Mutual Information Metrics for Fast Link Adaptation in IEEE 802.11n

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Abstract—We investigate link quality metrics (LQMs) based on mutual information (MI) for fast link adaptation (FLA) in IEEE 802.11n wireless local area network (WLAN) with convolutional coding and higher order modulations. The LQMs are scalar quantities that map the receiver’s post-decoding behaviour in a frequency-selective channel into an equivalent behaviour in an AWGN channel. From this metric the expected packet error rate (PER) can be predicted. Two LQMs are presented that are derived from the SINRs of all sub-carriers and streams: the effective SNR and the effective mutual information. The effective mutual information is the mean mutual information including a correction factor which improves the PER estimation accuracy in various highly frequency-selective channels. The FLA algorithm dynamically selects a modulation and coding scheme (MCS) that maximizes the throughput (TP), while keeping the average PER over time below a target value. Furthermore, we present methods of searching for the most suitable MCS. Practical performance bounds are obtained by means of simulations. We show that both investigated LQMs yield accurate PER estimates and that the resulting TP of the FLA algorithm used in a 2×2 MIMO bit-interleaved coded modulation (BICM) OFDM system that performs channel estimation is only 1.7 dB from the performance bound of the TP.

Index Terms - Adaptive modulation and coding, channel state information, fading, feedback delay, link quality metrics, PER estimation.

I. INTRODUCTION

The upcoming IEEE 802.11n Wireless Local Area Networks (WLAN) [1] standard offers higher throughput (TP) and supports larger range than the current IEEE 802.11a/b/g standards by employing multiple-input and multiple-output (MIMO) techniques with orthogonal frequency division multiplexing (OFDM). The link quality of the wireless fading channel varies over time, so adaptive modulation and coding (AMC), also known as fast link adaptation (FLA), can be employed to increase the TP by making use of good channel conditions when they occur. It was shown in [2] that AMC can increase the TP of a wireless communication system tremendously. In IEEE 802.11n the FLA algorithm can adapt the modulation and coding scheme (MCS) [1]. The key elements in FLA are prediction of the packet error rate (PER) of different candidate MCSs and selection of the MCS maximizing the TP, while keeping the average PER over time below a target value for the current channel conditions. The main difficulty arises due to the unequal SNR levels in the different sub-carriers and spatial streams. The particular SNR levels, induced by specific channel conditions, strongly influences the decoder performance. Since the same question is of fundamental importance in system level simulations, this topic has widely been studied in the literature, considering methods such as uncoded BER/RawBER [3], effective SNR [4], mutual information (MI) [5][6] and PER indicator [7]. To keep complexity low, scalar quantities are sought, from which the link performance can be estimated.

Contributions

We propose an alternative mean MI bit mapping (MMIBM) LQM to be used for FLA and compare it to the MI effective SNR mapping (MIESM) [6]. We compare both measures by means of realistic link-level simulations including channel estimation and feedback delay. Furthermore, we develop a methodology to obtain performance bounds for any FLA algorithm, i.e. an upper bound for TP and a lower bound for PER, and study the tightness of the TP performance bound.

Notation

We denote the k-th sub-carrier index by [k], the set of subsequent realizations of x in time or sub-carrier by {x}, a matrix by A, and its conjugate transpose by AH. The Q×Q identity matrix is written as I_Q. The expectation operator and the variance operator are denoted by E{·} and var {·} respectively. We denote the set of complex numbers by C.

II. SYSTEM MODEL

Fig. 1 shows the simplified block diagram of a MIMO bit-interleaved coded modulation (BICM) OFDM system model which uses N_T transmit antennas and N_R receive antennas. The information bit stream {b} is encoded with a convolutional encoder having generators [133, 171] in octal representation and basic code rate R_c = 1/2. The coded bits may be punctured, provided the code rate needs to be increased to 2/3, 3/4 or 5/6, depending on the used MCS. The interleaved and spatially parsed bits are mapped to complex symbols using Gray mapping. Supported modulations are BPSK, QPSK, 16QAM and 64QAM. The complex symbols {x} are modulated using OFDM with N_SD = 52 data sub-carriers over the mandatory bandwidth of 20 MHz. The input-output relation for the k-th sub-carrier, k = 1, · · · , N_SD, can be represented as [8]

\[ y[k] = H[k|x[k] + n[k]], \] (1)
where \( y[k] \in \mathbb{C}^{N_R} \) is the received symbol vector, \( x[k] \in \mathbb{C}^{N_{SS}} \) is the transmitted symbol vector, \( n[k] \in \mathbb{C}^{N_R} \) is a complex additive white Gaussian noise vector with covariance matrix \( N_0 I_N \). The variable \( N_{SS} \) denotes the number of spatial streams and \( H[k] \in \mathbb{C}^{N_R \times N_{SS}} \) describes the effective channel matrix for tone \( k \), including the channel impulse response, the cyclic delay values and the spatial expansion [1]. Spatial expansion aims at transmitting symbol streams over the \( N_T \) Tx antennas. Notice that some of these streams can be identical.

The elements of the class of candidate MCSs to be selected are used in this paper is

\[
\text{PL} \left( \psi_{\text{MCS}, \text{PL}}(LQM) \right) \approx \frac{\text{PER}_{\text{AWGN}}(LQM)}{\psi_{\text{MCS}, \text{PL}}(LQM)}
\]

This metric captures the effective MMI per symbol. It consists of the sum of two terms:

\[
\text{PL} \left( \psi_{\text{MCS}, \text{PL}}(LQM) \right) = \text{PER}_{\text{FadingChannel}} \left( \{ \text{SINR} \} \right)
\]

In the following four sections, we investigate two LQMs, their associated PER estimation accuracy, the MCS search mechanism, and present performance bounds of the FLA algorithm.

A. Link Quality Metrics

We investigate the following two LQMs which are based on the mean MI (MMI) between the coded bits \( \{ c \} \) at the output of the puncturer in the Tx and the corresponding LLRs \( \{ L(c) \} \) at the output of the de-interleaver in the Rx.

1) Mean Mutual Information per coded Bit Mapping (MMIBM):

This metric captures the effective MMI per symbol.
\[ T_{\text{eff}}^{\text{symbol}} = \frac{1}{N_{\text{SS}}N_{\text{SD}}} \sum_{j=1}^{N_{\text{SS}}} \sum_{k=1}^{N_{\text{SD}}} T_{\text{symbol}}^{\text{symbol}} \left( \text{SINR}_j[k] \right) \]

\[ + \lambda \left[ \frac{1}{N_{\text{SS}}} \sum_{j=1}^{N_{\text{SS}}} \text{var}_k \left\{ T_{\text{symbol}}^{\text{symbol}} \left( \text{SINR}_j[k] \right) \right\} \right] \]

where \( T_{\text{symbol}}^{\text{symbol}}(\text{SINR}_j[k]) \) is the MMI of all bits in a symbol for the \( j \)-th spatial channel and \( k \)-th sub-carrier. This figure depends only on the modulation type and SNR. It is provided in Table I for the modulation schemes used in IEEE 802.11n. Plots of the mapping functions giving the PER-vs.-MMI are depicted in Figure 4.

![Fig. 4. PER of the BICM system operating in the AWGN channel with the mapping function \( \psi_{\text{MCS,PL}} = 1024 \) (MMI) obtained from a quadratic log-linear regression. PER curves corresponding to the same used code rates are bundled by ellipses.](image)

In (4) a correction term is introduced that captures the high dynamics of the per symbol MI measures for each sub-carrier, which occur in frequency selective fading channels. The correction term is based on the variance of the MI across the sub-carriers. The MMI per coded bit can be used without correction term to estimate the PER of MCSs operating in fading channels [5]. However, simulations show that the MMI per coded bit does not accurately estimate the PER of IEEE WLAN fading channels due to the high dynamics of the MIs (or the corresponding \( \{\text{SNR}\} \)). It can be seen in Fig. 5 that the mean square error (MSE) is significantly reduced after including the correction term in (4). The correction factor \( \lambda \) is optimized for each MCS by performing a least-squares fit in the \( \log(\text{PER}) \) domain [11] for in total 45 IEEE channel realizations of both channel models B (RMS delay spread 15 ns) and E (RMS delay spread 100 ns). These two models are considered to cover a larger variation of channel realizations. Ideally this factor should be independent of the type of channel model.

Considering an equivalent Gaussian channel for each sub-carrier after MMSE equalization [12], the MMI per symbol has closed-form solutions for BPSK and QPSK [5]. So, for example \( T_{\text{symbol}}^{\text{symbol}}(\text{BPSK}) = J\left(\sqrt{8 \text{SNR}}\right) \), where the function \( J(\cdot) \) can be found in [13]. For the high-order modulation schemes 16QAM and 64QAM, \( T_{\text{symbol}}^{\text{symbol}} \) does not have a closed-form solution. In [5] the MMI-vs.-SNR relation for 16QAM and 64QAM is found by numerical integration of the LLR histogram and the result is approximated with a linear combination of \( J(\cdot) \) functions with different scaling arguments. A simple and more elegant method consist in computing \( T_{\text{symbol}}^{\text{symbol}} \) using the reliability of the LLRs [14] and then perform a non-linear least-squares fit (cf. [10]) to obtain the coefficients of the approximation model. The resulting approximations are listed in Table I.

![Fig. 5. Estimated PER-vs.-measured PER without and with correction factor for MCS = 0 (BPSK), \( R_c = 1/2 \), \( 1 \times 1 \) MIMO system, channel model B. This metric is an effective SNR \( \text{SNR}_{\text{eff}} \) based on the MI for a given MCS [6]:](image)

\[ \text{SNR}_{\text{eff}} = \gamma \left[ \frac{1}{N_{\text{SS}}N_{\text{SD}}} \sum_{j=1}^{N_{\text{SS}}} \sum_{k=1}^{N_{\text{SD}}} J \left( \frac{\text{SINR}_j[k]}{\gamma} \right) \right]^2 \]

where \( \gamma \) is optimized for each MCS in a similar way as the parameter \( \lambda \) in the MMIBM is optimized. The per subcarrier and stream \( \text{SINR}_j[k] \) is given in (2) and \( J^{-1}(y) \) can be found in [6]. The PER of the instantaneous fading channel realization is estimated by mapping \( \text{SNR}_{\text{eff}} \) to a corresponding PER in an AWGN channel using a mapping function \( \psi_{\text{MCS,PL}}(\text{LQM}) \) corresponding to the waterfall curves in the AWGN channel (cf. [10]).

### B. PER Estimation Accuracy

The PER estimation block can be validated by comparing the measured PER with the estimated PER in AWGN and/or flat fading channel. For these (scaled) AWGN channels, the PER can be exactly determined by \( T_{\text{eff}}^{\text{symbol}} \) (or \( \text{SNR}_{\text{eff}} \)) because (4) and (5) become the MI (or SNR) in the (scaled) AWGN channel. For more detailed information we refer to [10]. It is required that the calculated LQMs shall be quite accurate because of the steepness of the PER curve for the AWGN
channel. An inaccuracy of 1 dB can lead to a PER estimate shifted 1.5 decades away from the true PER. Fig. 6 shows a scatter plot of the pairs of measured and estimated PERs for different realizations of the channel models B and E using the considered LQMs. It can be seen that the MIESM and MMIBM methods exhibit nearly the same PER estimation accuracy.

![Fig. 6. Estimated PER-vs.-measured PER for MCS = 13 (64QAM, $R_c = \frac{5}{6}$), 2 × 2 MIMO, channel model B and MCS = 5 (64QAM, $R_c = \frac{2}{3}$), 1 × 1 MIMO, channel model E. The correction factors $\gamma$ and $\lambda$ are found using simulation measurements from the channel models B and E. One data set is used both for finding the correction factors and computing the PER values depicted in the figure.](image)

C. MCS Search

The selected MCS fed back to the Tx is the MCS with the maximum TP, while keeping the PER below a given threshold. This discrete optimization problem can be formulated as

$$\max_{\text{MCS} \in \Omega} TP(\text{MCS}) \quad \text{s.t.} \quad PER(\text{MCS}) \leq PER_{\text{threshold}}$$

(6)

where the variable $PER_{\text{threshold}}$ is selected such that $PER_{\text{target}}$ can be met on average over time. One way of solving the problem is an exhaustive search across $\Omega$. Another search method with a smaller computational burden is obtained by first ordering the MCSs in decreasing order of their TP and then evaluating the PER of the MCSs one by one in that list. The first MCS that fulfills the PER constraint in (6) is the sought solution. This implies a search set size equal to 16 when $N_T = 2$ [1]. Hence, both search methods are applicable with a fairly low number of MCSs.

D. Performance Bounds

The proposed algorithm provides performance bounds for any FLA algorithm, i.e. an upper bound on the TP and a lower bound on the PER, for a considered communication system and set of candidate MCSs. The two bounds are obtained using the following performance bound algorithm refer to as PBA: within one cycle, in which the channel realization is constant, the PBA individually scans the MCSs in the above defined order, i.e. starting with the MCS with the largest TP. In our case this MCS is 64QAM with $R_c = \frac{5}{6}$. At each step of the PBA the corresponding MCS is used to send a packet across the fixed channel. If the transmission fails, i.e. the packet is received incorrectly, the PBA steps to the next MCS in the ordered list and transmission of another packet is performed with the new corresponding MCS. Notice that the latter MCS has equal or lower TP than the former one. This step continues until the transmitted packet is received successfully. Information on the outcome of the last transmission (packet error event and TP) are then saved. It may happen that the PBA steps to the last MCS in the list. In this case the outcome of the transmission is saved regardless whether the packet transmission is successful or not. The PBA then starts a new cycle with a new channel realization.

The performance bounds are practically not reachable but are tight enough to be of practical use as we will see in Section IV. The proposed performance bounds can be obtained with lower complexity compared to the genie knowledge method [15] because it is not necessary to simulate the PER for all MCSs, SNRs and channel realizations. System settings, such as the channel type, packet length, etc., are fixed to ensure a correct comparison between the performance bounds and the simulation results.

Now we prove that the PBA yields an upper bound on the TP. Let $\hat{m} = f_{\text{FLA}}(\{H\},SNR) \in \Omega$ be the selection function of any FLA algorithm for the channel $\{H\}$ and SNR. The TP of this FLA algorithm can be written as

$$TP_{\text{FLA}}(SNR) = \mathbb{E}_\{H\}\left\{ TP(\hat{m}) (1 - PER(\hat{m}, \{H\},SNR)) \right\}$$

(7)

where the functions $TP(\hat{m})$ and $PER(\hat{m}, \{H\},SNR)$ are respectively the maximum TP of the MCS $\hat{m}$ and the PER for the MCS $\hat{m}$, channel realization $\{H\}$ and SNR value $SNR$. The TP resulting from using the PBA is

$$TP_{\text{PBA}}(SNR, \Omega) = \mathbb{E}_\{H\}\left\{ \sum_{m \in \Omega} TP(m) P_s(m, \Omega, \{H\},SNR) \right\}$$

(8)

where $P_s(m, \Omega, \{H\},SNR)$ denotes the probability that the MCS with index $m$ is selected by the PBA for the MCS set $\Omega$, the channel realization $\{H\}$ and SNR. Although $\{H\}$ is fixed, the random realization of the thermal noise casts successful decoding as a probabilistic process. Consider now the partition $\Omega = \Omega_{\text{high}} \cup \{\hat{m}\} \cup \Omega_{\text{low}}$. As given in Fig. 7, $\Omega_{\text{high}}$ is the set of MCSs with TP higher than the $TP(\hat{m})$ and $\Omega_{\text{low}}$ is the set of MCSs with TP lower than or equal to the $TP(\hat{m})$. Now, the evaluation of the PBA algorithm on the set $\hat{\Omega} = \{\hat{m}\} \cup \Omega_{\text{low}}$ will be

$$TP_{\text{PBA}}(SNR, \Omega) \geq TP_{\text{PBA}}(SNR, \hat{\Omega})$$

$$\quad = \mathbb{E}_\{H\}\left\{ TP(\hat{m}) P_s(\hat{m}, \hat{\Omega}, \{H\},SNR) \right\}$$

$$\quad + \mathbb{E}_\{H\}\left\{ \sum_{m \in \Omega_{\text{low}}} TP(m) P_s(m, \hat{\Omega}, \{H\},SNR) \right\}.$$  

(9)

The first term in (9) is the TP of any FLA algorithm as given in (7) because the first evaluated MCS in the PBA using the set $\hat{\Omega}$ is $\hat{m}$ and then $P_s(\hat{m}, \hat{\Omega}, \{H\},SNR) = (1 - PER(\hat{m}, \{H\},SNR))$. Now,

$$TP_{\text{PBA}}(SNR, \Omega) \geq TP_{\text{PBA}}(SNR, \hat{\Omega}) \geq TP_{\text{FLA}}(SNR)$$

(10)
holds, because \( P_{\gamma}(m, \Omega, \{\mathbf{H}\}, \text{SNR}) \geq 0 \), and \( TP_{\text{PBA}} \) is indeed an upper bound of TP for any FLA algorithm. Equality holds if \( P_{\gamma}(\tilde{m}, \Omega, \{\mathbf{H}\}, \text{SNR}) = 1 \) and \( P_{\gamma}(m, \Omega, \{\mathbf{H}\}, \text{SNR}) = 0, \forall m \in \Omega_{\text{high}} \cup \Omega_{\text{low}} \). This means that for this situation the upper bound on the TP is tight in the sense that the bound can be reached by an FLA algorithm. Fig. 7 illustrates this situation.

Fig. 7. An example setup that leads to a tight upper bound on the TP for any FLA algorithm. If \( \text{PER} \in \{0, 1\} \), then one MCS \( m \) will be selected with probability \( P_{\gamma}(m, \Omega, \{\mathbf{H}\}, \text{SNR}) = 1 \) by the PBA. This corresponds to a sharp transition from \( \text{PER} = 1 \) to \( \text{PER} = 0 \) in the \( \text{PER} \)-vs.-SNR plot for the different MCSs.

However, for a practical system the setup will look as depicted in Fig. 8, where the PER is a smooth decreasing function of the SNR. Then, there will generally be more than one MCSs \( m \) with \( P_{\gamma}(m, \Omega, \{\mathbf{H}\}, \text{SNR}) > 0 \) for a given \( \{\mathbf{H}\} \) and SNR, which corresponds to contributions from the MCSs with larger and smaller TPs relative to the MCS \( \tilde{m} \), and hence the upper bound is not be reachable by any FLA algorithm.

Fig. 8. This setup leads to an upper bound on the TP which is not reachable by any FLA algorithm. The PER is a smooth decreasing function for the different MCS. Then, there will generally be more than one MCS \( m \) with \( P_{\gamma}(m, \Omega, \{\mathbf{H}\}, \text{SNR}) > 0 \) for fixed \( \{\mathbf{H}\} \) and SNR.

So, the setup with the steepest PER-vs.-SNR curve yields the tightest bound. We observe the following:

(i) For stronger codes, the PER curves drop faster, leading to a tighter upper bound. Moreover, the higher the diversity is the tighter the bound.

(ii) If there do not exist many MCSs with a high probability to be selected by the PBA, i.e. with high \( P_{\gamma}(m, \Omega, \{\mathbf{H}\}, \text{SNR}) \), then the bound will be tight. So, the more MCSs to choose from, the less tight the bound is. This can be concluded by the fact that the TP performance bound is reached at very low or high SNR where there is only one MCS likely to be selected; see Fig. 9.

Similarly, it can be shown that the PBA provides a lower bound on the PER for any FLA algorithm.

IV. NUMERICAL RESULTS

For the evaluation of the FLA algorithm we utilize a simulator fully compliant to the IEEE 802.11n standard [1] and the IEEE channel models [9]. We apply channel estimation based on the long training field provided in the preamble. The channel estimates are smoothed before being inserted in (2) for the computation of \( \text{SNR}_j[k] \). The optimized correction factors are found by considering channel realizations from both channel model B and E (see Section III-A1 and Fig. 6), but only channel model B is considered for the simulations.

A. Throughput Results

Simulation results of the TP and PER are reported in Fig. 9. The performance bounds are obtained by using the PBA described in Section III-D. TPs simulation results are also obtained for all candidate MCSs considered individually and the envelopes of these curves are also depicted. The PER constraint envelope is computed by combining PER curves of individual MCSs maintaining a PER less than 1%. Within one cycle of the evaluation of the FLA algorithm, a single data packet with an MCS suggested by the Rx is used for the statistics. Neither feedback delay nor packet errors in the MCS feedback (MFB) request or response is considered. It is observed that the TP and PER curves of the FLA algorithm with MMIBM and MIESM are lying on top of each other. The TP performance curves of both LQMs are at most 1.7 dB shifted to the right of the TP performance bound. Moreover, a high gain can be observed in terms of TP from the adaptive selection of MCS compared to the PER constraint envelope. This gain is most pronounced for channels with low diversity, such as channel model B, which has lower frequency diversity than channel model E. For channels exhibiting high diversity the FLA algorithm approaches the selection of the same (fixed) MCS. This means that, no gain will be observed by using adaptive MCS compared to a fixed MCS selection.

B. MCS Feedback Delay

The performance of FLA is evaluated with respect to MCS feedback (MFB) delay. The MFB delay is the time required for the MCS selected in the FLA algorithm to be effectively utilized at the Rx. We consider only MIESM-based FLA to assess the impact of feedback delay. The result for MMIBM-based FLA is very similar and not shown due to the lack of space. Fig. 10 shows the TP and PER-vs.-MFB delay. The TP loss rate and the PER increase as the MFB delay increases for both SNR 30 dB and 20 dB. Furthermore, the degradation of PER to a non-acceptable level, like 5% (depending on the QoS requirements), around 4.5 – 5.5 ms MFB delay happens faster than the loss in TP for the same MFB delay. In general, it is hard to know the MFB delay in advance since it is unconstrained. So, an outer loop on the top of the FLA algorithm based on long-term PER statistics can be added to maintain the PER target. This can be accomplished by adapting the PER selection threshold in the MCS search without affecting the fast concept of FLA.
Fig. 9. TP and PER-vs.-SNR with channel estimation at the Rx: Channel model B with bandwidth 20 MHz.

Fig. 10. TP and PER-vs.-MFB delay for MIESM with channel estimation at the Rx: Channel model B with bandwidth 20 MHz.

V. CONCLUSION

We have considered two mutual information metrics, i.e. the Mean Mutual Information per coded Bit Mapping (MMIBM) and the Mutual Information Effective SNR Mapping (MIESM), for the purpose of fast link adaptation (FLA) in an IEEE 802.11n communication system operating in frequency selective multiple-input and multiple-output MIMO channels. Our findings indicate that introducing a (metric-specific) correction factor for MMIBM considerably improves the accuracy of the packet error rate (PER) estimation. The correction factor depends on the modulation and coding scheme (MCS), but are valid for a wide class of channel models. The PER estimators based on the modified MMIBM and MIESM perform very closely. Furthermore, we proposed an algorithm to obtain performance bounds by simulations. The throughput (TP) that results from using the FLA algorithm is only 1.7 dB from the TP performance bound. It was discussed that adaptive schemes are more beneficial than fixed MCS schemes for channels with low diversity. The dependency on feedback delay was also investigated. We observed that the PER increases rapidly to an unacceptable level, which can be counteracted by adjusting the PER threshold using an outer loop.

REFERENCES