MODULATED LAPPED TRANSFORM GENERALIZED SIDELOBE CANCELLER

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ABSTRACT

Continuing improvements in analog and digital technology has made elemental digital beamforming (DBF) for sensor arrays a reality. Weak signal reception enhancement can be achieved through the adaptive suppression of potentially time-varying interference in operational environments. Utilizing all the degrees of freedom (DoF's) of an adaptive elemental DBF array with a large number of elements is often prohibitive in terms of computational complexity and speed of adaptation. Partially adaptive suboptimal architectures which reduce the DoF's with minimal performance degradation are required. One such architecture based on the Generalized Sidelobe Canceller (GSC) with a wavelet-based spatial blocking matrix utilizing minimal length M-band K-regular wavelets has been previously proposed. [1] The work presented here improves upon the wavelet-based GSC in terms of cancellation performance and computational efficiency by using the Modulated Lapped Transform (MLT) to implement the blocking matrix.

1. INTRODUCTION

Radar, sonar, wireless communications, radio astronomy, seismology, and ultrasonics are often required to detect weak signals in the presence of interference. The interference is typically located in an angular direction different from that of the desired signal. Sensor arrays can be utilized to capitalize on this discriminant by forming spatial filters through a process known as "beamforming". These filters pass the desired signal and actually provide Signal-to-Noise Ratio (SNR) enhancement (relative to a single sensor) for detection. At the same time, undesired interference is attenuated. Adaptive beamformers improve upon fixed or data independent beamforming by adapting to the interference environment at hand to reject or "null" interference in a statistically optimum fashion while preserving or enhancing signals impinging upon the array from the desired direction or Line-of-Sight (LOS). By measuring the second order statistics of the received data, adaptation is achieved by optimizing the weighted sum of the received signals from each element.

Sensor arrays typically have receive elements uniformly spaced with a separation on the order of half the wavelength of the highest frequency the array is to receive. This allows the mainbeam of the array to be scanned in angle without grating lobes. For enhanced weak signal detection, high gain arrays with many elements are needed. To minimize system receive noise figure and reduce the requirements on the receiver system, there is a desire to digitize the received signals as close to the sensor elements as possible. Elemental digital beamforming (DBF) arrays actually have a digital receiver behind each receive element. The weighted digital output of these receivers are coherently combined, either deterministically or adaptively, to culminate the receive beam.

Adaptive techniques can update DBF weights continuously with a gradient-descent algorithm, such as the Least Mean Square (LMS) algorithm. The goal of the algorithm is to converge to the optimal weights over time. However, block adaptation techniques utilizing data-domain methods for Sample Matrix Inversion (SMI) are more common in the radar and sonar communities, where the speed of convergence is crucial and adaptive beamforming is done with digital signal processors.

Adaptive weights are usually linearly constrained to prevent suppression of signals from desired directions, especially when there is a lack of knowledge of these signals. Constrained adaptation can be achieved with the Linearly Constrained Minimum Variance (LMCV) beamformer. An alternate LMCV implementation known as the Generalized Sidelobe Canceller (GSC) allows the unconstrained optimization of adaptive weights. This is achieved by the use of a "blocking" matrix to reject signals from the desired LOS from the adaptation process.

As previously mentioned, DBF arrays may contain many elements to achieve a specified gain. The maximum flexibility of an adaptive array results from using all the elements as degrees of freedom (DoF's). This is usually prohibitive for a large number of elements due to the computational task of estimating the second order statistics of the data and the desire to respond to a potentially non-stationary interference environment. As a result, suboptimal partially adaptive arrays are required that reduce the DoF's while maximizing flexibility.

Finite Impulse Response (FIR) filtering techniques used in time-sample digital signal processing is analogous to sensor array beamforming. Discrete time samples being converted to the frequency domain is similar to converting spatial samples from a sensor array into the angular (i.e., spatial) domain. In [1], this analogy was applied to the
design of a partially adaptive beamformer. Specifically, minimal length M-band K-regular wavelet filter banks were proposed for the design of GSC blocking matrices. This results in a reduced DoF adaptation and faster convergence.

The work presented here improves upon the wavelet-based GSC by using M-band Modulated Lapped Transform (MLT) filter banks for GSC blocking matrix design. Convergence and cancellation performance of the MLT-based GSC is examined using both the LMS and SMI algorithms.

2. GENERALIZED SIDELOBE CANCELLER

Assume a N-element uniform linear receive array. Each receive sensor has a matched filter receiver which receives a narrowband signal, which is modulating a given carrier of wavelength $\lambda$ for which the receiver is tuned, and outputs the complex-valued modulating signal at base band. The array is pre-steered to particular angular direction $\theta$ with the appropriate time delays as given in (1). $c$ is the speed of propagation. For narrowband signals, the time-delay can be implemented with a phase shift. This phase shift is described by the steering vector $s(\theta) = \left[ e^{\frac{2\pi d_n \cos(\theta)}{\lambda}} \ldots e^{\frac{2\pi d_n \sin(\theta)}{\lambda}} \right]$. The output of each element receiver, or antenna channel, is digitally sampled.

At a particular sampling instant $k$, the spatial snapshot $x[k]$ is represented by a complex-valued, $N$-element column vector.

\[
\Delta \tau_n = \frac{d_n \cdot \sin(\theta)}{c}
\]

where

\[
d_n = \Delta d \cdot \left[ n - \frac{N+1}{2} \right], \text{ for } n = 1, \ldots, N
\]

The GSC is an implementation of the LCMV beamformer. Illustrated in Fig. 1, the spatial snapshot is $x[k]$ fed to two paths. The top path implements the "quiescent" beamformer (i.e., the data independent beamformer optimized for SNR in a thermal-noise-only environment) under the constraints the array is to provide the desired gain at the desired LOS. The result is the "desired" signal. The bottom path "blocks" the desired signal from $x[k]$ resulting in $u[k]$. Adaptive weights $w_n[k]$ are computed via unconstrained minimization of the error power $E[h_k'] = E[w_n'[k] \cdot x[k] - w_n'[k] \cdot u[k]]$.

The LMCV constraints can include $(S-1)$-order derivative constraints, as shown in (2), to add robustness in the presence of pointing angle (i.e., steering vector) inaccuracies. The resulting quiescent beamformer weights are

\[
w_f = C_s \cdot (C_s^T \cdot C_s)^{-1} \cdot f \in \mathbb{R}^{N \times 1}
\]

\[
C_s = (S-1) \text{ order derivative constraint matrix} \in \mathbb{R}^{N \times S}
\]

\[
c_i = \left[ \begin{array}{c}
1 - n_i \\
2 - n_i \\
\vdots \\
(N - n_i)
\end{array} \right], \text{ for } i = 0, 1, \ldots, S-1
\]

\[
f = \left[ \begin{array}{c}
1 \\
0 \\
\vdots \\
0
\end{array} \right] \text{ desired response vector} \in \mathbb{R}^{N \times 1}
\]

The optimal blocking matrix $B$ is $B = \text{null}(C_s^T)$ such that

\[
w_{a,OPTIMAL} = R^{-1} \cdot p_B \in \mathbb{C}^{(S-1)\times N}
\]

where $R = E[B^H \cdot x[k] \cdot x'[k] \cdot B]$

\[
p_B = E[B^H \cdot x[k] \cdot x'[k] \cdot w_f]
\]

For comparisons to fully-adaptive elemental beamformers, the effective element weighting vector achieved by the GSC is $w = w_f - B \cdot w_f$.

In this paper, two methods of adapting the weight vector $w[k]$ are discussed: LMS and SMI.

3. WAVELET FILTER BANK BLOCKING MATRIX

[1] investigates the use of wavelet filter banks in GSC blocking matrix design for partially adaptive arrays. The benefit of this approach is that the GSC blocking matrix function and a weight reduction transform are combined together. The goal is to reduce the number of weights that must be adapted, thus reducing computational complexity and increasing the rate of convergence, while still providing near-optimal interference suppression. The wavelet-based GSC blocking matrix forms a set of orthogonal beams and allows interference cancellation to take place in beam-space.

The wavelet-based GSC blocking matrix is derived from a $M$-band analysis filter bank consisting of an unitary "scaling" filter and the corresponding $M$-1 "wavelet" filters. Since the array is pre-steered, the desired LOS corresponds to a DC spatial frequency. The blocking matrix must reject this low frequency component. Therefore, the $M$-1 "wavelet" filters are used.

The $M$-band filter bank can be described by the impulse responses of its subband filters. Let each filter be of length $L$, where $L = M \cdot K$ for the minimal length $M$-band $K$-regular filters and $L = 2 \cdot M$ for the MLT filters. The filter bank can be described by (6), where $h_m[n]$ represents the filter coefficients for $n = 0, 1, \ldots, L-1$.

\[
h_m[n] = [h_m[0], h_m[1], \ldots, h_m[L-1]]', \text{ for } m = 0, 1, \ldots, M-1
\]

Only the high pass filters are used to form the matrix $H_m$, which is a \(\left(\frac{N-L}{M}\right) + 1 \times N\) matrix, for $m = 1, 2, \ldots, M-1$, and given in (7). $\left(\frac{N-L}{M}\right)$ must be an integer. $\theta_m$ is a $M \times 1$ null vector. [1]
beamformer in the presence of steering vector errors at the derivative constraint matrix to maintain a robust adaptive vector. This feature is enabled by using a 2-band Haar wavelet analysis filter bank, where $h_1[n] = [1, -1]$, in (7) and (8). This wavelet filter is 1-regular.

The conventional Griffiths-Jim GSC [4] blocking matrix can be realized by using a 2-band Haar wavelet analysis filter bank, where $h_1[n] = [1, -1]$, in (7) and (8). This wavelet filter is 1-regular.

[1] utilized the $M$-band $K$-regular minimal length wavelet filter bank with sufficiently high regularity as a means to block the first ($K$-1)-order Taylor series expansion components of the desired signal's steering vector. This feature is enabled by using a ($K$-1)-order derivative constraint matrix to maintain a robust adaptive beamformer in the presence of steering vector errors at the expense of peak array directivity. To maximize peak array directivity for weak signal detection, derivative expense of peak array directivity. To maximize peak array directivity, analysis filters are given in (9).

This wavelet filter is 1-regular. The MLT filter bank was introduced in [2]. The analysis filters are given in (9).

$$h_u[n] = \frac{2}{M} \cos \left( \frac{\pi}{M} m \frac{1}{2} \right) \cos \left( \frac{\pi}{M} n \frac{1}{2} \right)$$

where $h_1[n] = -\sin \left( \frac{\pi}{2M} \left( n + \frac{1}{2} \right) \right)$ and $m = 0, 1, \ldots, M - 1$ and $n = 0, 1, \ldots, 2 - M - 1$

MLT filter banks were considered in light of the conclusions of [1], which suggest that wavelet filters with large $M$ and small $K$ should be chosen for superior interference cancellation performance. However, computational complexity increases for such choices of $M$ and $K$. MLT filter banks are 1-regular and computationally efficient. Fig 2 shows the spatial response of a 4-band MLT blocking matrix.

For a baseline, an optimal GSC implementation of the LCMV beamformer, where the blocking matrix is $B = \text{null}(C^H)$, and a conventional Griffiths-Jim GSC beamformer were simulated. Both of these configurations have 24x23 blocking matrices. GSC beamformers with blocking matrices based on the 3-band, 2-regular to 5-regular minimal length wavelets, 4-band, 2-regular to 5-regular minimal length wavelets, and the 3-, 4-, and 6-band MLT's were compared. Table 1 provides the dimensionality of the blocking matrices.

**TABLE 1 BLOCKING MATRIX DIMENSIONS**

<table>
<thead>
<tr>
<th>Min Lgth</th>
<th>3-band</th>
<th>4-band</th>
<th>MLT</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-regular</td>
<td>24 x 14</td>
<td>24 x 15</td>
<td>3-band</td>
</tr>
<tr>
<td>3-regular</td>
<td>24 x 12</td>
<td>24 x 12</td>
<td>4-band</td>
</tr>
<tr>
<td>4-regular</td>
<td>24 x 10</td>
<td>24 x 9</td>
<td>6-band</td>
</tr>
<tr>
<td>5-regular</td>
<td>24 x 8</td>
<td>24 x 6</td>
<td>8-band</td>
</tr>
</tbody>
</table>

First, each beamformer was characterized by its optimal adaptive beamforming performance in terms of Signal-to-Interference-plus-Noise Ratio (SINR) loss for a single 30 dB jammer as a function of jammer angle. A comparison of the SINR loss for the LCMV, Griffiths-Jim, minimal length 4-band, 2-regular, and 4-band MLT GSC's is shown in Fig. 3.

The baseline LCMV and Griffiths-Jim beamformers show excellent performance for optimal weights as expected. Wavelet-based beamformers with higher regularity resulted in increased SINR loss near the mainbeam due to the smoother transition band of the spatial blocking filter. MLT beamformers had better interference rejection performance near the mainbeam because of their single regularity. Increased number of subbands had a positive impact on optimal performance.

To evaluate the beamformers’ convergence rate, a scenario with five 20 dB jammers, arranged at angles of -64.7°, -24.8°, -17.4°, 43.3°, and 34.5°, was simulated.

The convergence rate of the beamformers using the LMS algorithm to continuously update adaptive weights is shown in Fig. 4. SINR loss within 3 dB of the optimal value is considered acceptable. The optimal LCMV has an excellent convergence rate. For $M$-band $K$-regular minimal length wavelet-based GSC's, higher regularity adversely affects convergence rate, while more bands improve the rate. [1] The MLT improved the rate approaching that of the fully-adaptive LCMV as the number of bands was increased. This improvement is due to the reduction of the DoF's while reducing the eigenvalue spread of the covariance matrix.

The training sample support required for the SMI algorithm for the various blocking matrix configurations is shown in Fig. 5. The sample support is normalized to the number of array elements for a fair comparison. For the $M$-band $K$-regular minimal length wavelet-based blocking matrices the higher the number of bands, the less...
sample support was required. Also the higher the regularity, the less sample support required. This amount of the reduction depended upon the number of subbands. The MLT blocking matrices all consistently reduced the required sample support by about half that required by the optimal LMCV and Griffiths-Jim GSC beamformers.

5. REFERENCES

