Effective Channel Shortening by Modified MSSNR Algorithm for Simplified UWB Receiver

Syed Imtiaz Husain†, Jinhong Yuan† and Jian Zhang‡,¶

†School of Electrical Engineering and Telecommunications, University of New South Wales, Sydney, Australia.
‡Research School of Information Sciences and Engineering, Australian National University, Canberra, Australia.
¶National ICT, Australia.

E-mails: imtiaz.syed@student.unsw.edu.au, j.yuan@unsw.edu.au, andrew.zhang@nicta.com.au

Abstract—In this paper, we present a modified version of the maximum shortening signal to noise ratio (MSSNR) algorithm for channel shortening in a time hopping (TH) pulse position modulated (PPM) ultra wideband (UWB) communication system. The proposed algorithm introduces two additional UWB channel related parameters in the optimization problem along with the conventional energy criterion. This modification significantly improve the performance of the conventional MSSNR algorithm and enables it to handle the extreme nature of channel shortening needed in UWB systems. We also derive a lower bound for bit error rate (BER) as a comparison benchmark. The proposed algorithm does not need any training or channel estimation and outperforms the conventional MSSNR algorithm in terms of different comparative parameters.

Index Terms—Channel shortening, RAKE receiver, ultra wide bandwidth (UWB), MSSNR.

I. INTRODUCTION

Channel shortening equalizers or time domain equalizers (TEQs) have been used in communication systems since early 1970s [1], [2]. Most of the recent applications of TEQs are specifically developed for multicarrier modulation (MCM) systems [3]-[5] to mitigate the intersymbol interference (ISI) produced due to inadequate cyclic prefix (CP). These algorithms are used to suppress just few channel taps outside the CP length and exploit some of those parameters explicitly available in MCM systems.

A major problem encountered in UWB systems is to capture enough multipaths through a RAKE receiver [7] to maintain a sufficient signal to noise ratio (SNR) for further signal processing. TEQs can also greatly simplify an UWB receiver structure by reducing the effective channel delay spread [6], [9]. In UWB systems, the problem appears in its extreme form where a large number of channel taps are needed to be suppressed and most of the channel energy should be compressed within just few taps. Hence, the underlying theory and assumptions for most of the existing algorithms become inappropriate when applied to UWB. Therefore, new or modified channel shortening algorithms exploiting UWB features are required to address the specific needs of UWB systems.

Due to the impulse like pulses used in UWB systems, the received pulse is very similar to the channel impulse response (CIR). This unique UWB phenomenon makes an approximate channel estimation possible from the received pulse. Also, UWB channels exhibit some well defined statistical properties. These channel parameters can be used to define effective channel shortening. Here, we use some UWB channel properties to make the proposed algorithm more robust and effective in UWB channels.

Remaining of the paper is organized as follows: In section II, we briefly discuss the UWB channel models used in this paper. Section III discusses the probability of error lower bound for an ideal RAKE receiver. Section IV describes the assumed system architecture and associated mathematical model. Section V explains the underlying assumptions and mathematical structure of the proposed modified MSSNR algorithm. Simulation results are shown in section VI and section VII finally concludes the discussion.

II. CHANNEL MODEL

Based on the measurements carried out by different researchers for a wide variety of propagation scenarios, IEEE 802.15 Study Group 3a finalized four standard models for UWB channels [8]. In this paper we use these standard channel models, namely CM 1 to CM 4, to develop the system architecture and evaluate its performance. These channel models are modified versions of Saleh-Valenzuela (S-V) model and generated to fit different propagation scenarios. They take, in general, the following mathematical form:

$$h(t) = X \sum_{l=0}^{L} \sum_{k=0}^{K} \alpha_{k,l} \delta(t - T_l - \tau_{k,l})$$

$$= \sum_{m=0}^{M-1} h_m \delta(t - \tau_m),$$

where $\alpha_{k,l}$ are the multipath gain coefficients, $T_l$ is the delay of the $l^{th}$ cluster, $\tau_{k,l}$ is the delay of $k^{th}$ multipath component relative to the $l^{th}$ cluster arrival time $T_l$, $L$ is the number of clusters, $K$ is the number of multipaths within a cluster and $X$ represents the log-normal shadowing. Eq. (2) is the simplified
form of (1) where the multipath gain coefficients $h_m$ and their arrival times $\tau_m$ are assumed to have absorbed all the statistical properties of $\alpha_{k,t}$, $T_i$ and $\tau_{k,t}$.

### III. Probability of Error Lower Bound

Assuming $g(t)$ is the transmitted pulse shape, for a binary time hopping (TH) UWB signal employing pulse position modulation (PPM), a symbol transmitted by the $j$th user at time $t$ can be given by:

$$s_j(t) = \sum_{i=0}^{N-1} g(t - iT_f - c_j,iT_c - a_j\Delta),$$  \hfill (3)

where $T_f$ is pulse repetition period, $N$ is the number of repetitions, $T_c$ is TH chip period, $c_j,i$ is randomly generated TH sequence for the $j$th user, $a_j \in \{0,1\}$ and $\Delta$ is the delay to represent a binary symbol.

The received symbol from the $j$th user at the channel output is:

$$r_j(t) = \sum_{i=0}^{N-1} \sum_{m=0}^{M-1} h_{j,m}^i p_{j,m}^i(t - iT_f - c_j,iT_c - a_j\Delta - \tau_{j,m}^i) + n(t),$$  \hfill (4)

where $h_{j,m}^i$ are the channel coefficients for the $j$th user in the $i$th multipath. The corresponding arrival time, $\tau_{j,m}^i$ is the associated distorted version of $g(t)$, $n(t)$ is the additive white Gaussian noise (AWGN) and we assume that each user is experiencing same channel length.

It is appropriate to assume that the channel does not change during a binary symbol period. Therefore, in the above equation we can drop the summation over $i$ and its corresponding superscripts. Hence in a multiuser environment with $N_u$ simultaneous active users, the received signal can be written as:

$$r(t) = \sum_{j=1}^{N_u} \sum_{m=0}^{M-1} h_{j,m} p_{j,m}(t - T_f - c_j T_c - a_j\Delta - \tau_{j,m}) + n(t),$$  \hfill (5)

$$r(t) = \sum_{j=1}^{N_u} h_j p_j + n(t) = \sum_{j=1}^{N_u} h_j + n(t),$$  \hfill (6)

where $h_j = [h_{j,0}, h_{j,1}, \ldots, h_{j,M-1}]$ and $p_j = \text{diag}[p_{j,0}(\cdot), p_{j,1}(\cdot), \ldots, p_{j,M-1}(\cdot)].$

We consider a perfect rake receiver which is synchronized to the user $\bar{u}$ with $M$ fingers and can capture all resolvable multipaths in $r(t)$. Let $v(t)$ be the template waveform available at the rake correlators such that:

$$\rho_{j,q} \triangleq \int_{-\infty}^{\infty} v(t)p_{j,q}(\cdot)dt.$$  \hfill (7)

Based on (7) we define a correlation matrix as follows:

$$R_j \triangleq \text{diag}[\rho_{j,0}, \rho_{j,1}, \ldots, \rho_{j,M-1}].$$  \hfill (8)

The output of the rake receiver is:

$$r_{\text{out}}(t) = \sum_{j=1}^{N_u} h_j R_j + n_j,$$  \hfill (9)

where $n_j = \int_{-\infty}^{\infty} v(t)n(t)dt$.

In an ideal synchronization with the user $\bar{u}$, $R_{\bar{u}} = I_M$ whereas $R_j = O_M, \forall j \neq \bar{u}$. This will lead to a lower bound of bit error rate (BER):

$$P_e = \mathbb{E} \left[ Q \left( \sqrt{\frac{\sum_{m=0}^{M-1} h_{\bar{u},m}^2 \sigma^2}{\sigma^2}} \right) \right],$$  \hfill (10)

where $\mathbb{E}(\cdot)$ and $Q(\cdot)$ are expectation and $Q$ function respectively and $\sigma^2$ is the noise power.

The BER in (10) will be used as a bench mark for performance analysis in multiuser AWGN environments.

### IV. System Architecture

Since the transmitted pulse $g(t)$ is very narrow in time, a single pulse transmitted in a single user environment will quite accurately reveal the CIR at the receiver. This is a characteristic feature of UWB systems and not available in other systems where pulse/bit duration is comparable to channel delay spread. Though this assumption is not perfectly valid in a multiuser environment but it still provides a basis to develop the TEQ. Hence, a TEQ which can shorten the received signal is also capable of shortening the CIR. Therefore the TEQ can be evaluated and updated using the received pulse information. There is no need of separate training or channel estimation.

Let a TEQ be present at the receiver front end. The received signal with $N_u$ users in an AWGN environment is given by:

$$r_m(t) = \sum_{j=1}^{N_u} \sum_{m=0}^{M-1} h_{j,m}\delta(t - \tau_{j,m})g(\beta - t)d\beta + n(t),$$  \hfill (11)

where we assumed that the CIR does not exist when $t < 0$.

Let the TEQ shorten the channel of user $\bar{u}$ to first $\ell$ multipaths. In this case, the above equation can be split into two parts with respect to channel taps as follows:

$$r_m(t) = \sum_{j=1}^{N_u} \sum_{m=0}^{\ell-1} h_{j,m}\delta(t - \tau_{j,m})g(\beta - t)d\beta + \mathfrak{r}_{1}^{(\ell_1)} + \mathfrak{r}_{1}^{(\ell_1)},$$  \hfill (12)

Or alternatively for the user $\bar{u}$ each part in (12) can be given in matrix form as:

$$\mathfrak{r}_{1}^{(\ell_1)} \triangleq \mathbf{G}_{1,\bar{u}}^{(p,\ell_1)} \mathbf{h}_{1,\bar{u}}^{(q,\ell_1)},$$

and

$$\mathfrak{r}_{2}^{(q,\ell_1)} \triangleq \mathbf{G}_{2,\bar{u}}^{(q,M-\ell)} \mathbf{h}_{2,\bar{u}}^{(q,M-\ell)} + \mathbf{n}_{(q,1)},$$  \hfill (13)

where $\mathbf{G}_{1,\bar{u}}$ and $\mathbf{G}_{2,\bar{u}}$ are the convolution matrices of appropriate order for user $\bar{u}$ transmitted signal vector $\mathbf{g}_{\bar{u}}, \mathbf{h}_{1,\bar{u}}$ and $\mathbf{h}_{2,\bar{u}}$ are the splitted parts of channel vector $\mathbf{h}_{\bar{u}}$. The parentheses show the order of each matrix or vector such that $p = \ell + b - 1$, $q = M - \ell + b - 1$, where $b$ is the length of transmitted signal vector $\mathbf{g}_{\bar{u}}$. It is worthy to note that the effective noise component $\mathbf{n}$ is added only in $\mathfrak{r}_{2}$ to simplify the expression.
V. Modified MSSNR Algorithm

Let \( w \) be the TEQ such that:

\[
\begin{bmatrix}
  w_0 & w_1 & w_2 & \cdots & w_{d-1}
\end{bmatrix}^T,
\]  

where \( d \) is the number of TEQ taps.

If the TEQ is inserted before the RAKE reception then the received signal applied to the RAKE is:

\[
r = (R_1^{(\eta,d)} + R_2^{(\eta',d)})w,
\]  

where \( R_1^{(\eta,d)} \) and \( R_2^{(\eta',d)} \) are the convolution matrices of \( r_1^{(p,1)} \) and \( r_2^{(q,1)} \) respectively, \( \eta = d + p - 1 \) and \( \eta' = d + q - 1 \).

The algorithm which deals with channel shortening in a very primitive sense is the maximum shortening signal to noise ratio (MSSNR) algorithm [10]. It formulates a single Rayleigh quotient to be optimized which takes the following mathematical form:

\[
w_{\text{opt}} = \arg \max_w \frac{w^H B w}{w^H A w},
\]  

where \( A \) and \( B \) represent the channel energy outside and inside the shortened channel window, respectively. The right hand side of the above equation is referred to as the shortening signal to noise ratio (SSNR). The solution to this problem is to minimize the denominator with a constrained numerator. The constraint is applied to avoid some trivial solutions. Thus, an optimum TEQ is the generalized eigenvector corresponding to the smallest generalized eigenvalue of the matrix pair \((A,B)\). This solution assumes that \( d < \ell \). Violation of this assumption results in a situation where \( B \) cannot be decomposed through Cholesky factorization. In case of UWB channels, we need a comparatively larger \( d \) which could efficiently shorten the dense multipath channel. This makes \( d > \ell \) and thus the conventional MSSNR algorithm does not work. A variant of MSSNR algorithm is introduced in [11] which works the other way around. It maximizes the numerator with constrained denominator. In this case a TEQ with \( d > \ell \) is possible. Therefore, in this paper, we modify the scheme in [11] to work in UWB scenarios.

In the proposed algorithm, we define a unique Rayleigh quotient to be maximized. From (15), the signal energy within the shortened channel window and outside can be given by:

\[
\lambda_{\text{win}} \triangleq w^T[R_1^{(\eta,d)}]^T R_1^{(\eta,d)} w
\]

\[
\lambda_{\text{wall}} \triangleq w^T[R_2^{(\eta',d)}]^T R_2^{(\eta',d)} w.
\]

To efficiently shorten the channel, an optimum \( w \) will maximize \( \lambda_{\text{win}} \) keeping \( \lambda_{\text{wall}} \) constant.

Since UWB CIRs exhibit a sort of exponentially decaying profile, the signal amplitude is larger in beginning and reduces with time. As we assumed the shortened channel window is spanning over first \( \ell \) multipaths, some statistical parameters associated to the shortened channel window can be optimized to improve TEQ performance. Here, we introduce two additional parameters in the optimization problem:

Let us first define the “conditional variance” of \( r_1^{(p,1)} \) on \( r_2^{(q,1)} \) as:

\[
\gamma \triangleq \mathbb{E}[(\gamma \gamma^T)],
\]

where

\[
\gamma \triangleq r_1^{(p,1)} - \mu_r u^{(p,1)},
\]

in which \( \mu_r \) and \( u^{(p,1)} \) contains all elements equal to 1. In other words, the conditional variance of \( r_1^{(p,1)} \) on \( r_2^{(q,1)} \) can be regarded as the second moment of \( r_1^{(p,1)} \) but with the mean of \( r_2^{(q,1)} \).

This parameter gives a measure of the squared difference in the mean amplitudes within and outside the shortened channel window. A higher value is desired for effective channel shortening. Thus, the first parameter to be included in optimization is \( \psi \):

\[
\psi \triangleq \mathbb{E}[\Theta^{(\eta,d)}]^T \Theta^{(\eta,d)} w,
\]

where \( \Theta^{(\eta,d)} \) is the convolution matrix of \( \theta^{(p,1)} \).

It is to be noted that the conditional variance of \( r_2^{(q,1)} \) on \( r_1^{(p,1)} \) provides a similar measure to (19). But, it would result in a larger matrix \( \Theta^{(\eta',d)} \) of order \( \eta' \times d \). Obviously, with the same TEQ length, it is difficult to optimize a larger matrix. Thus, the efficiency of the TEQ will be degraded. That is why the latter case is not included in the optimization.

The second parameter included in the optimization is the gradient of \( r_1^{(p,1)} \):

\[
\xi^{(p,1)} \triangleq \nabla r_1^{(p,1)},
\]

where \( \nabla \) is the gradient operator.

This parameter exploits the decaying characteristics of the received signal within the shortened channel window. A slower signal decay will ensure a flat CIR within the shortened window. This eventually increases the channel energy inside the shortened window and leads to an efficiently shortened channel. Similar reasons as described earlier preclude \( r_2^{(q,1)} \) in (22).

Hence, this parameter is included as:

\[
\zeta \triangleq \mathbb{E}[(\Xi^{(\eta,d)})^T \Xi^{(\eta,d)}] w,
\]

where \( \Xi^{(\eta,d)} \) is the convolution matrix of \( \xi^{(p,1)} \).

At this point, it is worthy to explain the significance of assuming the shortened channel window over first \( \ell \) multipaths. In general, it could be defined anywhere in the CIR as in [10] and [11]. In fact, due to exponentially decaying UWB channel profiles, \( \theta^{(p,1)} \) and \( \xi^{(p,1)} \) may be irrelevant to define efficient channel shortening if the shortened window does not exist at the beginning of CIR. Also, this assumption makes the optimization solution computationally easy and effective as the optimum TEQ does not need to modify the CIR significantly to shorten the channel.

Based on (17), (18), (21) and (23) we define the following optimization problem for the proposed algorithm:

\[
w_{\text{opt}} = \arg \max_w \frac{\lambda_{\text{win}}}{\lambda_{\text{wall}} + \psi + \zeta}.
\]

Substituting the values \( \lambda_{\text{win}}, \lambda_{\text{wall}}, \psi \) and \( \zeta \) in (24), we get:

\[
w_{\text{opt}} = \arg \max_w \frac{w^T (R_1^T R_1) w}{w^T (R_2^T R_2 + \Theta^T \Theta + \Xi^T \Xi) w}.
\]
where parentheses depicting the order of each matrix have been removed for concise representation.

The above equation poses a traditional optimization problem as in (16) with:

\[ B = R_{1}^{T}R_{1} \]

and

\[ A = R_{2}^{T}R_{2} + \Theta^{T}\Theta + \Xi^{T}\Xi. \]

Hence:

\[ w_{opt} = (\sqrt{A})^{-1}a_{max} \]

where \( a_{max} \) is the eigenvector corresponding to maximum eigenvalue of \( (\sqrt{A})^{-1}B(\sqrt{A})^{-1} \) and \( \sqrt{A} \) is the Cholesky factor of \( A \).

If \( \bar{u} \) is the user of interest, the probability of error in a multiuser environment can be given as:

\[ P_{e} = Q\left( \sqrt{w_{opt}^{T}H_{1,\bar{u}}H_{1,\bar{u}}w_{opt}} - \sum_{j=1}^{N_{u}}w_{opt}^{T}H_{1,j}H_{1,j}w_{opt} + \sigma^{2} \right), \]

where \( H_{1,\bar{u}} \) and \( H_{1,j} \) are convolution matrices of \( h_{1,\bar{u}}^{(\ell,1)} \) and \( h_{1,j}^{(\ell,1)} \) respectively.

VI. SIMULATION RESULTS

Extensive simulations are performed to obtain a fair comparison between the proposed algorithm and the conventional MSSNR algorithm in standard UWB channel models. The comparison is done in terms of captured energy of the channel and BER.

Figure 1 shows the channel captured energy within a shortened channel window of first 20 multipaths in a single user and noise free environment. For CM1 and CM2 \( d=50 \) and for CM3 and CM4 \( d=75 \).

The length of shortened channel window and TEQ is chosen in such a way that the performance of TEQ is consistent. The channel can be shortened to further less taps if the length of TEQ is increased. The proposed algorithm performs well above the conventional MSSNR, especially in less dense multipath channels. For example, in CM1 and CM2 the proposed algorithm captures nearly as double channel energy as the MSSNR algorithm.

The additional optimization parameters \( \psi \) and \( \zeta \) have interestingly different effects on TEQ performance if introduced separately to the conventional MSSNR optimization problem. The gradient parameter \( \zeta \) improves the TEQ performance more significantly as compared to the conditional variance parameter \( \psi \). The reason for this dominating effect is that \( \zeta \) is linear and only related to the shortened channel window. On the contrary, \( \psi \) is quadratic and evaluated from both the shortened and residual channel. The presence of any significant multipaths away from the shortened window will not affect \( \zeta \) but influence \( \psi \) in a quadratic fashion. This situation may lead to spuriously
low conditional variance and degrade TEQ’s performance.

Figure 2 depicts the comparative performance of both algorithms in a multiuser AWGN environment with $N_u = 10$. Remaining of the factors remain same as in the previous case. A similar trend can be observed, the performance gain is larger in less dense multipath channels. It is also observed that MSSNR algorithm’s performance gradually decreases with increasing SINR in a multiuser environment whereas the proposed algorithm performs gradually better. The performance enhancement is a result of the inclusion of channel statistics in the optimization problem.

Figure 3 illustrates the BER performance of both algorithms with respect to lower bound derived in (10). It is important to note that the SINR is evaluated on the basis of transmitted signal energy for comparison purposes. The results of simulations clearly indicate that the proposed algorithm outperforms the MSSNR algorithm in terms of energy capture and BER.

VII. CONCLUSION

In this paper, we presented a modified version of the MSSNR algorithm which exploits some characteristic features of UWB communications systems and channel statistics for performance improvement. It is shown through simulations that these modifications enhance the performance of algorithm in UWB environment. Hence, it enables a RAKE structure with less number of fingers but still capturing good percentage of channel energy. Reduction in the number of fingers in the RAKE front end greatly simplifies the signal processing that follow. Such a RAKE receiver is not only simple from analysis and design point of view but incurs less manufacturing cost from hardware implementation perspective.

REFERENCES