FORCE OPTIMIZATION OF GRASPING BY ROBOTIC HANDS*

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Abstract: It is important for robotic hands to obtain optimal grasping performance in the meanwhile balancing external forces and maintaining grasp stability. The problem of force optimization of grasping is solved in the space of joint torques. A measure of grasping performance is presented to protect joint actuators from working in heavy payloads. The joint torques are calculated for the optimal performance under the frictional constraints and the physical limits of motor outputs. By formulating the grasping forces into the explicit function of joint torques, the frictional constraints imposed on the grasping forces are transformed into the constraints on joint torques. Without further simplification, the nonlinear frictional constraints can be simply handled in the process of optimization. Two numerical examples demonstrate the simplicity and effectiveness of the approach.

Key words: Force optimization  Grasping force  Robot hand

0 INTRODUCTION

Dexterous manipulation using multi-fingered robotic hands has been an active area of research in the past years. A.M. Okamura, N. Smaby and M. R. Cutkosky made a rather comprehensive overview about the topic[1]. One of the key issues on grasp of robotic hands is to obtain optimal grasping quality. When dexterous robotic hands manipulate and grasp objects, there are usually multiple choices for contact locations. In addition, for each choice of contact locations, there are many solutions for applying grasping forces by controlling joint torques of fingers. To achieve stable grasps, grasping forces are required to balance the external wrench applied on the object while satisfying friction constraints at all contact points. When optimizing grasping forces, one of the main difficulties is the nonlinear frictional constraint. Some researchers solve this problem based on a linear programming formulation by means of linearizing the friction constraint[2,3]. This results in conservative solutions. Y. Nakamura, K. Nagai and T. Yoshikawa proposed a nonlinear programming approach for the optimization of grasping forces[4]. It can only allow off-line solution with current computing resources. Buss, Hashimoto and Moore made an important observation that the nonlinear friction constraint is equivalent to positive definiteness of certain symmetric matrices. They developed an algorithm of optimization efficient enough for real time computation[5]. Li Han, J.C.Trinkle, and Zexiang Li further cast the friction constraint into linear matrix inequalities and formulate the problem of optimization as a set of convex involving the linear matrix inequalities[6].

All of the proposed optimization methods have in common that the optimal variables are grasping forces. This results in two problems: ①The friction constraints are complicated to solve;②The dimension of the optimization problem increases with the number of contact points. In the case when a finger contacts the object in multiple locations, the complexity of the optimization increases a lot. The number of contact points may change with different tasks of grasping, but the number of joints is fixed for a given hand. Therefore, we solve the optimization problem in the space of joint torques. The dimension of optimization problem depends, then, on the number of joints instead of the number of contact points. However, we need to transform the frictional constraints imposed on the grasping forces into the constraints on joint torques. This is achieved by using the results from our previous work, in which the grasping forces are formulated into an explicit function of the joint torques and the external wrench[7,8]. For any given configuration of grasp, the output torque of each joint actuator may be different. It is desired to accomplish the task of grasping with smaller torques of the joint actuators. In this paper, we define an objective function to make the actuators working at as smaller torques as possible.

1 PROBLEM OF OPTIMIZATION

Consider an m-fingered robotic hand grasping and manipulating an object. If the hand-object system is in static or the manipulating velocity is slow enough, we can handle the system in static domain. That means the contact forces provided by the robotic fingers must balance the external wrench exerted on the object to achieve grasp stability. This can be formulated as the following equilibrium equation

\[ w = -Gt, \]  

where  
\[ t \] —The vector of contact forces exerted on the object,  
\[ t = \begin{bmatrix} f^T \ m^T \end{bmatrix}^T \in \mathbb{R}^n \]  
\[ w \] —The external wrench applied on the object,  
\[ w \in \mathbb{R}^6 \]
\[ G \] —The grasp matrix,  
\[ G \in \mathbb{R}^{m \times n} \]  
\[ n \] —The dimension of the space of contact
forces. Eq. (1) can be considered as constraints imposed on the contact forces to make a grasp stable. Here we call it balance constraint. In general, the dimension of the space of contact forces is greater than that of external wrench \( w \), which is six in three dimensional case. Therefore there is indefinite number of solutions for the contact force vector \( t \) satisfying Eq. (1). It is necessary to search for the optimal solutions of grasping forces. If the contact forces are taken as optimal variables, the dimension of the optimization problem increases with the number of contact points. In the case when a finger contacts the object in multiple locations, the complexity of the optimization increases greatly. We note that the number of contact points may change with different tasks of grasping, but the number of joints is fixed for a given hand. Moreover, it is the joint torques of the fingers that provide the required contact forces. Therefore, we solve the optimization problem in the space of joint torques. However, we need to transform the balance constraint imposed on the grasping forces into the constraints on the joint torques. To do this the relationship between \( \tau \) and \( t \) must be developed. Considering the balance of robotic fingers, the following equation must be satisfied,

\[
\tau = J^T t
\]  

(2)

where \( \tau \) — The vector of joint torques, \( \tau \in \mathbb{R}^m \),

\( J \) — The Jacobian matrix of the hand,

\( J \in \mathbb{R}^{6 \times m} \),

\( m \) — The number of joints.

Combining Eqs. (1) and (2), we can formulate the grasping forces into an explicit function of the joint torque and the external wrench. The approach was developed in Refs.[7,8], and is summarized in the following.

Combining Eqs. (1) and (2), we obtain

\[
At = b
\]  

(3)

where

\[
A = \begin{bmatrix} -G \\ J^T \end{bmatrix} \in \mathbb{R}^{(6+n) \times m} \quad b = \begin{bmatrix} w \\ \tau \end{bmatrix}
\]

The vector of contact forces \( t \) can be formulated as

\[
t = N_B x_B + N_C x_C + N_B x_B + N_A x_A
\]  

(4)

where \( N_A, N_B, N_C \) and \( N_D \) are the matrices whose columns span the null spaces of the matrices \( A, B, C \) and \( D \), respectively. And,

\[
B = \begin{bmatrix} G \\ N_A^T \end{bmatrix} \quad C = \begin{bmatrix} J_T^T \\ N_C^T \end{bmatrix} \quad D = \begin{bmatrix} N_B^T \\ N_D^T \end{bmatrix}
\]

\( x_A, x_B, x_C, x_D \) are free vectors, whose dimensions are \( n - \text{rank}(A) \), \( n - \text{rank}(B) \), \( n - \text{rank}(C) \), and \( n - \text{rank}(D) \), respectively. Substituting Eq. (4) into Eq. (3), the free vectors \( x_B, x_C \), and \( x_D \) can be determined. Then, substituting them back into Eq. (4), we obtain

\[
t = M_1 w + M_2 \tau + N_A x_A
\]

where \( M_1 \in \mathbb{R}^{n \times 6} \), \( M_2 \in \mathbb{R}^{n \times m} \).

For a rigid hand-object system, we have \( x_A = 0 \). Thus, the vector of contact forces \( t \) can be expressed into the following explicit function of the joint torque vector \( \tau \)

\[
t = M_1 w + M_2 \tau
\]  

(5)

In addition to the above constraint, the joint torque vector is constrained by the physical limits of motor outputs. In general, the payload on each joint actuator is different and the maximum output of each joint actuator is also different. Assume that the \( i \)th joint torque \( \tau_i \) is limited by the upper bound constant \( \tau_{i\text{max}}, i = 1, \Lambda, m \). We use the ratio of the actual joint torque to the upper bound constant to measure the grasp quality,

\[
k_i = \frac{\tau_i}{\tau_{i\text{max}}}, i = 1, 2, \Lambda, m
\]  

(6)

In order to protect the joint actuators from working in a heavy load, the value of \( \max(k_i) \) is expected to be minimal. Therefore the objective function of the optimization problem is defined as

\[
\min(\max(k_i))
\]  

(7)

The vector of joint torques must satisfy the constraints

\[
\tau - \tau_{\text{min}} \geq 0
\]

\[
-\tau + \tau_{\text{max}} \geq 0
\]  

(8)

where
\[ \boldsymbol{\tau} = \begin{bmatrix} \tau_1 \\ \mathbf{M} \\ \tau_{m1} \\ \mathbf{M} \\ \tau_{mn1} \end{bmatrix}, \quad \boldsymbol{\tau}_{\text{min}} = \begin{bmatrix} \tau_{\text{imin}} \\ \mathbf{M} \\ \tau_{\text{min}} \\ \mathbf{M} \end{bmatrix}, \quad \boldsymbol{\tau}_{\text{max}} = \begin{bmatrix} \tau_{\text{imax}} \\ \mathbf{M} \\ \tau_{\text{max}} \\ \mathbf{M} \end{bmatrix} \]

2 FORMULATION OF FRICTIONAL CONSTRAINTS

In addition to the balance constraints and the constraints of joint torque limit, the joint torques are required to provide contact forces that satisfy the friction constraints. For our purpose, we must transform the frictional constraints imposed on the contact forces into the constraints on the joint torques.

We focus on two contact models that are frequently used: hard-finger and soft-finger contacts. Fig.1 shows the contact force, \[ \mathbf{t}_i = [f_{ix}, f_{iy}, f_{iz}, m_{iz}]^T, \]
for soft-finger contact model at the \( i \)th contact point. \( f_{iz} \) is the normal force pointed toward the object, and \( f_{ix} \) and \( f_{iy} \) are the tangential forces, and \( m_{iz} \) is the torsion moment about the contact normal. For the hard-finger contact, the friction model is assumed to conform to Coulomb’s law

\[ \sqrt{f_{ix}^2 + f_{iy}^2} \leq \mu_i f_{iz}, \quad f_{iz} > 0 \]  

where \( \mu_i \) denotes the friction coefficient of the \( i \)th contact point. For soft-finger contact, experimental results have shown the following relation\[\text{[9]}\]

\[ \frac{1}{\mu_i} \sqrt{f_{ix}^2 + f_{iy}^2} + \frac{1}{\mu_{ii}} |m_{iz}| \leq f_{ix}, f_{iz} > 0 \]

where \( \mu_{ii} \) is the proportional constant between the torsion and shear limits.

In order to apply Eq. (5), we use the vector of contact forces to express Eqs. (9) and (10). The contact force \( \mathbf{t}_i \) lies in the friction cone when it satisfies the friction constraints. Assume the radius of the bottom circle of the friction cone is \( r \) (Fig.1). As shown in Fig.2, we write

\[ \mathbf{r} = \mu_i f_{iz} \]  

According to Eq. (9), we obtain

\[ -r \cos \theta_i \leq f_{ix} \leq r \cos \theta_i \]
\[ -r \sin \theta_i \leq f_{iy} \leq r \sin \theta_i \]
\[ f_{iz} > 0 \]

where

\[ \theta_i = \arctg(f_{iy}/f_{ix}) \]

Assume

\[ \mu_{ix} = \mu_i \cos \theta_i, \mu_{iy} = \mu_i \sin \theta_i \]

Substituting Eqs. (11) and (14) into Eq.(12), there exits

\[ \mathbf{S}_i \mathbf{t}_i \geq 0 \]

where

\[ \mathbf{S}_i = \begin{bmatrix} -1 & 0 & \mu_{ix} \\ 1 & 0 & \mu_{ix} \\ 0 & -1 & \mu_{iy} \\ 0 & 1 & \mu_{iy} \end{bmatrix} \cdot \mathbf{t}_i = \begin{bmatrix} f_{ix} \\ f_{iy} \end{bmatrix} \]

\[ i = 1,2, \Lambda ,n \]

Let

\[ \mathbf{S} = \text{Blockdiag}(\mathbf{S}_i), \]

We have

\[ \mathbf{S}_i \geq 0 \]
where \( t = \begin{bmatrix} t_1^T & t_2^T & \Lambda & t_n^T \end{bmatrix}^T \). Substituting Eq.(5) into Eq.(18), the friction constraint of the hard-finger contact can be transformed into the following

\[
SM_2 \tau + SM_1 \omega \geq 0 \quad (19)
\]

For soft-finger contact, we rewrite Eq.(10) as the following

\[
\sqrt{f_i^2 + f_j^2} \leq \mu_i f_i - \mu_j m_i \quad f_i > 0 \quad (20)
\]

Note that the right side of the inequality is positive. This means \( |m_i| \leq \mu_i f_i \). Therefore

\[
\mu_i f_i - m_i > 0 \\
\mu_i f_i + m_i > 0
\]

Define

\[
r = \mu_i f_i + \alpha \frac{\mu_i}{\mu_i} m_i
\]

where \( \alpha = -1 \) when \( m_i \geq 0 \), otherwise \( \alpha = 1 \). It is noted that Eq. (20) has the same form as Eq. (9). Let

\[
\mu_{ix} = \alpha \frac{\mu_i}{\mu_i} \cos \theta_i, \mu_{iy} = \alpha \frac{\mu_i}{\mu_i} \sin \theta_i
\]

Substituting Eqs.(13), (14), (22) and (23) into Eq.(12), and combining with Eq.(21), we can write the same equation as Eq.(15). However, the matrix \( S_i \) and the components of the contact force vector \( t_i \) are different. They are given as

\[
S_1 = \begin{bmatrix}
-1 & 0 & \mu_{ix} & \mu_{ix} \\
1 & 0 & \mu_{ix} & \mu_{ix} \\
0 & -1 & \mu_{iy} & \mu_{iy} \\
0 & 1 & \mu_{iy} & \mu_{iy} \\
0 & 0 & 1 & 0 \\
0 & 0 & \mu_{ii} & -1 \\
0 & 0 & \mu_{ii} & 1
\end{bmatrix}, t_1 = \begin{bmatrix} f_{ix} \\ f_{iy} \\ f_{iz} \\ m_{iz} \end{bmatrix}
\]

In summary, the optimization problem can be stated as

\[
\begin{aligned}
\min & (\max(k_i)) \\
\text{subjected to:} & \begin{cases}
SM_2 \tau + SM_1 \omega \geq 0 \\
\tau - \tau_{\text{min}} \geq 0 \\
-\tau + \tau_{\text{max}} \geq 0
\end{cases}
\end{aligned}
\]

3 ALGORITHM OF OPTIMIZATION

The algorithm of the optimization is as follows.

Step 1. Input the joint torque limits \( \tau_{\text{min}} \) and \( \tau_{\text{max}} \), and the valid initial joint torque vector \( \tau_0 \).

Step 2. Input the grasp matrix \( G \), Jacobian matrix \( J \), and the external wrench \( \omega \). Calculate the matrices \( M_1 \) and \( M_2 \).

Step 3. Input friction coefficient \( \mu_i \) and the proportional constant \( \mu_0 \).

Step 4. Calculate the contact force vector \( \tau \) using Eq.(5).

Step 5. Construct the matrices \( S_i \) and \( S \) using Eqs. (13), (14), (16), and (17) or (13), (14), (21) and (17).

Step 6. Calculate a new value of the joint torque vector \( \tau \) using appropriate numerical method.

Step 7. If the desired optimal precision is satisfied, the present \( \tau \) is the optimal solution. Otherwise, repeat from step 4 until the optimal precision is reached.

4 NUMERICFAL EXAMPLES

The method of hybrid penalty function is used in our numerical examples.

Example 1:

In summary, the optimization problem can be stated as

\[
\begin{aligned}
\min & (\max(k_i)) \\
\text{subjected to:} & \begin{cases}
SM_2 \tau + SM_1 \omega \geq 0 \\
\tau - \tau_{\text{min}} \geq 0 \\
-\tau + \tau_{\text{max}} \geq 0
\end{cases}
\end{aligned}
\]

Fig.3 shows a two-fingered grasp with a hard-finger contact model. There are three joints. At the illustrated configuration,

\[
G = \begin{bmatrix}
-1 & 0 & 1 & 0 & 1 & 0 \\
0 & -1 & 0 & 1 & 0 & -1 \\
0 & 1 & -1 & 0 & 0 & -1
\end{bmatrix}
\]
\[
\begin{bmatrix}
1 & 0 & -2 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
-2 & 0 & 0 \\
-0.5 & -0.5 & 0.5 \\
-1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0.5 & -0.5 & -0.5
\end{bmatrix}
\]

\[
\begin{bmatrix}
-1 & 0 & -2 \\
-0.5 & 0.5 & -0.5 \\
-1 & 0 & -1 \\
0 & 1 & 0 \\
0 & 0 & -1 \\
0.5 & 0.5 & 0.5
\end{bmatrix}
\]

Assume the friction coefficient is \( \mu_i = 0.5 \), \( i = 1, 2, 3 \) and the joint torque limits are \( \tau_{\text{limit}} = [\pm 5 \ \pm 3 \ \pm 5]^T \). For the external wrench, \( \mathbf{w} = [0 \ -1 \ 0]^T \), a valid initial joint torque is \( \tau_0 = [3 \ 2 \ -3]^T \). The corresponding cost index is 0.6. The final optimal solution is \( \tau_{\text{opt}} = [1 \ 0 \ -1]^T \). The maximal cost index is 0.2.

Example 2:

Fig. 4 shows a two-armed grasp with a soft-finger contact model. Each arm has two joints.

\[
\begin{bmatrix}
5 & 0 & -1 & 0 & 0 & 0 \\
0 & -1 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & -1 & 0 & -1 & 0 \\
0 & -1 & 0 & 0 & 1 & 0 
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{1}{6} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{3} & 0 & 0 & 0 & -\frac{1}{3} \\
0 & 0 & 0 & 1 & 0 & -\frac{1}{2} \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{5}{12} & 0 & -\frac{1}{12} & 0 \\
0 & -\frac{1}{3} & 0 & 0 \\
0 & 0 & 0 & 0 \\
\frac{1}{12} & 0 & -\frac{5}{12} & 0 \\
0 & 0 & 0 & \frac{1}{3} \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

At the illustrated configuration,
external wrench, $w = \begin{bmatrix} 0 & -1 & 1 & 0.5 & 0 \end{bmatrix}^T$, a valid initial joint torque is $\tau_0 = \begin{bmatrix} 9.5 & 2.5 & -9.5 & -1.1 \end{bmatrix}^T$. The corresponding cost index is 0.95. The final optimal solution is $\tau_{opt} = \begin{bmatrix} 5.4 & 1.0 & -5.4 & -0.29 \end{bmatrix}^T$. The maximal cost index is 0.54.

5 CONCLUSIONS

The problem of grasping force optimization for dexterous hands is solved in the space of joint torques. The grasping forces satisfying the equilibrium equations are formulated into a linear function of the joint torques and the external wrench. The frictional constraints on the grasping forces are transformed into the constraints on the joint torques. The approach is demonstrated by minimizing the ratio of the actual joint torques to the maximum output of the motors. The optimal solution means that the hand can perform the given task of grasping with the smallest joint torques. Therefore, it allows the saving of energy and the increased payload of the hand.

Because the optimal variables are the joint torques, the approach avoids the change of the problem dimension with the change of contact points. This is an advantage over the most existing approaches in which the optimal variables are grasping forces. The nonlinear frictional constraints are handled in the optimization process without using any approximation. This makes a full use of friction forces at the contacts. Furthermore, the approach provides a new way to implement the force control of hands in which the desired joint torques are commanded to obtain the optimal grasping.

References


Biography

Li jiting (1967—), she received the B.Sc. degree and M.S. degree in mechanical engineering from Dalian University of Science and Technology in 1989 and 1992, respectively. She works in Beijing University of Aeronautics and Astronautics. Her research interests include the theory of mechanisms, robot kinematics and dynamics, and dexterous manipulation.

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机器人灵巧手的抓持力优化

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摘要：对于机器人灵巧手而言，在平衡外力及保证稳定抓持的同时希望获得最优的抓持性能。本文在关节空间解决抓持力的优化问题，提出了一个抓持性能评价指标以使各关节驱动器在尽可能小的载荷下工作。所计算的关节力矩同时满足摩擦约束及物理极限的约束。通过将抓持力推导为关节力矩的显函数，抓持力所受到的摩擦约束转化为关节力矩的约束，因而优化过程中非线性的摩擦约束无需进一步简化。通过两个数值算例说明该方法的简洁及有效。

叙词：力优化，抓持力，机器人，灵巧手

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