Fusion of Decisions Modeled as Weak Signals in Wireless Sensor Networks

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Abstract—Distributed detection has newly received research interest due to the success of the emerging wireless sensor network (WSN) technology. To deal with the problem of distributed detection for the WSN having the energy constraint, the fusion of decisions modeled as weak signals is studied. By using the weak signal model and additive non-Gaussian noise channels in the canonical parallel fusion scheme, we propose an asymptotic fusion rule applicable for wide classes of noise probability density functions (pdfs). In the particular case of a known pdf, an optimal detection rule is given. Both asymptotic analysis and Monte Carlo simulation are used to examine the performance of the proposed detection fusion rule.

I. INTRODUCTION

The weak signal approach is widely used in signal detection problems for additive noise environments [1]. The design of the locally optimal detector (LOD) for weak signals is considered as a good alternative to the fully optimal detector in the Neyman-Pearson sense because of easier implementation, and since the LOD is only required to have near-optimum detection power for weak signals, in that for strong signals it can usually provide satisfactory performance [2]. In order to realize low intercept probability and high anti-jamming capability, the modern communication system is designed to minimize and disguise the transmitted signal by spreading it in time and frequency, and also a transmitter may be designed to transmit only the energy required for reliable detection [1]. These features result in the weak signal approach that can be directly applied to detection problems in WSNs, since these research issues are mainly focused on the energy efficiency.

The problem of distributed detection using multiple sensors has been extensively studied. The recent success of emerging WSN technology encourage researchers to develop distributed algorithms in this field. In [3], the authors review the classical framework for decentralized detection, and basing on this they discuss alternative frameworks for detection in sensor networks including several design and optimization issues, where one of the addressed important issues is wireless channel considerations. The distributed detection and decision fusion problems in the WSNs incorporated with communication channel layer is studied in [4], [5]. Several early works deal with weak signals in distributed detection, but the weak signal is only used to model sensor observations [6]-[8]. In order to consider the problem of energy constraint in WSNs, we apply the weak signal approach to the system model used in [4].

The detection dealing with weak signals is mostly studied in the non-Gaussian noise case [2], [9] and this fact is more attractive to the WSNs, since the applications of WSNs include various communication environments to be modeled as the non-Gaussian noise. Concurrently with weak signals, thus, we integrate non-Gaussian noise channels into the fusion model. In this paper, we propose an asymptotic fusion rule of a general form directly applicable for any probability density function (pdf) from a wide class of non-Gaussian noise pdfs.

The remainder of this paper is organized as follows. In Section II, we formulate the parallel fusion problem using the weak signal approach and derive the asymptotic fusion rule with the particular case of the optimal maximum likelihood fusion. In Section III, the asymptotic performance analysis is conducted for the Neyman-Pearson setting. Numerical results including both analysis and simulation examples are obtained to show the asymptotic optimality of the proposed fusion rule in Section IV. Finally, we conclude in Section V.

II. THE ASYMPTOTIC FUSION RULE

The parallel fusion model consisting of \( K \) sensor nodes and the fusion center is depicted in Fig. 1, where each sensor node observes a common physical phenomenon and makes a preliminary decision \( u_k \in \{0, \theta\} \) corresponding to the two hypotheses, either \( H_0 \) (target-absent) or \( H_1 \) (target-present). For each sensor, the false alarm and detection probabilities are given by \( P[u_k = \theta|H_0] = P_{f_k} \) and \( P[u_k = \theta|H_1] = P_{d_k} \), respectively. At the fusion center, the received decision from the \( k \)th sensor node is given by

\[
y_k = h_k u_k + n_k, \quad k = 1, \ldots, K, \tag{1}
\]

where \( h_k \) is a real valued fading envelope with \( h_k > 0 \), and \( n_k \) is the additive channel noise with pdf \( f(x) \). In this model, the sensor decision \( \theta \) is the weak signal in the sense that its amplitude decreases with \( K \) as \( \theta = \theta_K = \nu/\sqrt{K} \) with the finite constant \( \nu > 0 \). In what follows, we also assume that the noise pdf \( f(x) \) is symmetric about zero and unimodal.

In this fusion model, given the conditional independence assumption of local observations, the likelihood ratio is written...
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As

$$\Lambda(\bar{y}) = \prod_{k=1}^{K} \frac{P_{dk}f(y_k - h_k\theta) + (1 - P_{dk})f(y_k)}{P_{fk}f(y_k - h_k\theta) + (1 - P_{fk})f(y_k)},$$

where $\bar{y} = [y_1, \ldots, y_K]^T$ is the vector of observations.

Assume that $K \to \infty$ and consequently $\theta \to 0$, then

$$\Lambda(\bar{y}) \sim \prod_{k=1}^{K} \frac{P_{dk}[f(y_k) - h_k\theta f'(y_k)] + (1 - P_{dk})f(y_k)}{P_{fk}[f(y_k) - h_k\theta f'(y_k)] + (1 - P_{fk})f(y_k)}$$

$$= \prod_{k=1}^{K} \frac{1 + P_{dk}h_k\psi_{ML}(y_k)\theta}{1 + P_{fk}h_k\psi_{ML}(y_k)\theta} = \Lambda_A$$

where $\psi_{ML}(x) = -f'(x)/f(x)$ is the maximum likelihood (ML) score function.

By taking logarithm on both sides of the $\Lambda_A$ and by the well-known asymptotic relation $\log(1 + x) \sim x$ as $x \to 0$, we get for the limit case of the $\log \Lambda_A$

$$\log \Lambda_A = \sum_{k=1}^{K} \log(1 + P_{dk}h_k\psi_{ML}(y_k)\theta)$$

$$- \sum_{k=1}^{K} \log(1 + P_{fk}h_k\psi_{ML}(y_k)\theta)$$

$$\sim \sum_{k=1}^{K} (P_{dk} - P_{fk})h_k\theta \psi_{ML}(y_k).$$

Further, with the norming factor $1/\sqrt{K}$, the asymptotically optimal statistic takes the form

$$T_K(\bar{y}) = \frac{1}{\sqrt{K}} \sum_{k=1}^{K} (P_{dk} - P_{fk})h_k\psi_{ML}(y_k).$$

(2)

Thus, the asymptotic fusion rule can be written as follows

$$T_K(\bar{y}) \overset{H_1}{\underset{H_0}{\rightarrow}} \lambda_{1-\alpha},$$

where $\lambda_{1-\alpha}$ is the threshold value providing the required rate of the false alarm probability $P_F = \alpha$.

### III. PERFORMANCE ANALYSIS

In this section, we analyze the asymptotic performance of the proposed fusion rule. In the Neyman-Pearson setting, the detection probability is determined by the given threshold value providing the required false alarm probability. Thus, our aim is to obtain the asymptotic results for the false alarm and detection probabilities, and the threshold value.

In the Neyman-Pearson setting, the false alarm probability is defined as

$$P_F = P\{T_K > \lambda_{1-\alpha} | H_0\}.$$ 

By the central limit theorem (CLT), the test statistic $T_K$ is asymptotically normal with mean $E[T_K | H_0]$ and variance $\sigma^2(T_K | H_0)$. To obtain the false alarm probability, we compute $E[T_K | H_0]$ and $\sigma^2(T_K | H_0)$. Given $h_k$, from (2) it follows that

$$E[T_K | H_0] = \frac{1}{\sqrt{K}} \sum_{k=1}^{K} h_k(P_{dk} - P_{fk})E[\psi(y_k) | H_0].$$

Since $\theta \to 0$ as $K \to \infty$, the following expansion holds

$$\psi(n_k + h_k \theta) \sim \psi(n_k) + h_k \theta \psi'(n_k).$$

Under $H_0$, therefore, we get that

$$\psi(y_k) = \begin{cases} \psi(n_k) & \text{with probability } 1 - P_{fk} \\ \psi(n_k) + h_k \theta \psi'(n_k) & \text{with probability } P_{fk} \end{cases}$$

and

$$E[\psi(y_k) | H_0] = E[\psi(n_k)] + P_{fk}E[\psi'(n_k)].$$

Since, $n_k$ is an i.i.d random variable, the expectation takes the form

$$E[\psi(n_k)] = \bar{\psi} + P_{fk}h_k \frac{\nu}{\sqrt{K}} \bar{\psi}'$$

where $\bar{\psi} = \int \psi(n)f(n)dn = 0$ and $\bar{\psi}' = \int \psi'(n)f(n)dn$.

Hence,

$$E[T_K | H_0] = \frac{1}{K} \sum_{k=1}^{K} P_{fk}(P_{dk} - P_{fk})h_k^2 \nu \bar{\psi}'.$$

and denoting

$$H_{fK} \overset{K}{=} \frac{1}{K} \sum_{k=1}^{K} P_{fk}(P_{dk} - P_{fk})h_k^2$$

we get that

$$E[T_K | H_0] = \nu \bar{\psi}^2 H_{fK}.$$ 

Next we compute $\sigma^2(T_K | H_0)$ as follows

$$\sigma^2(T_K | H_0) = E[T_K^2 | H_0] - (E[T_K | H_0])^2,$$

where

$$E[T_K^2 | H_0] = \frac{1}{K} \sum_{i=1}^{K} \sum_{j=1}^{K} h_i h_j (P_{di} - P_{fi})(P_{dj} - P_{fj})E[\psi(y_i)\psi(y_j) | H_0].$$
By using asymptotic expansions with elementary but tedious transformations, it can be shown that

\[ E[T_K^2|H_0] = \frac{1}{K} \sum_{k=1}^{K} (P_{dk} - P_{fk})^2 h_k^2 \psi^2 + \nu^2 (\psi^2)^2 H_K^2, \]

where \( \psi^2 = \int \psi^2(n) f(n) dn \). Thus, the variance \( \sigma^2(T_K|H_0) \) is given by

\[ \sigma^2(T_K|H_0) = \psi^2 H_K, \]

where

\[ H_K = \frac{1}{K} \sum_{k=1}^{K} (P_{dk} - P_{fk})^2 h_k^2. \]

As \( K \to \infty \), we get that \( H_{fK} \to H_f \) and \( H_K \to H \). Here we assume that the corresponding limits exist either in the common or in the probabilistic sense. Then, by the CLT, the asymptotic expression for the false alarm probability takes the following form

\[ P_F = \lim_{K \to \infty} P \{ T_K > \lambda_{1-\alpha}|H_0 \} = 1 - \Phi \left( \frac{\lambda_{1-\alpha} - \nu \psi H_f}{\sqrt{\psi^2 H}} \right) = \alpha, \tag{3} \]

where \( \Phi(z) = (2\pi)^{-1/2} \int_{-\infty}^{z} \exp(-t^2/2) dt \) is the Gaussian cumulative. Further, the threshold value is given by

\[ \lambda_{1-\alpha} = \nu \psi H_f + \sqrt{\psi^2 H} \Phi^{-1}(1 - \alpha). \tag{4} \]

Under the alternative \( H_1 \), we similarly get that

\[ E[T_K|H_1] = \nu \psi \frac{1}{K} \sum_{k=1}^{K} P_{dk}(P_{dk} - P_{fk}) h_k^2 = \nu \psi H_dK \]

and

\[ \sigma^2(T_K|H_1) = \psi^2 H_d. \]

Finally, the asymptotic expression for the detection probability is given by

\[ P_D = \lim_{K \to \infty} P \{ T_K > \lambda_{1-\alpha} \} = 1 - \Phi \left( \phi^{-1}(1 - \alpha) - \frac{\nu \psi (1 - \nu \psi H_f)}{\sqrt{\psi^2 H}} \right). \tag{5} \]

IV. NUMERICAL RESULTS

In this section, we evaluate the asymptotic optimality of the proposed fusion rule, and compare both the analytical and simulation results in several examples. For simplicity, we assume that each sensor node has the same false alarm and detection probabilities, \( P_d \) and \( P_f \), respectively.

Throughout this paper, we assume that the fading channel pdf is Rayleigh with unit variance

\[ f_R(h) = \frac{h}{a^2} e^{-h^2/2a^2}, \]

where \( h \geq 0 \) and \( a > 0 \).

To verify asymptotic optimality, firstly, we consider Gaussian noise with the pdf

\[ f_G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-x^2/(2\sigma^2)}. \]

For Gaussian noise, the ML score function is given by

\[ \psi_G(x) = x/\sigma^2. \]

From (5) it follows that the detection probability is given by

\[ P_{DG} = 1 - \Phi \left( \phi^{-1}(1 - \alpha) - \frac{\nu}{\sigma} (P_d - P_f) \sqrt{2a} \right). \]

Also from (4), we can get the threshold value as

\[ \lambda_{1-\alpha} = \frac{1}{\sigma} (P_d - P_f) \sqrt{2a} \left( \nu P_f \sqrt{2a} - \phi^{-1}(1 - \alpha) \right). \]

Assuming that the Rayleigh fading has unit power, i.e., \( E[h^2] = 1 \), we get that

\[ P_{DG} = 1 - \Phi \left( \phi^{-1}(1 - \alpha) - \frac{\nu}{\sigma} (P_d - P_f) \right), \]

and

\[ \lambda_{1-\alpha} = \frac{1}{\sigma} (P_d - P_f) \left( \nu P_f - \phi^{-1}(1 - \alpha) \right). \]

For the Laplace noise with the pdf

\[ f_L(x) = (2s)^{-1} \exp(-|x|/s), \]

the score function is given by \( \psi_L(x) = 1/s \text{ sign}(x) \) with the detection probability as

\[ P_{DL} = 1 - \Phi \left( \phi^{-1}(1 - \alpha) - \frac{\nu}{s} (P_d - P_f) \right). \]

The detection probability for the Cauchy noise with the pdf

\[ f_C(x) = \gamma / (\pi (x^2 + \gamma^2)) \]

and the score function \( \psi_C(x) = 2x/ (x^2 + \gamma^2) \) is given by

\[ P_{DC} = 1 - \Phi \left( \phi^{-1}(1 - \alpha) - \frac{\nu}{\gamma \sqrt{2}} (P_d - P_f) \right). \]
Fig. 3. ROC curves for the Gaussian noise at SNR = 0, 10, and 20 dB with $K=100$.

For simulation, the detection and false alarm probabilities of each sensor are taken as $P_d = 0.5$ and $P_f = 0.05$, respectively. In order to use the conventional signal-to-noise ratio (SNR), we set the variances of the Gaussian and Laplace pdfs equal to unit, namely, $\sigma^2 = 1$ and $s = 1/\sqrt{2}$. Since the conventional SNR is not applicable for the Cauchy pdf, we utilize the Geometric SNR (GSNR) defined in [10], therefore, we have $\gamma = \sqrt{2C_g}$ where $C_g \approx 1.78$. Thus, the total SNR of the network is given by $\text{SNR} = \nu^2$.

Fig. 2 shows asymptotic optimality of the proposed fusion rule for the fixed SNR equal to 20 dB and for the fixed false alarm probability $P_F = 0.01$. When the number of sensor nodes increases, the detection probability of the asymptotic fusion rule tends to the optimal maximum likelihood rate. Fig. 3 gives the receiver operating characteristic (ROC) curves obtained by both Monte Carlo simulation and by the aforementioned asymptotic results for the Gaussian noise. Since we use the weak signal approach, it is quite natural that the smaller SNR, the better match between theory and experiment.

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The ROC curves for the Cauchy noise is shown in Fig. 4, and these results are also similar to the Gaussian example from the viewpoint of asymptotic optimality. The ROC curves for the Laplace noise are shown in Fig. 5, and the results for the asymptotic fusion rule are obtained by using ML score function for the Laplace density. In this example, the asymptotic optimality is also achieved at 0 dB SNR. However, in this case, the discordance at 10 dB SNR is larger than the results for the Gaussian and the Cauchy cases because of the growing inaccuracy when using the CLT for these kinds of pdfs which are not bell-shaped.

The numerical results obtained from these three examples represent the asymptotic optimality of the proposed fusion rule utilizing the ML score function, where analytical and experimental results match quite well.

V. CONCLUSION

Fusion of decisions modeled as weak signals in WSNs is studied in this paper. By incorporating the weak signal model and the non-Gaussian noise channels into the parallel fusion scheme, the asymptotic fusion rule using the ML score function is proposed. Within the weak signal approach, the proposed fusion rule with the ML score function is approaching optimum when the number of sensor nodes becomes large. In the Neyman-Pearson setting, the performance characteristics of fusion such as the false alarm and detection probabilities together with the threshold value are analytically obtained in the closed form. The asymptotic optimality of the proposed rule is justified by numerical results obtained from both theoretical and experimental results for the Gaussian, Laplace, and Cauchy noises.
In detection problems, it is often desired to address the robustness issue directly related to the choice of a score function under the conditions of uncertainty of noise pdf models. Although this issue is beyond the scope of our paper, we note that the obtained results allow to consider robust fusion schemes based on the maximin approach when the channel noise pdfs are only partially known.

ACKNOWLEDGMENT

This work was supported in part by a grant (ADD080601) from basic research program of the Agency for Defense Development (ADD), and was partially supported by the World Class University (WCU) program at GIST through a grant provided by the Ministry of Education, Science and Technology (MEST) of Korea (Project No. R31-2008-000-10026-0).

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