Resource Flexibility and Capital Structure

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Abstract

This paper examines how the optimal investment in a portfolio of flexible and nonflexible capacity is affected by the firm’s capital structure and, vice versa, how a firm’s resource flexibility affects its optimal capital structure. We consider a levered two-product firm that invests in the optimal mix of product-flexible and product-dedicated capacity in the presence of demand uncertainty. Taking the debt level as given, financial leverage leads to underinvestment and substitution of flexible capacity with nonflexible capacity due to the shareholder-debtholder agency conflict. However, when the firm chooses capital structure optimally, trading off the tax benefit and the agency cost associated with debt, the relation between flexibility and leverage changes. Namely, full resource flexibility eliminates the asset substitution as well as the underinvestment problem, induces the first-best capacity investment, and increases the optimal leverage. This is in sharp contrast to the common wisdom that real flexibility exacerbates the agency cost of leverage by allowing shareholders to shift investment or production toward risky ventures. The key managerial implication is that firms investing in product-flexible capacity can expect more favorable credit terms and, therefore, should demand more debt not only because flexibility mitigates risk but also because it mitigates the agency cost of debt.

Keywords: flexibility, capacity, leverage, capital structure, agency, asset substitution, underinvestment

1 Introduction

This paper bridges the operations and corporate finance literatures by examining the relation between a firm’s resource flexibility, also known as product or mix flexibility, and its capital structure. We consider a two-product levered firm that invests in product-flexible and product-dedicated capacity in the presence of product demand uncertainty. When choosing the optimal level of each

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type of capacity, the firm has to trade off the cost of excess capacity against the opportunity cost of lost sales. While flexible capacity is more expensive than nonflexible capacity, it is also more valuable because it can be used to produce either product depending on the realized demand. A typical example would be a car manufacturer that uses a combination of debt and equity financing to build several assembly lines, some of which will be able to produce multiple car platforms.

We first examine how the optimal capacity investment depends on an exogenously given debt level. Financial leverage distorts the firm’s capacity investment because of the agency conflict between shareholders and debtholders. Namely, the shareholders of a levered firm ignore the capacity investment payoff that accrues to debtholders in the event of bankruptcy, and therefore, the capacity investment maximizing shareholder value does not maximize total firm value.

This has two implications. First, we show that leverage reduces the total capacity investment. This is because a levered firm ignores the marginal revenue of capacity received by debtholders in bankruptcy states, very much in the spirit of the underinvestment theory of Myers (1977). Second, we show that leverage increases the optimal level of nonflexible capacity and decreases the optimal level of flexible capacity. Because a levered firm ignores the magnitude of loss in bankruptcy states, it undervalues the operational hedge provided by resource flexibility. As a result, a levered firm substitutes flexible capacity with the more risky nonflexible capacity consistent with the asset substitution theory of Jensen and Meckling (1976). This means that with an exogenously given debt level, a negative relation between financial leverage and resource flexibility is expected.

With this understanding of how debt affects the optimal capacity investment, we endogenize the capital structure decision. Namely, we assume that prior to investing in capacity, the firm issues the optimal amount of fairly-priced debt, trading off the tax benefit and the agency cost associated with it. Assuming a flat corporate profit tax rate, the benefit of debt stems from the tax shield provided by the risky interest. The agency cost of debt stems from the fact that debtholders anticipate the firm’s distorted investment strategy and price the debt accordingly. The loss of firm value resulting from the shareholder-debtholder agency conflict is thus ultimately borne by shareholders through the higher cost of external financing.

We show that when it is optimal to invest in some nonflexible capacity, the optimal capital structure generally involves debt as well as equity due to the trade-off between the tax benefit and the agency cost of debt. When, however, the market and technology parameters justify investing exclusively in flexible capacity, the agency cost of debt disappears and it is optimal to use only debt financing. Whereas it is not surprising that a firm investing in a single flexible resource cannot en-

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3See e.g., Goyal, Netessine and Randall (2007) and the examples therein.
gage in asset substitution, it is less obvious that resource flexibility eliminates the underinvestment problem as well.

The intuition is as follows. The shareholder-debtholder agency conflict arises because shareholders ignore the marginal investment payoff in default states. This conflict exists only when there is a positive probability that the firm will default while some of its resources have a positive marginal revenue accruing to debtholders. This is the case when the firm defaults due to low demand for one product at the same time as the demand for the other product is high enough so that some of the firm’s resources are fully utilized. When it is optimal to invest exclusively in flexible capacity, this is not an issue. Whenever a fully flexible firm defaults, it is because its single flexible resource is underutilized and thus has zero marginal revenue from the shareholders’ as well as from the debtholders’ points of view. Therefore, under full resource flexibility, the shareholder-debtholder agency conflict does not exist, the firm chooses the first-best capacity level and maximizes its value by relying entirely on debt financing.

Although the operations literature on the optimal investment in flexible capacity is extensive (e.g., Fine and Freund, 1990; Van Mieghem, 1998; Chod and Rudi, 2005; and Goyal and Netessine, 2007), it has been implicitly assuming that firms are financed entirely by equity. An exception is Boyabatli and Toktay (2006 and 2007), who study a firm that chooses whether to invest in a flexible or nonflexible technology while relying on external financing, but they do not consider the shareholder-debtholder agency conflict. There are several studies that model inventory decisions in a framework similar to ours and do consider external financing (e.g., Xu and Birge, 2005; and Kouvelis and Zhao, 2010) but they don’t address the issue of resource flexibility. Our model of the optimal investment in a portfolio of flexible and nonflexible capacity builds on the seminal work of Van Mieghem (1998) but allows the capacity investment to be financed by equity as well as debt.

Our contribution to the flexibility literature is twofold. First, we show how the optimal investment in flexible and nonflexible capacity depends on financial leverage. Second, whereas the operations literature has long recognized the value of flexibility in mitigating the mismatch between supply and demand, our paper is the first to identify an additional benefit of resource flexibility. Namely, we demonstrate that full resource flexibility also eliminates the shareholder-debtholder agency conflict which generally prevents a levered firm from choosing the first-best capacity in-

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4Boyabatli and Toktay (2006) assume the cost of external financing to be exogenous, and focus on the interaction between flexibility and hedging. Boyabatli and Toktay (2007) relax the assumption of exogenous financing cost but they assume that the firm’s equity is given, whereas we allow the firm to raise the optimal amount of both equity and debt.
vestment. The key managerial implication is that firms investing in product-flexible capacity can expect more favorable credit terms and, therefore, should demand more debt not only because flexibility mitigates risk but also because it mitigates the agency cost of debt.

We also contribute to the corporate finance literature that studies interactions between a firm’s capital structure and real investment. Although this literature has recognized real flexibility, or real options, as one of the determinants of capital structure (e.g., Myers, 1977; Leland, 1998; and MacKay, 2003), it has not addressed resource flexibility, arguably the most extensively studied form of real flexibility in the operations literature. A common feature of real flexibilities considered in the aforementioned finance literature is that they exacerbate the shareholder-debtholder agency conflict by providing shareholders with opportunities to shift investment and production towards risky ventures once debt is in place. “Anticipating risk shifting and asset substitution, lenders respond to real flexibility with tight credit terms, causing flexible firms demand less debt” (MacKay, 2003, p.1135). The opposite is true about resource flexibility. Because the real option embedded in resource flexibility, i.e., the option to use flexible capacity for production of either product, is exercised only after demand risk has been realized, it cannot be used by shareholders to shift investment or production toward risky ventures. On the contrary, investment in a single flexible resource as opposed to investment in a mix of flexible and dedicated resources does not allow any asset substitution and, furthermore, it eliminates underinvestment. To sum up, we contribute to the capital structure literature by showing that unlike other forms of real flexibility previously examined in this literature, resource flexibility mitigates the agency cost of debt and, therefore, is expected to be positively related to financial leverage.

The rest of the paper is organized as follows. In the next section, we examine the optimal capacity investment of an unlevered firm, which we later use as a benchmark to assess the impact of leverage. In Section 3, we show how leverage and the resulting agency conflict between shareholders and debtholders distort the firm’s investment in flexible and nonflexible capacity. Section 4 endogenizes the firm’s choice of capital structure. In Section 5, we examine the impact of a firm’s inherent flexibility on its optimal capital structure. Section 6 summarizes our findings. All proofs are provided in Appendix.

\(^5\)For a recent survey of this literature, see Franck and Huyghebaert (2004).
2 Unlevered firm

In the following, $\mathbb{E}$ denotes expectation with respect to the risk-neutral probability measure; the risk-free interest rate is normalized to zero; a prime denotes transpose; and the terms “increasing” and “decreasing” are used in a weak sense.

We consider a two-product firm that invests in the capacity of two product-dedicated resources and one product-flexible resource, labeled 1, 2 and 3, respectively. In modeling the capacity investment problem, we generally follow the seminal work of Van Mieghem (1998), although we assume all cost, revenue and demand distribution parameters to be equal for the two products. While product demand is uncertain, the firm chooses the vector of capacity levels $K = (K_1, K_2, K_3)'$. We assume a constant marginal capacity investment cost and denote the vector of unit capacity costs as $c = (c_N, c_N, c_F)'$, where subscripts $N$ and $F$ refer to nonflexible (product-dedicated) and flexible resources, respectively. For now, we assume that $c_N < c_F < 2c_N$ to avoid the scenarios in which the firm invests only in one type of capacity.

Once product demand is realized, the firm chooses the vector of production quantities $x = (x_{11}, x_{22}, x_{13}, x_{23})'$, where $x_{ij}$ is the output of product $i$ produced using resource $j$. The marginal cost of production is also assumed to be constant. After production is complete, the output is sold at a predetermined price, the firm is liquidated, and capacity has no residual value. Let $p$ be the unit contribution margin, i.e., the output price net of the unit production cost.

The output vector is constrained by the existing capacity as well as by the realized demand. We assume the demand vector $D = (D_1, D_2)$ to be drawn from an arbitrary distribution with a continuous density $f$ over $\mathbb{R}_+^2$. Given the optimal output decision, the firm’s operating profit, i.e., the sales revenue net of the production cost, equals

$$
\pi(D, K) = \max_{x \in \mathbb{R}_+^4} p(x_{11} + x_{22} + x_{13} + x_{23}),
$$

subject to $x_{ii} + x_{i3} < D_i, i = 1, 2,$

$$x_{ii} < K_i, i = 1, 2,$$

and $x_{13} + x_{23} < K_3$. \hspace{1cm} (1)

To write the operating profit explicitly as a function of the existing capacity and the realized demand, we partition the state space of the demand vector, $\mathbb{R}_+^2$, into four events corresponding to

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6The above modeling framework in which a firm chooses resource capacity while facing stochastic demand and a predetermined output price is known as the newsvendor problem and it is the most common approach to modeling the capacity investment problem in the operations literature. There are about 6,700 articles listed by Google Scholar that contain the terms “capacity” and “newsvendor” (or “newsboy”).
four possible solutions of (1):

\[ \Omega_0 (K) = \{ D \geq 0 : D_1 + D_2 < K_1 + K_2 + K_3, D_i \leq K_i + K_3, i = 1, 2 \}, \]
\[ \Omega_1 (K) = \{ D \geq 0 : D_1 > K_1 + K_3, D_2 \leq K_2 \}, \]
\[ \Omega_2 (K) = \{ D \geq 0 : D_2 > K_2 + K_3, D_1 \leq K_1 \}, \]
\[ \Omega_3 (K) = \{ D \geq 0 : D_1 + D_2 \geq K_1 + K_2 + K_3, D_i > K_i, i = 1, 2 \}. \]  

(2)

These events are illustrated in Figure 1a. If $D \in \Omega_0$, both demands are relatively low and can be fully satisfied with the existing capacity. If $D \in \Omega_1 (\Omega_2)$, demand for product 2 (1) is relatively low and can be fully satisfied with the corresponding product-dedicated capacity whereas demand for product 1 (2) is so high that it cannot be fully satisfied even when the entire flexible capacity is allocated to this product. Finally, if $D \in \Omega_3$, both demands are so high that although each type of capacity is fully utilized, some demand is lost. Depending on which of the four events takes place, the operating profit becomes

\[ \pi (D, K) = \begin{cases} 
   p (D_1 + D_2) & \text{if } D \in \Omega_0 \\
   p (K_1 + K_3 + D_2) & \text{if } D \in \Omega_1 \\
   p (D_1 + K_2 + K_3) & \text{if } D \in \Omega_2 \\
   p (K_1 + K_2 + K_3) & \text{if } D \in \Omega_3 
\end{cases}. \]  

(3)

Before demands are known, the firm chooses a capacity vector that maximizes the firm value, denoted as $V$, which is equal to the expected operating profit less the capacity investment cost, i.e.,

\[ V = \mathbb{E} \pi (D, K) - c'K. \]  

(4)

The optimal capacity vector that maximizes the value of an unlevered firm, $K^{UL}$, is characterized in the next proposition. The symmetry of product parameters together with the uniqueness of the optimal capacity vector implies that it is always optimal to set both nonflexible capacity levels as equal, and we can therefore simplify notation by letting $K_N \equiv K_1 = K_2$ and $K_F \equiv K_3$. Furthermore, we let $\Omega_{12} \equiv \Omega_1 \cup \Omega_2$, $\Omega_{123} \equiv \Omega_1 \cup \Omega_2 \cup \Omega_3$, etc.

**Proposition 1** The optimal capacity investment of an unlevered firm is characterized by the following necessary and sufficient conditions:

\[ p \Pr (\Omega_{13}) = c_N \]  

(5)

and

\[ p \Pr (\Omega_{123}) = c_F. \]  

(6)
The optimality conditions set the expected marginal operating profit of each type of capacity equal to its marginal cost. The marginal operating profit of either type of capacity is the unit contribution margin, \( p \), if this type of capacity is fully utilized, and zero otherwise. Capacity dedicated to product 1 (2) is fully utilized if demand for product 1 (2) is relatively high, namely, if \( D_2 > D_1 \) (\( D_2 > D_3 \)). Flexible capacity is fully utilized if either product demand is relatively high, namely, if \( D_2 > D_1, D_2 > D_3 \). Next we consider the optimal capacity investment of a levered firm.

3 Leverage and optimal capacity investment

In this section, we consider the same capacity investment problem faced by a levered firm and examine how leverage affects the firm’s investment policy. For now, we take the amount of the firm’s debt as given. Let \( B \) be the face value of the debt and let \( r \) be the interest so that the issue price of the debt is \( B - r \). In other words, the firm borrows \( B - r \) promising to repay \( B \). We assume that the issue price of the debt is sufficiently low so that the optimal capacity investment requires raising additional equity, i.e., \( c'K^* > B - r \). Finally, we assume that the firm is fully controlled by shareholders who maximize shareholder value, which is defined as the expected terminal value of equity net of the equity raised.
Once production is complete and the firm generates operating profit $\pi(D, K)$ given by (3), one of the following two events can occur depending on the realized demand:

(a) If $\pi(D, K) \geq B$, the firm is able to repay the debt and the terminal value of equity is the operating profit net of the debt obligation, $\pi(D, K) - B$.

(b) If $\pi(D, K) < B$, the firm is unable to repay the debt, it declares bankruptcy, and the terminal value of equity is zero.

We denote the sets of demand realizations corresponding to these two events as $\Omega_a$ and $\Omega_b$, respectively. Superimposing these two events on the four events defined in (2) so that e.g., $\Omega_{0a} \equiv \Omega_0 \cap \Omega_a$, we can distinguish the following seven events:

\[
\begin{align*}
\Omega_{0a} &= \{D \geq 0 : B/p \leq D_1 + D_2 < 2K_N + K_F, \text{ and } D_i < K_N + K_F, i = 1, 2\}, \\
\Omega_{0b} &= \{D \geq 0 : D_1 + D_2 < B/p, \text{ and } D_i < K_N + K_F, i = 1, 2\}, \\
\Omega_{1a} &= \{D \geq 0 : K_N + K_F < D_1, \text{ and } B/p - K_N - K_F < D_2 < K_N\}, \\
\Omega_{1b} &= \{D \geq 0 : K_N + K_F < D_1, \text{ and } D_2 < B/p - K_N - K_F\}, \\
\Omega_{2a} &= \{D \geq 0 : K_N + K_F < D_2, \text{ and } B/p - K_N - K_F < D_1 < K_N\}, \\
\Omega_{2b} &= \{D \geq 0 : K_N + K_F < D_2, \text{ and } D_1 < B/p - K_N - K_F\}, \\
\Omega_3 &= \{D \geq 0 : D_1 + D_2 \geq 2K_N + K_F, \text{ and } D_i > K_N, i = 1, 2\}.
\end{align*}
\]  

These events are illustrated in Figure 1b, in which the shaded area corresponds to the demand realizations resulting in bankruptcy. Note that when the entire capacity is fully utilized, the firm cannot go bankrupt, i.e., $\Omega_3 = \Omega_{3a}$.

Because the terminal value of equity is $\pi(D, K) - B$ if $D \in \Omega_a$ and it is zero otherwise, the shareholder value equals

\[V = \Pr(\Omega_a) \mathbb{E}(\pi(D, K) - B|\Omega_a) - c'K + B - r.\]  

As is apparent from the objective function (8), once the debt has been issued, a limited liability firm is only concerned with the investment payoff in non-bankruptcy states. In the next proposition, we characterize the optimal capacity investment that maximizes the shareholder value (8) for a given debt level. Unless otherwise stated, we assume throughout the paper that the capacity investment problem has an interior solution, i.e., it is optimal to invest in both flexible and nonflexible capacity.
Proposition 2 For a given debt level $B$, the optimal capacity investment $K^*(B)$ is characterized by the following optimality conditions:

\[ p \Pr(\Omega_{13a}) = c_N, \quad (9) \]
\[ p \Pr(\Omega_{123a}) = c_F. \quad (10) \]

The conditions characterizing the optimal capacity investment of a levered firm are similar to those characterizing the optimal capacity investment of an all-equity firm except that a levered firm considers only non-bankruptcy states of the world, i.e., the states when $D \in \Omega_a$. The next proposition shows what this means for the optimal levels of flexible, nonflexible and total capacity.

Proposition 3

(i) Financial leverage decreases the optimal level of flexible capacity, i.e., $K_F^* \leq K_{UL}^*$, and increases the optimal level of nonflexible capacity, i.e., $K_N^* \geq K_{UL}^*$.

(ii) Financial leverage decreases the optimal level of total capacity, i.e., $K_F^* + 2K_N^* \leq K_{UL}^* + 2K_{UL}^*$, as well as the total capacity investment, i.e., $c_F K^* \leq c_{UL}^*$.

Part (i) of Proposition 3 is a manifestation of the asset substitution problem (Jensen and Meckling, 1976). Because a levered firm is not concerned with the magnitude of loss realized in bankruptcy states, it pursues a more risk-seeking investment strategy, and therefore substitutes the less risky flexible capacity with the more risky nonflexible capacity.

To develop a sharper intuition into the trade-off between flexible and nonflexible capacity, we can combine the conditions characterizing the optimal capacity investment in the following relation:

\[ c_F - c_N = \begin{cases} p \Pr(\Omega_i) & \text{if the firm is unlevered} \\ p \Pr(\Omega_{ia}) & \text{if the firm is levered} \end{cases}, \quad i = 1, 2. \quad (11) \]

If $D \in \Omega_i$, $i = 1, 2$, demands for the two products are so uneven that there is an excess of capacity dedicated to the low-demand product and a shortage of capacity available to produce the high-demand product. In this case, replacing one unit of the nonflexible capacity dedicated to the low-demand product with one unit of the flexible capacity would generate an additional revenue $p$ while increasing the capacity investment cost by $c_F - c_N$. Condition (11) ensures that the marginal cost of such an increase in flexibility is equal to its expected marginal benefit. Because a levered firm considers only non-bankruptcy outcomes, it ignores the marginal benefit of flexibility that accrues to debtholders and, as a result, underinvests in flexibility.

Part (ii) of Proposition 3 is in the spirit of Myers (1977), who suggested that leverage leads to underinvestment because investment transfers wealth from shareholders to debtholders. Because the
shareholders of a levered firm disregard the marginal investment payoff which accrues to debtholders in bankruptcy states, a levered firm underinvests in capacity relative to an all-equity firm.

The next proposition stipulates that all the effects of leverage on the capacity investment established in Proposition 3 are monotone in the amount of debt.

**Proposition 4**  
(i) The optimal level of nonflexible capacity increases, whereas the optimal level of flexible capacity decreases in the debt level, i.e., \( \frac{dK_N(B)}{dB} \geq 0 \) and \( \frac{dK_F(B)}{dB} \leq 0 \).

(ii) The optimal level of total capacity as well as the total capacity investment decrease in the debt level, i.e., \( \frac{d(K_F(B)+2K_N(B))}{dB} \leq 0 \) and \( \frac{dc(K(C,B))}{dB} \leq 0 \).

The implication of Proposition 4 is that a higher amount of debt leads, ceteris paribus, to a less flexible capacity mix and a lower overall capacity level. Importantly, the negative relation between leverage and flexibility assumes that debt is given exogenously. If the amount of debt is chosen optimally, the relation between leverage and flexibility changes, as we show in the following two sections.

## 4 The optimal capital structure

In this section, we endogenize the firm’s choice of capital structure. Namely, we allow the firm to issue the optimal amount of debt prior to investing in capacity.\(^7\) The decision timeline, which is illustrated in Figure 2, is important. Because the firm chooses its capacity after debt has been issued, its capacity choice is distorted by asset substitution and underinvestment, as discussed in the previous section. If, in contrast, the firm could commit to a given capacity prior to issuing fairly-priced debt, e.g., by means of bond covenants or delegated monitoring, it would have an incentive to choose capacity that maximizes the total firm value. As is common in the literature, we consider the more realistic scenario in which a firm cannot credibly commit to a given capacity investment until debt financing has been secured.\(^8\)

The cost of issuing debt stems from the distortion of the firm’s investment strategy discussed above. One of the most important benefits of debt financing is the tax shield provided by the interest payment. To capture this benefit, we assume that the firm’s profit is subject to corporate

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\(^7\)We assume that the firm is not allowed to use the borrowed capital to pay out a dividend before realizing operating profit. This means that the optimal borrowing never exceeds the optimal capacity investment cost.

\(^8\)For example, Smith and Warner (1979) find that bond covenants involving extensive direct restrictions on production/investment policy are expensive to employ and are not empirically observed.
firm issues debt firm invests in capacity firm chooses output mix firm repays the debt or defaults

demand is realized

time

Figure 2: The sequence of events.

tax with a flat rate $t$. Because we consider a single-period model, we assume that no tax carry-backs and carry-forwards are allowed, as is standard in the literature (e.g., DeAngelo and Masulis, 1980 and Dotan and Ravid, 1985).

After product demand is realized and the firm generates operating profit $\pi(D, K)$, which is again given by (3), one of three scenarios can take place:

(a) If $\pi(D, K) > c'K + r$, the firm earns a positive profit that is subject to tax. The terminal value of equity is $\pi(D, K) - B - t(\pi(D, K) - c'K - r)$. The debtholders receive the full face value of the debt.

(b) If $B \leq \pi(D, K) \leq c'K + r$, the firm incurs a loss and, thus, is not subject to tax, but it is able to repay the debt in full. The terminal value of equity is $\pi(D, K) - B$. The debtholders receive the face value of the debt.

(c) If $\pi(D, K) < B$, the firm is unable to repay the full face value of the debt, declares bankruptcy, and is taken over by the debtholders. Equity becomes worthless, whereas the debtholders receive the firm’s operating profit $\pi(D, K)$. For analytical tractability, we assume there is no cost of bankruptcy.

We denote the sets of demand realizations corresponding to these three events as $\Omega_a$, $\Omega_b$, and $\Omega_c$, respectively. Superimposing these two events on the four events corresponding to the various capacity allocations (2), we can distinguish the following ten states of the world, which are
Figure 3: Partitioning of the state space of the demand vector into the ten regions defined in (12). The darkly shaded area corresponds to demand realizations resulting in bankruptcy; the lightly shaded area corresponds to demand realizations resulting in a loss but not bankruptcy.

illustrated in Figure 3:

\[
\begin{align*}
\Omega_{0a} & = \{ \mathbf{D} \geq 0 : (c'\mathbf{K} + r)/p \leq D_1 + D_2 < 2K_N + K_F, \text{ and } D_i < K_N + K_F, i = 1, 2 \}, \\
\Omega_{0b} & = \{ \mathbf{D} \geq 0 : B/p \leq D_1 + D_2 < (c'\mathbf{K} + r)/p, \text{ and } D_i < K_N + K_F, i = 1, 2 \}, \\
\Omega_{0c} & = \{ \mathbf{D} \geq 0 : D_1 + D_2 < B/p, \text{ and } D_i < K_N + K_F, i = 1, 2 \}, \\
\Omega_{1a} & = \{ \mathbf{D} \geq 0 : K_N + K_F \leq D_1, \text{ and } (c'\mathbf{K} + r)/p - K_N - K_F \leq D_2 < K_N \}, \\
\Omega_{1b} & = \{ \mathbf{D} \geq 0 : K_N + K_F \leq D_1, \text{ and } B/p - K_N - K_F \leq D_2 < (c'\mathbf{K} + r)/p - K_N - K_F \}, \\
\Omega_{1c} & = \{ \mathbf{D} \geq 0 : K_N + K_F \leq D_1, \text{ and } D_2 < B/p - K_N - K_F \}, \\
\Omega_{2a} & = \{ \mathbf{D} \geq 0 : K_N + K_F \leq D_2, \text{ and } (c'\mathbf{K} + r)/p - K_N - K_F \leq D_1 < K_N \}, \\
\Omega_{2b} & = \{ \mathbf{D} \geq 0 : K_N + K_F \leq D_2, \text{ and } B/p - K_N - K_F \leq D_1 < (c'\mathbf{K} + r)/p - K_N - K_F \}, \\
\Omega_{2c} & = \{ \mathbf{D} \geq 0 : K_N + K_F \leq D_2, \text{ and } D_1 < B/p - K_N - K_F \}, \\
\Omega_{3} & = \{ \mathbf{D} \geq 0 : D_1 + D_2 \geq 2K_N + K_F, \text{ and } D_i \geq K_N, i = 1, 2 \}. 
\end{align*}
\]

(12)

Given the three possible scenarios outlined above, we can write the shareholder value, i.e., the expected terminal value of equity net of the equity initially raised, as

\[
V = \Pr(\Omega_a) \mathbb{E}(\pi - B - t(\pi - c'\mathbf{K} - r)|\Omega_a) + \Pr(\Omega_b) \mathbb{E}(\pi - B|\Omega_b) - c'\mathbf{K} + B - r. 
\]

(13)
The optimal capacity vector, \( K^* (B) \), which maximizes shareholder value (13) for a given level of debt, is characterized in the next Proposition.

**Proposition 5** The optimal capacity investment \( K^* (B) \) is characterized by the following first-order conditions:

\[
(1 - t) p \Pr (\Omega_{13a}) + p \Pr (\Omega_{1b}) = c_N (1 - t \Pr (\Omega_a)), \quad (14)
\]

and

\[
(1 - t) p \Pr (\Omega_{123a}) + p \Pr (\Omega_{12b}) = c_F (1 - t \Pr (\Omega_a)). \quad (15)
\]

These conditions are similar to the optimality conditions (9)-(10) except that the marginal revenue as well as the marginal cost of capacity are diminished by tax where applicable.

Having characterized the optimal capacity investment, we are ready to consider the firm’s choice of the optimal debt level. We assume that debt is fairly priced, i.e., its issue price is equal to its expected value.\(^9\) Recall that debtholders are repaid the face value of the debt, \( B \), if \( D \in \Omega_{ab} \), and they receive the firm’s operating profit, \( \pi (D, K) \), if \( D \in \Omega_c \). Therefore, fair pricing requires that

\[
B - r = \Pr (\Omega_{ab}) B + \Pr (\Omega_c) \mathbb{E} (\pi|\Omega_c). \quad (16)
\]

The firm chooses the optimal amount of debt to maximize shareholder value (13) where the capacity investment \( K^* (B) \) is given by (14)-(15) and the interest \( r (B, K^* (B)) \) satisfies the fair-pricing condition (16). We characterize the optimal debt level in the next proposition.

**Proposition 6** The optimal debt level \( B^* \) is characterized by the following optimality condition:

\[
t \Pr (\Omega_a) \Pr (\Omega_c) + \frac{d \left( K^* (B) + K^* (B) \right)}{dB} \Pr (\Omega_{12c}) (1 - t \Pr (\Omega_a)) = 0, \quad (17)
\]

where \( K^* (B) \) satisfies (14) and (15).

The optimality condition (17) captures the trade-off involved in debt financing. An increase in debt level has two effects. First, it reduces the expected tax liability, which is reflected by the first term in (17). The marginal tax benefit of debt, \( t \Pr (\Omega_a) \Pr (\Omega_c) \), is large when the tax rate \( t \) is high; when the probability that the firm earns positive profit and thus pays tax, \( \Pr (\Omega_a) \), is high; and when the probability of default, \( \Pr (\Omega_c) \), and thus the tax-deductible interest, are high.

Second, an increase in the debt level exacerbates the distortion of the firm’s capacity investment due to the agency conflict between shareholders and debtholders. The marginal agency cost of debt

\(^9\) This is the case if agents are risk-neutral or if the expectation is taken with respect to the risk-neutral probability measure (e.g., Harrison and Kreps, 1979).
is captured by the second term in (17), which is always negative. The nature of the agency conflict is the following. If $D \in \Omega_{12c}$, demand for one product is so low that the firm goes bankrupt while demand for the other product exceeds the entire flexible and nonflexible capacity available to this product, $K^*_N + K^*_F$. A higher debt level exacerbates the underinvestment problem, i.e., the firm installs less capacity available to satisfy the high-demand product, $\frac{d(K^*_N + K^*_F)}{dB} \leq 0$. This reduces the operating profit that accrues to debtholders in the bankruptcy state $\Omega_{12c}$ by $p\left| \frac{d(K^*_N + K^*_F)}{dB} \right|$. When debt is fairly priced, this ultimately backfires at shareholders because debtholders anticipate the firm’s behavior and increase the interest by $p\left| \frac{d(K^*_N + K^*_F)}{dB} \right|Pr(\Omega_{12c})$. Finally, the term $(1 - tPr(\Omega_a))$ reflects the fact that this increase in the cost of external financing does not reduce the shareholder value by the same amount because the interest is tax deductible. To sum up, the optimal capital structure is such that the marginal tax benefit and the marginal agency cost associated with debt are equal.

Before we examine how the optimal leverage depends on resource flexibility, it is useful to consider as a benchmark an alternative scenario in which the firm chooses capacity prior to issuing debt. Because issuing debt changes the firm’s objective, this would only be the case if the firm could credibly commit to its capacity choice, for example, through bond covenants or sharing control with debtholders.

**Proposition 7** If the firm chooses capacity prior to issuing debt, the optimal capacity investment is characterized by the following optimality conditions:

$$pPr(\Omega_{13}) = c_N,$$  \hspace{1cm} (18)

and

$$pPr(\Omega_{123}) = c_F,$$  \hspace{1cm} (19)

and it is financed entirely by debt, i.e., $B - r = c^'K$.

If a firm chooses capacity prior to issuing debt, it has an incentive to take into account non-bankruptcy as well as bankruptcy states of the world because the latter affects the cost of borrowing, which is yet to be determined. In this case, there is no agency conflict between the firm’s shareholders and debtholders, the firm chooses the first-best capacity investment, and it relies exclusively on debt financing to maximize the tax benefit of debt. Note that the first-best capacity investment is unaffected by corporate tax because of the tax shield provided by the maximal leverage.

In the next section, we examine how the optimal capital structure depends on the firm’s inherent flexibility.
5 Resource flexibility and optimal leverage

The optimal mix of flexible and nonflexible capacity is determined endogenously and depends on the various model parameters and, in particular, on the cost of flexible capacity $c_F$ relative to the cost of nonflexible capacity $c_N$. So far, we have considered the situation in which $c_N < c_F < 2c_N$ and the firm optimally invests in flexible as well as nonflexible capacity. We have seen that in this case the firm generally relies on a mix of equity and debt financing trading off the tax benefit and the agency cost associated with leverage. In the following, we show that this is not the case under full resource flexibility.

Suppose that flexibility is costless, i.e., $c_F = c_N$, so that it is optimal to invest only in flexible capacity. In reality, a firm may invest exclusively in flexible capacity even if its unit cost exceeds the unit cost of nonflexible capacity provided that the cost difference is small and there is a fixed cost associated with investing in each type of capacity. We characterize the case of full flexibility in the next proposition.

**Proposition 8** Suppose that $c_F = c_N$ so that it is optimal to invest exclusively in flexible capacity. The optimal capacity investment is characterized by the following optimality condition:

$$p \Pr(\Omega_3) = c_F,$$

and it is financed entirely by debt, i.e., $B - r = c_F K_F$.

Proposition 8 has the following implication:

**Proposition 9** Under full resource flexibility, there is no agency cost associated with leverage, the firm chooses the first-best capacity investment, and it maximizes the tax benefit of debt by relying exclusively on debt financing.

While it is not surprising that a firm does not face the asset substitution problem when it invests in a single flexible asset, it is less obvious that resource flexibility eliminates the underinvestment problem as well. Recall that the agency conflict associated with debt stems from the fact that a levered firm ignores the marginal revenue generated by the fully utilized resources in bankruptcy states. Under full flexibility, i.e., with a single flexible resource, this is not an issue. Whenever a fully flexible firm goes bankrupt, its single flexible resource is underutilized and thus has zero marginal revenue from the shareholders’ as well as the debtholders’ points of view. As a result, there is no shareholder-debtholder agency conflict associated with leverage, the firm chooses the
first-best capacity investment, and it relies entirely on debt financing. Furthermore, because of
the tax shield provided by full external financing, the optimal capacity investment is unaffected by
corporate tax.

In the next proposition we show that in the other extreme case, i.e., when a firm invests
exclusively in two nonflexible and thus equally risky resources and, therefore, cannot engage in
asset substitution, the agency cost of leverage persists nevertheless.

**Proposition 10** Suppose that $c_F \geq 2c_N$ so that it is optimal to invest exclusively in nonflexible ca-
pacity. The optimal capacity investment and debt level are characterized by the following optimality
conditions:

\[
(1 - t)p \Pr(\Omega_{13a}) + p \Pr(\Omega_{1b}) = c_N(1 - t \Pr(\Omega_a)), \tag{21}
\]

\[
\text{and } t \Pr(\Omega_a) \Pr(\Omega_c) + p \frac{dK_N^*(B)}{dB} \Pr(\Omega_{12c})(1 - t \Pr(\Omega_a)) = 0. \tag{22}
\]

Even when investing in two equally risky resources, a levered firm chooses a capacity investment
that deviates from the first-best, and therefore, its optimal capital structure has to trade off the
tax benefit and the agency cost of debt. This is again because the shareholders ignore the marginal
revenue of the capacity dedicated to the high-demand product that would accrue to the debtholders
upon bankruptcy.

To sum up, the shareholder-debtholder agency conflict exists whenever a firm relies on multiple
resources dedicated to different products and may go bankrupt while some of its resources are fully
utilized. This is illustrated in Figure 4 where bankruptcy states are indicated by the shaded areas,
and their subsets in which some resources are fully utilized, $\Omega_{1c}$ and $\Omega_{2c}$, are indicated by the darker
areas. A fully flexible firm can go bankrupt only when its single flexible resource is underutilized
and thus provides zero marginal revenue to shareholders as well as to debtholders. Thus, under full
flexibility, $\Omega_{12c} = \emptyset$ and the agency conflict associated with leverage does not exist.

### 6 Conclusions

This paper examines the relation between resource flexibility and capital structure. We consider
a levered firm that invests in the optimal capacity of product-flexible and product-dedicated re-
sources in the presence of demand uncertainty and produces the optimal output mix once demand
uncertainty is resolved. Our model extends that of Van Mieghem (1998) by allowing the capacity
investment to be financed by equity as well as debt.
Figure 4: Partitioning of the demand state space defined in (7) in the case of no flexibility (a), partial flexibility (b), and full flexibility (c). The darkly shaded areas, $\Omega_{ic}, i = 1, 2$, correspond to demand realizations that induce agency conflict between shareholders and debtholders.

When the debt level is given exogenously, leverage reduces the total capacity investment consistent with the underinvestment theory of Myers (1977), and induces the firm to substitute flexible capacity with nonflexible capacity in line with the asset substitution theory of Jensen and Meckling (1976). However, when the firm chooses its capital structure optimally, trading off the tax benefit and the agency cost of debt, the relation between leverage and flexibility is reversed. In particular, when it is optimal to invest exclusively in flexible capacity, the agency cost associated with debt disappears and the firm has incentives to choose the first-best capacity investment and to rely exclusively on debt financing. In other words, our model predicts a positive relation between the employment of product-flexible technology and financial leverage.

This finding is in sharp contrast to the notion that real flexibility exacerbates the asset substitution problem by allowing shareholders to shift investment or production toward risky ventures (e.g., Myers, 1977; Leland, 1998; and MacKay, 2003). The real option embedded in resource flexibility is exercised after demand uncertainty has been resolved and, therefore, cannot be used by shareholders to increase risk. On the contrary, full resource flexibility eliminates the asset substitution as well as the underinvestment problem. The managerial implication is that firms investing in product-flexible technology can expect more favorable credit terms and, therefore, should rely more on debt financing.
Appendix

**Proof of Proposition 1:** The objective (4) is jointly concave in $K_N$ and $K_F$, and the optimal solution is thus given by the first-order condition $\nabla K V = 0$, which can be written as (5)-(6). □

**Proof of Proposition 2:** The optimal capacity vector must satisfy the first-order condition, $\nabla K V = 0$. Differentiating (8) with respect to $K_N$ and $K_F$ gives

$$\frac{\partial V}{\partial K_N} = 2p \Pr(\Omega_{13a}) - 2c_N \quad \text{and} \quad \frac{\partial V}{\partial K_F} = p \Pr(\Omega_{123a}) - c_F,$$

respectively. Setting the derivatives equal to zero gives (9)-(10). □

**Proof of Proposition 3:** From Proposition 1, the optimal capacity investment of an unlevered firm must satisfy

$$\Pr(\Omega_{123}(K_{UL})) = c_F/p,$$

and $\Pr(\Omega_{13}(K_{UL})) = c_N/p$

$$\Rightarrow \Pr(\Omega_{3}(K_{UL})) = (2c_N - c_F)/p,$$

and $\Pr(\Omega_{1}(K_{UL})) = (c_F - c_N)/p.$

From Proposition 2, the optimal capacity investment of a levered firm must satisfy

$$\Pr(\Omega_{123a}(K^*, B^*)) = c_F/p,$$

and $\Pr(\Omega_{13a}(K^*, B^*)) = c_N/p$

$$\Rightarrow \Pr(\Omega_{3}(K^*)) = (2c_N - c_F)/p,$$

and $\Pr(\Omega_{1a}(K^*, B^*)) = (c_F - c_N)/p.$

Therefore,

$$\Pr(\Omega_{123}(K_{UL})) = \Pr(\Omega_{123a}(K^*, B^*)), \quad (23)$$

$$\Pr(\Omega_{3}(K_{UL})) = \Pr(\Omega_{3}(K^*)), \quad (24)$$

and $\Pr(\Omega_{1}(K_{UL})) = \Pr(\Omega_{1a}(K^*, B^*)). \quad (25)$

We first prove that $K_F^* \leq K_{UL}^*$ by contradiction. Suppose $K_F^* > K_{UL}^*$. If $K_N^* \geq K_{UL}^*$, then $\Pr(\Omega_{123}(K_{UL})) > \Pr(\Omega_{123a}(K^*, B^*))$, which contradicts (23). Therefore, $K_N^* < K_{UL}^*$. If also $K_N^* + K_F^* \geq K_{UL}^* + K_{UL}^*$, then $\Pr(\Omega_{1}(K_{UL})) > \Pr(\Omega_{1a}(K^*, B^*))$, which contradicts (25). Therefore, $K_N^* + K_F^* < K_{UL}^* + K_{UL}^*$. This implies $2K_N^* + K_F^* < 2K_{UL}^* + K_{UL}^*$, which in turn implies $\Pr(\Omega_{3}(K_{UL})) < \Pr(\Omega_{3}(K^*))$, which contradicts (24). Therefore, our premise that $K_F^* > K_{UL}^*$ cannot be true, which proves that $K_F^* \leq K_{UL}^*$. 

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Next, we prove that $K_N^* \geq K_N^{UL}$ by contradiction. Suppose that $K_N^* < K_N^{UL}$. Using the fact that $K_F^* \leq K_F^{UL}$, this implies that $2K_N^* + K_F^* < 2K_N^{UL} + K_F^{UL}$, which in turn implies that $\Pr(\Omega_3(K^{UL})) < \Pr(\Omega_3(K^*))$, which contradicts (24). Therefore, our premise that $K_N^* < K_N^{UL}$ cannot be true, which proves that $K_N^* \geq K_N^{UL}$.

We prove that $K_F^* + 2K_N^* \leq K_F^{UL} + 2K_N^{UL}$ also by contradiction. Suppose that $K_F^* + 2K_N^* > K_F^{UL} + 2K_N^{UL}$. Using the fact that $K_N^* \geq K_N^{UL}$, this implies that $\Pr(\Omega_3(K^{UL})) > \Pr(\Omega_3(K^*))$, which contradicts (24). Therefore, our premise that $K_F^* + 2K_N^* > K_F^{UL} + 2K_N^{UL}$ cannot be true. Finally, the facts that leverage results in a lower total capacity and a higher proportion of the cheaper nonflexible capacity means that it also results in a lower capacity investment. □

Proof of Proposition 4: From the proof of Proposition 2, the optimal capacity vector $K^*(B)$ satisfies

$$\nabla K V = \left( \frac{\partial V}{\partial K_N}, \frac{\partial V}{\partial K_F} \right)' = 0,$$  

(26)

where

$$\frac{\partial V}{\partial K_N} = 2p \Pr(\Omega_{13a}) - 2c_N$$

$$= 2p \int_{K_N}^{K_N+K_F} \int_{2K_N+K_F-x}^{\infty} f(x,y) \, dy \, dx + 2p \int_{K_N}^{\infty} \int_{B/p-K_N-K_F}^{\infty} f(x,y) \, dy \, dx$$

$$- 2c_N,$$  

(27)

and

$$\frac{\partial V}{\partial K_F} = p \Pr(\Omega_{123a}) - c_F$$

$$= p \int_{K_N}^{K_N+K_F} \int_{2K_N+K_F-x}^{\infty} f(x,y) \, dy \, dx + p \int_{K_N+K_F}^{\infty} \int_{B/p-K_N-K_F}^{\infty} f(x,y) \, dy \, dx$$

$$+ p \int_{K_N+K_F}^{\infty} \int_{B/p-K_N-K_F}^{K_N} f(x,y) \, dy \, dx - c_F.$$  

(28)

Implicit differentiation of (26) gives

$$\frac{dK_F}{d\beta} = \frac{\partial^2 V}{\partial B \partial K_N} \frac{\partial^2 V}{\partial K_N \partial K_F} - \frac{\partial^2 V}{\partial K_N^2} \frac{\partial^2 V}{\partial B \partial K_F}$$

and

$$\frac{dK_N^*}{d\beta} = \frac{\partial^2 V}{\partial K_F \partial K_N} \frac{\partial^2 V}{\partial B \partial K_F} - \frac{\partial^2 V}{\partial B \partial K_N} \frac{\partial^2 V}{\partial K_F^2},$$

(29)

where

$$\|\nabla^2 K V\| = \frac{\partial^2 V}{\partial K_N^2} \frac{\partial^2 V}{\partial K_F^2} - \frac{\partial^2 V}{\partial K_N \partial K_F} \frac{\partial^2 V}{\partial K_N \partial K_F}$$

is the determinant of the Hessian of $V(K)$. Because the objective function, $V(K)$, must be locally
concave at optimal $K^*$, we have $\|\nabla^2 K V\| \geq 0$ at $K^*$. Differentiating (27) and (28) gives

$$\frac{\partial^2 V}{\partial K^2} = 2p \int_{K+N}^\infty f(x, B/p - K - K_F) \, dx - 2p \int_{B-2K}^{K+N} f(K + K_F, y) \, dy$$

$$- 2p \int_{K+N}^\infty f(K, y) \, dy - 4p \int_{K+N}^{K+N+K_F} f(x, 2K + K_F - x) \, dx,$$

$$\frac{\partial^2 V}{\partial K \partial K_F} = 2p \int_{K+N}^\infty f(x, B/p - K - K_F) \, dx - 2p \int_{B-2K}^{K+N} f(K + K_F, y) \, dy$$

$$- 2p \int_{K}^{K+N} f(x, 2K + K_F - x) \, dx,$$

$$\frac{\partial^2 V}{\partial B \partial K_F} = \frac{\partial^2 V}{\partial B \partial K_F} = -2 \int_{K+N}^\infty f(x, B/p - K - K_F) \, dx.$$

Substituting the above expressions into (29), we obtain

$$\frac{dK^*_F}{dB} = -\|\nabla^2 K V\|^{-1} 4p \int_{K+N}^\infty f(x, B/p - K - K_F) \, dx$$

$$\times \left( \int_{K}^{K+N+K_F} f(x, 2K + K_F - x) \, dx + \int_{K+N+K_F}^\infty f(x, K) \, dx \right) \quad (30)$$

$$\leq 0,$$

$$\frac{dK^*_N}{dB} = \|\nabla^2 K V\|^{-1} 2p \int_{K+N}^\infty f(x, B/p - K - K_F) \, dx$$

$$\times \int_{K}^{K+N+K_F} f(x, 2K + K_F - x) \, dx \quad (31)$$

$$\geq 0,$$

$$\frac{d(K^*_F + 2K^*_N)}{dB} = -\|\nabla^2 K V\|^{-1} 4p \int_{K+N}^\infty f(x, B/p - K - K_F) \, dx \int_{K+N+K_F}^\infty f(x, K) \, dx$$

$$\leq 0.$$

Finally, the facts that $\frac{dK^*_f}{dB} \geq 0$, $\frac{dK^*_N}{dB} \leq 0$, and $\frac{d(K^*_F + 2K^*_N)}{dB} \leq 0$ imply that $\frac{d(c'K^*(B))}{dB} \leq 0$. □

**Proof of Proposition 5:** The proof is analogous to the proof of Proposition 2. □

**Proof of Proposition 6:** The optimal debt level $B^* = \arg \max_B V(B, K^*(B), r(B, K^*(B)))$ where $V$ is given by (13), $K^*(B)$ is given by (14)-(15), and $r(B, K^*(B))$ is given by (16). Because
\[ \frac{\partial V}{\partial K_N} = \frac{\partial V}{\partial K_F} = 0, \]
we have
\[
\frac{dV}{dB} = \frac{\partial V}{\partial B} + \frac{dr}{dB} \frac{\partial V}{\partial r} = \frac{\partial V}{\partial B} + \left( \frac{\partial r}{\partial B} + \frac{\partial K_N}{\partial B} \frac{\partial r}{\partial K_N} + \frac{\partial K_F}{\partial B} \frac{\partial r}{\partial K_F} \right) \frac{\partial V}{\partial r}
= t \Pr (\Omega_a) \Pr (\Omega_c) + (1 - t \Pr (\Omega_a)) p \Pr (\Omega_{12c}) \frac{d(K_N + K_F)}{dB}.
\]

Thus, the first-order condition characterizing the optimal debt level \( B^* \), \( dV/dB = 0 \), can be written as (17). \( \square \)

**Proof of Proposition 7:** Once capacity has been chosen, the firm chooses its debt level to maximize the shareholder value \( V \) given by (13) where \( r \) is implied by the fair-pricing constraint (16). Combining (13) and (16), the firm’s objective can be written as
\[
V = \Pr (\Omega_a) \mathbb{E}(\pi - t (\pi - c'K - r) | \Omega_a) + \Pr (\Omega_{bc}) \mathbb{E}(\pi | \Omega_{bc}) - c'K. \tag{32}
\]
Differentiating (32) with respect to \( B \) gives
\[
\frac{dV}{dB} = \frac{\partial r}{\partial B} \frac{\partial V}{\partial r} = t \Pr (\Omega_a) \Pr (\Omega_c) > 0.
\]

Because the firm’s objective is monotonically increasing in \( B \), it is optimal to choose the maximum debt level \( B \). From (16), we have \( d(B - r)/dB = \Pr (\Omega_{ab}) \geq 0 \), i.e., the issue price of debt is monotonically increasing in the face value of debt. Therefore, maximizing the face value of debt means maximizing its issue price. Because we do not allow the firm to pay out a dividend before profit is realized, the maximum issue price of debt equals the capacity investment cost, i.e., \( B - r = c'K \).

When choosing optimal capacity, the firm again maximizes \( V \) given by (13) where \( B \) and \( r \) are given by \( B - r = c'K \) together with (16). Note that because \( B - r = c'K \), we have \( \Pr (\Omega_b) = 0 \) and the firm’s objective and fair-pricing condition simplify into
\[
V = (1 - t) \Pr (\Omega_a) \mathbb{E}(\pi - B | \Omega_a), \tag{33}
\]
and
\[
\Pr (\Omega_a) B + \Pr (\Omega_c) \mathbb{E}(\pi | \Omega_c) - c'K = 0. \tag{34}
\]
Differentiating (33) with respect to \( K_N \) and \( K_F \) gives
\[
\frac{dV}{dK_N} = \frac{\partial V}{\partial K_N} + \frac{\partial B}{\partial K_N} \frac{\partial V}{\partial B} = 2 (1 - t) \Pr (\Omega_{13}) p - c_N),
\]
and
\[
\frac{dV}{dK_F} = \frac{\partial V}{\partial K_F} + \frac{\partial B}{\partial K_F} \frac{\partial V}{\partial B} = (1 - t) \Pr (\Omega_{123}) p - c_F).
\]
Therefore, the optimality condition for \( K^* \), \( \nabla_K V = 0 \), can be written as in (18)-(19). \( \square \)
Proof of Proposition 8: When $c_F = c_N$, then $K^*_N = 0$ and the shareholder value given by (13) becomes

$$V = \Pr (\Omega_a) \mathbb{E} (\pi - B - t (\pi - c_F K_F - r) | \Omega_a) + \Pr (\Omega_b) \mathbb{E} (\pi - B | \Omega_b) - c_F K_F + B - r.$$  (35)

Consider the second-stage problem of choosing the optimal level of flexible capacity for given $B$ and $r$. Suppose that the optimal capacity investment requires additional equity financing, i.e., $c_F K_F > B - r$. In this case, the optimal level of flexible capacity must satisfy the first-order condition, $dV/dK_F = 0$, which can be written as

$$(1 - t) p \Pr (\Omega_3) = c_F (1 - t \Pr (\Omega_a)).$$  (36)

Differentiating (35) with respect to $B$ where $K_F$ is given by (36) and $r$ is given by (16), we obtain

$$\frac{dV}{dB} = \frac{\partial V}{\partial B} + \frac{dr}{dB} \frac{\partial V}{\partial r} = t \Pr (\Omega_a) \Pr (\Omega_c) > 0.$$  

Because the firm’s objective is monotonically increasing in $B$, our premise that $c_F K_F > B - r$ cannot be true. In other words, it is optimal to exclusively use debt financing, i.e., $c_F K_F = B - r$.

When $c_F K_F = B - r$, then $\Pr (\Omega_b) = 0$ and the shareholder value simplifies into

$$V = \Pr (\Omega_a) \mathbb{E} (\pi - B - t (\pi - c_F K_F - r) | \Omega_a),$$  (37)

where $K_F = (B - r) / c_F$, and $r$ is given by (16). Knowing the optimal capacity as a function of $B$ and $r$, we can turn to the first-stage problem of choosing the optimal debt level. Differentiating (37) with respect to $B$ gives

$$\frac{dV}{dB} = \frac{\partial V}{\partial B} + \frac{\partial K_F}{\partial B} \frac{\partial V}{\partial K_F} + \frac{dr}{dB} \left( \frac{\partial V}{\partial r} + \frac{\partial K_F}{\partial r} \frac{\partial V}{\partial K_F} \right)$$

$$= - \Pr (\Omega_a) + \frac{1}{c_F} \Pr (\Omega_a) \Pr (\Omega_3) (1 - t) p + \Pr (\Omega_a) t.$$  

Thus, the first-order optimality condition for $B^*$, $dV/dB = 0$, can be written as (20). □

Proof of Proposition 9: The result follows directly from Proposition 8. □

Proof of Proposition 10: The condition (21) is obtained by setting $\partial V/\partial K_N = 0$ where $V$ is given by (13) with $K_F = 0$. The condition (22) is derived in the same way as the optimality condition (17) in the proof of Proposition 6. □

References


