 - p_i \cdot (p_{s,i}' S(z) + p_{c,i}' C(z))}
\]

\(S(Z) = ZT_{s,i}\) and \(C(Z) = ZT_{c,i}\) are the Z-transforms of the transmission and the collision period, respectively.

\(p_{s,i}'\) and \(p_{c,i}'\) are the probabilities of time-slot interruption of the AC\(i\) of a station, because of a successful transmission of another station, and because of collision of any stations, respectively.

The duration of the backoff procedure at a specific stage \(x\):

\[
D_{x,i}(z) = \begin{cases} 
\frac{1}{CW_{x,i}} \sum_{y=0}^{CW_{x,i}-1} D_{x,y,i}^{total}(z), & 0 \leq x \leq m_i' \\
D_{m_i',i}(z), & m_i' < x \leq m_i
\end{cases}
\]
MAC Delay Analysis (cont.)

The total backoff duration (over all \(x_i, y_j\)):

\[
BD_i(Z) = (1 - p_i) \cdot S_i(Z) \cdot \sum_{x=0}^{m} \left[ (p_i \cdot C_i(Z))^x \cdot \prod_{j=0}^{x} D_{j,i}(Z) \right] + (p_i \cdot C_i(Z))^{m+1} \cdot \prod_{j=0}^{m} D_{j,i}(Z)
\]

transmission delay multiplied by the delay encountered in the previous \((x_a, y_b)\) stages

delay until packet dropping

The mean and the variance of the MAC delay:

\[
E[M_i] = BD'_i(Z) \bigg|_{Z=1}
\]

\[
Var^2(M_i) = BD''(Z) \bigg|_{Z=1} + BD'_i(Z) \bigg|_{Z=1} - \left( BD'_i(Z) \bigg|_{Z=1} \right)^2
\]
To calculate the distribution \( d_{i,k} \) (\( k \): discrete time) of the MAC delay, we use the expression:

\[
BD_i(z) = \sum_{k=0}^{\infty} d_{i,k} z^k
\]

Using the Lattice-Poisson Algorithm (with error bound \( r^{2k} \)):

\[
d_{i,k} = \frac{1}{2kr^k} \sum_{h=1}^{2k} (-1)^h \text{Re} \left( BD_i(re^{i\pi h/k}) \right)
\]

The distribution \( d_{i,k} \) is derived by integration of \( BD_i(Z) \) over a circle with radius \( 0 < r < 1 \).
Each AC follows an MMPP/G/1/K queuing model with buffer capacity K and the wireless channel as the single server.

- The arrival process is characterized by the matrix $\Lambda_i = \text{diag}(\lambda_{i,1}, \lambda_{i,2}, \ldots, \lambda_{i,d})$.
- $Q_i$ is a $d \times d$ transition rate matrix of the process underlying the MMPP.
The one-step transition probability $P_i$ of the Imbedded Markov Chain (IMC) for each AC$i$ queue:

$$P_i = \begin{bmatrix}
U_i A_{i,0} & U_i A_{i,1} & \cdots & U_i A_{i,K-1} & \sum_{n=K-1}^{\infty} U_i A_{i,n} \\
A_{i,0} & A_{i,1} & \cdots & A_{i,K-2} & \sum_{n=K-1}^{\infty} A_{i,n} \\
0 & A_{i,0} & \cdots & A_{i,K-2} & \sum_{n=K-2}^{\infty} A_{i,n} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & A_{i,0} & A_i - A_{i,0}
\end{bmatrix}$$

where $A_{i,n}$ is a $d \times d$ matrix, whose $(k,l)$ element denotes the conditional probability of reaching phase $l$ of the MMPP and having $n$ arrivals at the end of a service time, starting from phase $k$, and $U_i = (A_i - Q)^{-1} \Lambda_i$.

Using $A_i = \sum_{n=0}^{\infty} A_{i,n}$ we calculate $A_{i,n}$ using an iterative procedure.
The steady-state probability distribution at the imbedded time points \( \pi_i = \pi_i P_i \) can be obtained by a matrix sequence \{C_{i,k}\}, such that \( \pi(k) = \pi(0) \cdot C_k \)

\[
C_{i,k+1} = \left[ C_{i,k} - U_i A_{i,k} - \sum_{v=1}^{k} C_{i,v} A_{i,k-v+1} \right] A_{i,0}^{-1}
\]

\[
\pi(0) \cdot \left[ \sum_{v=0}^{K-1} C_v + (I - U_i) A_i (I - A_i + eq_i)^{-1} \right] = q_i \quad C_{i,1} = (I - U_i A_i) A_{i,0}^{-1}
\]

The steady-state probability distribution of the queue’s length

\[
\begin{aligned}
p_{s,i}(0) & = \xi_i \pi_i(0) \left( \Lambda_i - Q_i \right) (E[BD_i])^{-1} \\
p_{s,i}(k) & = \xi_i \left[ \pi_i(k) + \sum_{v=0}^{k-1} \pi_i(v) U_i^{k-v-1} \cdot (U_i - I) \right] \\
p_{s,i}(K) & = q_i - \sum_{n=1}^{K-1} p_{s,i}(n)
\end{aligned}
\]

\( \xi_i = [1 - \pi_i(0) \left( \Lambda_i - Q_i \right)^{-1} (E[BD_i])^{-1} e]^{-1} \)

E[BD_i] is the mean MAC delay of AC_i
• The loss probability for an arbitrary AC $i$ packet is given by

$$P_{B,i} = 1 - \left( E[BD_i] q_i \Lambda_i e \right)^{-1} \left[ 1 + \pi_i(0) \left( \Lambda_i - Q_i \right)^{-1} (E[BD_i])^{-1} e \right]^{-1}$$

• We can calculate the mean queuing delay $E[Q_i]$ by using Little’s law and the effective arrival rate $\lambda_i^* = q_i \Lambda_i e$

$$E[Q_i] = \frac{\lambda_i^*}{1 - P_{B,i}} \sum_{k=1}^{K} k \cdot p_{s,i}(k) \cdot e$$

• Finally, the average end-to-end packet delay $E[D_i]$ is given by

$$E[D_i] = E[BD_i] + E[Q_i]$$
Numerical results

• WLAN of 5 or 10 mobile stations
• Mean packet length of 1024 bytes and Basic bit rate (1Mbps).
• Nodes are equally distributed to every class.
• The matrix $Q_i$ is the same for all ACs

$$Q_i = \begin{pmatrix} -2.5 \cdot 10^{-4} & 2.5 \cdot 10^{-4} \\ 5 \cdot 10^{-6} & -5 \cdot 10^{-6} \end{pmatrix}$$

<table>
<thead>
<tr>
<th>Application</th>
<th>$\text{AC}_3$</th>
<th>$\text{AC}_2$</th>
<th>$\text{AC}_1$</th>
<th>$\text{AC}_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Application</td>
<td>VoIP</td>
<td>Video</td>
<td>Best-Effort</td>
<td>Background</td>
</tr>
<tr>
<td>$CW_{\text{min}}$</td>
<td>7</td>
<td>15</td>
<td>31</td>
<td>31</td>
</tr>
<tr>
<td>$CW_{\text{max}}$</td>
<td>15</td>
<td>31</td>
<td>1023</td>
<td>1023</td>
</tr>
<tr>
<td>AIFS</td>
<td>SIFS+2</td>
<td>SIFS+2</td>
<td>SIFS+3</td>
<td>SIFS+7</td>
</tr>
<tr>
<td>$A_i=\text{diag}$</td>
<td>(0.4,0.1)</td>
<td>(0.5,0.4)</td>
<td>(0.6,0.3)</td>
<td>(0.6,0.4)</td>
</tr>
</tbody>
</table>
The greater number of mobile stations, the greater number of packet collisions; stations choose higher backoff stages, which results in longer delays.

We study the effect of the arrival rate to the end-to-end delay. All ACs have an arrival rate denoted by $\lambda_i = \text{diag}(\lambda_i, 0)$ (IPP).

<table>
<thead>
<tr>
<th>Total Delay (sec)</th>
<th>Arrival Rate $\lambda_i$ (packets/slot)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.05</td>
</tr>
<tr>
<td>$E[D_3]$</td>
<td>0.0023</td>
</tr>
<tr>
<td>$E[D_2]$</td>
<td>0.0018</td>
</tr>
<tr>
<td>$E[D_1]$</td>
<td>0.0015</td>
</tr>
<tr>
<td>$E[D_0]$</td>
<td>0.0013</td>
</tr>
</tbody>
</table>
Conclusion

• The performance of the IEEE 802.11e MAC layer is extensively investigated regarding the analysis of the delay.

• We calculated the transmission probability using elementary conditional probabilities.

• The mean, the variance and the PGF of the MAC delay of each AC are obtained using the Z-transform of the backoff duration.

• Using the characteristics of the MAC delay we calculated the mean queuing delay by considering an MMPP/G/1/K queue.

• In our future work we plan to create a proper simulator in order to evaluate the analytical results that were presented in this work.