Performance of Cooperative Spatial-interleaved Superposition Modulation in Fading Multiple-Access Channels

Tao Yang and Jinhong Yuan
School of electrical engineering and telecommunications
University of New South Wales, Australia

Abstract—In this paper, we propose a new superposition-modulation based cooperative multiple-access scheme. In this scheme, each user only superimpose the information from the previous user, where the full diversity is achieved by cooperative spatial-interleaving. Focusing on interference cancellation with minimum mean square error filtering at the destination, we derive a lower bound on the frame error probability of the scheme. We show that the bound can be used to predict the frame error rate of the scheme. Moreover, we show that full diversity gain is achieved and the detection complexity is reduced as compared to the previous scheme.

I. INTRODUCTION

In modern wireless communication systems, diversity techniques are adopted to combat the multi-path fading effects. Among those, spatial diversity has been the interest of research for many years. The concept of spatial diversity has been widely explored in the design of multiple-input multiple-output (MIMO) systems in which the physical antenna array can provide diversity, given that the antennae are separated far enough from each other. However, a physical array is not always applicable at a mobile station due to the constraints on its size and power consumption.

Nowadays, the research on employing user-cooperation to provide spatial diversity in wireless networks has evolved into one of the most exciting area in the wireless communication society. As a result of the broadcasting nature of wireless communication, each user in the network can overhear the information from the source node and provide assistance for the transmission to achieve the spatial diversity.

Previous information-theoretic analysis of outage behavior in [1] has shown that several time-division-duplex based orthogonal cooperative schemes achieve full diversity for small-rate region. In [2] and [3], a hybrid scheme which utilizes both decode-and-forward (DF) and amplify-and-forward (AF) has been discussed. Later in [4], a non-orthogonal AF and a dynamic DF are proposed and are shown to outperform the orthogonal AF or DF in terms of diversity-multiplexing trade-off (DMT). Comparing [1] and [4], the drawback of the schemes proposed in [1] is that employing orthogonal time slots for relaying is a suboptimal way of using the radio channel.

In [5], a bandwidth-efficient two-user cooperative multiple-access (CMA) protocol is proposed and is shown to outperform the DF schemes proposed in [1]. The fundamental idea in [5] is that for each users, its partner’s information is processed and superimposed onto its own fresh information. In [6] and [7], the authors extended the protocol in [5] to the scenario with an arbitrary number of users and provided the information-theoretic performance analysis. One feature of the protocol in [6] is that in order to achieve full diversity, each user is required to incorporate the information overheard from all other users. Therefore, the detection at either the source node or destination node is very complicated1 and the analysis of the power allocation for the scheme is tough.

In this paper, we propose a new and more practical CMA scheme based on superposition modulation for two-user and three-user applications. Different from [6], each user in the proposed scheme only superimpose the information transmitted in the previous symbol duration, where the full diversity is achieved by “cooperative spatial-interleaving” and channel coding. Focusing on interference cancellation (IC) with linear minimum mean square error (LMMSE) filtering at the destination, we derive a lower bound on the frame error probability of this scheme. We show that the performance bound can be used to predict the frame error rate (FER) of the scheme. Moreover, we show that full diversity gain can be achieved by the proposed scheme and the complexity is reduced with respect to the scheme in [6][7].

II. SYSTEM MODEL

In this section, we illustrate the system model of the proposed scheme with two and three users. We denote $b_A$, $b_B$ and $b_C$ the information sequences to be transmitted by user $A$, $B$ and $C$, respectively. Each information stream is independently encoded by a rate $R_c$ channel encoder. Then, the coded sequences are independently mapped to constellation symbols. We use $d_A(t)$, $d_B(t)$ and $d_C(t)$ to denote the $t$-th symbols to be transmitted by the three users, respectively.

In this paper, we assume that the transmission of each user undergoes independent quasi-static Rayleigh fading process. The channel coefficients between the users and the destination are denoted by $g_A$, $g_B$, $g_C$ which are circular symmetric complex Gaussian random variables with

1The detection dimension of Gaussian random variables with
zero means and unit variances. Moreover, the fading coefficients between users (inner-user channels) are denoted by $h_{AB}, h_{BC}, h_{CA}$ where the channel between any two users are symmetric. We assume that the fading coefficients of inner-user channels are also circular symmetric complex Gaussian random variables with zero means and variances $\sigma^2$. Furthermore, we consider that the perfect channel state information (CSI) is known by the receiver but is not available at the transmitter.

A. Cooperative Transmission for Two Users

![Frame work of 2 user cooperative transmission.](image)

In the first place, we depict the proposed cooperative transmission protocol for the scenario with two users. The framework is visualized in Fig. 1. In our protocol, we adopt a symbol-by-symbol demodulation-and-forward approach at the relay nodes. For the $t$-th symbol, user $B$ transmits a superposition of its own symbol and the estimated symbol from user $A$, that is

$$x_B(t) = \sqrt{1 - \gamma^2_{AB}d_B(t)} + \gamma_{AB}d_A(t)$$  \hspace{1cm} (1)$$

where $x_B(t)$ stands for the transmitted signal by user $B$, $d_A(t)$ is the estimate of symbol $d_A(t)$ by user $B$ and $\gamma_{AB}$ ($\gamma^2_{AB} < 0.5$) is the power allocation factor for the superposition modulation. In [1], the estimated symbol $d'_A(t)$ is generated by demodulating the symbol which is received by $B$ when $A$ is transmitting. For BPSK, the estimate becomes

$$d'_A(t) = \text{sign}[h^*_{AB}y_B(t)]$$ \hspace{1cm} (2)$$

where

$$y_B(t) = h_{AB}x_A(t) + n_B$$ \hspace{1cm} (3)$$

is the overheard signal by $B$ when $A$ is transmitting and $n_B$ is the AWGN noise at node $B$. In the next symbol duration, the same operation is used with the roles of $A$ and $B$ swapped.

Compared with [5], our scheme uses a symbol-by-symbol demodulation-and-forward approach, rather than the frame-by-frame decode-and-forward approach in [5]. Since the “flag bit” in [5] is not applicable in our scheme, to reduce the estimation error at the relay node in favor of cooperation, we choose [6]

$$\gamma_{AB} = \begin{cases} \Gamma, & \text{if } \rho_{AB} = \overline{\rho}|h^2_{AB}| \geq \Phi \\ 0, & \text{otherwise} \end{cases}$$ \hspace{1cm} (4)$$

where $\overline{\rho}$ is the average signal-to-noise ratio (SNR), $\Phi$ is an SNR threshold and $\Gamma$ is a fixed value for the power allocation. If the inner-user channel is weak, there is no cooperation. Otherwise, there is cooperation and $\gamma_{AB} = \Gamma$. The reason why we adopt symbol-by-symbol demodulation-and-forward is to enable “cooperative spatial-interleaving” which will be explained next.

B. Cooperative Transmission for Three Users

Now, we depict the proposed cooperative transmission protocol for the scenario with three users. Our purpose is to design a simple cooperative transmission scheme based on superposition modulation while achieving the full diversity gain.

![Frame work of 3 user cooperative transmission.](image)

![Frame work of 3 user cooperative transmission in [6] and [7].](image)

The frame work of the proposed scheme with three users is shown in Fig. 2 and that of [6] is shown in Fig. 3. In our proposed scheme, a user is helped by only one of the other two users in each phase of cooperation. However, by swapping the order of transmission of the other two users, this user is helped by two different users at two adjacent symbol duration. For example, user $A$’s information is relayed by user $B$ at duration $t = 1$ whereas it is relayed by user $C$ at duration $t = 2$. Similar cases apply to user $B$ and $C$. By using the transmission swapping, the coded digits of a codeword are relayed from different locations which provides spatial diversity. In this paper, we refer to this approach as cooperative spatial-interleaving (CI). From Fig. 2 and Fig. 3, it is apparent that the proposed scheme is simpler than that proposed in [6].

In a coded system, by adopting cooperative spatial-interleaving, different coded digits in the codeword may experience different relay (cooperative) paths. In addition,
thanks to the superposition cooperation, each single coded digit has virtually two paths by itself. This scheme can be directly extended to a system with \( K \) users. Intuitively, as long as the code free distance \( d_{free} \) is larger than \( K - 1 \), the scheme can achieve a full diversity order of \( K \).

Similar to the scenario with two users, the degree of cooperation of the three-user case is determined by the quality (instantaneous SNR) of the inner-user channels. The power allocation factor for the cooperative transmission of user \( i \) and user \( j \) is

\[
\gamma_{ij} = \begin{cases} \Gamma, & \text{if } \rho_{ij} = \frac{1}{h_{ij}^2} \geq \Phi \\ 0, & \text{otherwise} \end{cases}
\]  

(5)

where \( i, j \in \{ A, B, C \}, j \neq i \). If the effective SNRs of all the inner-user channels are smaller than the threshold specified in (5), the scheme falls back to the direct transmission strategy (\( \gamma_{AB} = \gamma_{BC} = \gamma_{CA} = 0 \)). If all the inner-user channels are of good quality, full cooperation will be enabled, that is \( \gamma_{AB} = \gamma_{BC} = \gamma_{CA} = \Gamma \). Moreover, there are other possibilities such as \( \gamma_{AB} = \gamma_{BC} = \Gamma, \gamma_{CA} = 0 \). At high SNRs, the probability of full cooperation approaches 1. The formulation of this three-user cooperation can be easily obtained from that of two-user scenario and we omit it in this paper.

### III. Linear Symbol-by-Symbol Detection at the Destination

In this section, we present a linear symbol-by-symbol detection algorithm for the proposed cooperative transmission protocol. Without loss of generality and for the ease of illustration, we assume that BPSK modulation and a half-rate repetition code are employed. Moreover, we focus on interference cancellation (IC) with linear minimum mean square error (LMMSE) filtering for the symbol-by-symbol detection. In particular, we derive the optimal filter coefficients for the IC-LMMSE algorithm. Then, we provide a lower bound on the frame error probability (FEP) of the scheme.

#### A. Two-User Case

![Block diagram of detection at the destination for two-user case](image)

1) **IC-LMMSE Algorithm**: From Fig. 1, we see that the information of \( d_A(t) \) is imparted by two successive symbols \( x_A(t) \) and \( x_B(t) \), whereas \( d_B(t) \) is conveyed by \( x_B(t) \) and \( x_A(t+1) \). Now, let us focus on the detection of \( d_A(t) \). The block diagram of the detection is shown in Fig. 4. To detect \( d_A(t) \), we use interference cancellation to take away \( \gamma_{AB} d_B(t-1) \) from \( x_A(t) \) and remove \( \sqrt{1 - \gamma_{AB}^2} d_B(t) \) from \( x_B(t) \). Then, LMMSE filtering is used to combine the information relevant to \( d_A(t) \).

At the destination \( D \), the received signal is represented as

\[
y_{DA}(t) = g_A \left[ \sqrt{1 - \gamma_{AB}^2} d_A(t) + \gamma_{AB} d_B(t-1) \right] + n_{DA}
\]  

(6)

\[
y_{DB}(t) = g_B \left[ \sqrt{1 - \gamma_{AB}^2} d_B(t) + \gamma_{AB} d_A(t) \right] + n_{DB}
\]  

(7)

For BPSK modulation, we have

\[
d_B(t-1) = s_B d_B(t-1)
\]  

(8)

and

\[
Pr(s_B = -1) = Pr(s_B = 1) = \frac{Q(\sqrt{\rho_{AB}}) + Q(\sqrt{\rho_{AB}(1-2\gamma_{AB}^2)})}{2}
\]  

(9)

where \( Q(\bullet) \) stands for Q-function. By using (8), (6) becomes

\[
y_{DA}(t) = g_A \left[ \sqrt{1 - \gamma_{AB}^2} d_A(t) + s_B \gamma_{AB} d_B(t-1) \right] + n_{DA}
\]  

(10)

and similarly,

\[
y_{DB}(t) = g_B \left[ \sqrt{1 - \gamma_{AB}^2} d_B(t) + s_A \gamma_{AB} d_A(t) \right] + n_{DB}
\]  

(11)

The interference cancellation yields

\[
r_{DA}(t) = y_{DA}(t) - g_A \gamma_{AB} \hat{d}_B(t-1)
\]  

(12)

\[
= \left\{ \begin{array}{ll} g_A \left[ \sqrt{1 - \gamma_{AB}^2} d_A(t) \right] + n_{DA} & \text{if } s_B = -1 \\ g_A \left[ \sqrt{1 - \gamma_{AB}^2} d_A(t) - 2 \gamma_{AB} d_B(t-1) \right] + n_{DA} & \text{otherwise} \end{array} \right.
\]

(13)

where \( \hat{d}_B(t-1) \) is the hard estimate on \( d_B(t-1) \). We assume that \( d_B(t-1) \) is perfectly recovered in the previous detection, that is \( \hat{d}_B(t-1) = d_B(t-1) \). Also, we have

\[
r_{DB}(t) = y_{DB}(t) - g_B \sqrt{1 - \gamma_{AB}^2} \cdot \text{sign}[g_B y_{DB}(t)]
\]

\[
= \left\{ \begin{array}{ll} g_B s_A \gamma_{AB} d_A(t) + n_{DB}, & \text{if } \text{sign}[g_B y_{DB}(t)] = d_B(t) \\ g_B [s_A \gamma_{AB} d_A(t) + 2 \sqrt{1 - \gamma_{AB}^2} d_B(t)] + n_{DB}, & \text{otherwise} \end{array} \right.
\]

(14)

and

\[
Pr\{\text{sign}[g_B y_{DB}(t)] \neq d_B(t)\} = Q\left(\sqrt{1 - 2\gamma_{AB}^2} \cdot \rho_{DB}\right) + Q\left(\sqrt{\rho_{DB}}\right) = Q_B.
\]

Now, let us derive the coefficients of the LMMSE filter [8][9], which is given by

\[
w_{AB} = \arg \min_{w_{AB}} \left\{ \left| d_A(t) - w_{AB} [r_{DA}(t), r_{DB}(t)]^T \right|^2 \right\}
\]

(15)
where the cross-correlation term is
\[ c = E \{ d_A(t) \cdot [r^*_D(t), r^*_B(t)] \} \] (16)
and the auto-correlation term is
\[ \Sigma = E \{ [r^*_D(t), r^*_B(t)]^T [r^*_D(t), r^*_B(t)] \}. \] (17)
By using (12) and (13) in (16), we get
\[ c = g_A \sqrt{1 - \gamma^2_{AB}} \cdot g_B \gamma_{AB} (1 - 2P_{AB}) \] (18)
where \( P_{AB} \) is defined in (9). Also, by substituting (12) and (13) in (17), we have
\[ \Sigma = \begin{bmatrix} U & K^* \\ K & V \end{bmatrix} + \sigma^2_D I_2 \] (19)
where
\[ U = |g_A|^2 (1 - \gamma^2_{AB} + 4P_{AB} \gamma^2_{AB}) \] (20)
\[ V = |g_B|^2 (\gamma^2_{AB} - 4Q_B \gamma^2_{AB} + 4Q_B) \] (21)
\[ K = g_A g_B \sqrt{1 - \gamma^2_{AB} \gamma_{AB} (1 - 2P_{AB})} \] (22)
and \( Q_B \) is defined in (14). In (19), \( I_2 \) is an identity matrix with dimension of two and \( \sigma^2_D \) is the AWGN noise variance at destination. It is worthy noting that when \( \gamma^2_{AB} = 0 \), which implies that the inner-user channel is weak, the filtering coefficients becomes \( w_{AB} = \begin{bmatrix} |g_A|^2 & g_A^* & 0 \end{bmatrix} \) and the scheme falls back to direct transmission.

After IC and LMMSE filtering, the output signal follows a Gaussian distribution with mean \([8][9]\)
\[ \mu_A = c \Sigma^{-1} c^H \] (23)
and variance
\[ \sigma^2_A = c \Sigma^{-1} c^H (1 - c \Sigma^{-1} c^H). \] (24)
where \((\bullet)^H\) stands for Hermitian transpose. The symbol log-likelihood ratio (LLR) w.r.t user \( A \) is written as
\[ \Lambda^s_A(t) = \frac{\mu_A}{\sigma_A} w_{AB} [r^*_D(t), r^*_B(t)]^T. \] (25)
After the symbol of \( A \) is recovered, the same procedure is used to generate the LLR \( \Lambda^b_B(B) \) for user \( B \).

For a repetition-coded system with \( R_c = \frac{1}{2} \), the bit LLR w.r.t user \( A \) is computed by
\[ \Lambda^b_A(I) = \Lambda^s_A(2l - 1) + \Lambda^s_A(2l) \] (26)
where \( l \in [1, 2, ..., L]^2 \) and \( L \) is the frame length.

2) **Lower Bound on the Frame Error Probability:** Now, we derive a lower bound on the frame error probability of the scheme. According to (23)-(26), the output bit-level SNR for a specific CMA channel is
\[ \Omega_{AB|h_{AB},g_{AB}} = \frac{2c \Sigma^{-1} c^H}{1 - c \Sigma^{-1} c^H} \] (27)
Then, the conditional bit error probability is
\[ P_b(d_A|h_{AB},g_{AB}) = Q \left( \sqrt{\Omega_{AB|h_{AB},g_{AB}}} \right). \] (28)
As a result, the conditional frame error probability is
\[ P_f(d_A|h_{AB},g_{AB}) = 1 - [1 - P_b(d_A)]^L \] (29)
and the unconditional frame error probability is
\[ P_f(d_A) = \int_{h_{AB},g_{AB}} 1 - [1 - P_b(d_A)]^L d_{h_{AB}} d_{g_{AB}} \] (30)
which can be numerically evaluated. Since we assumed that \( d_B(t - 1) \) is perfectly recovered in the above derivation, (30) provides a lower bound on the FER.

**B. Three-User Case**

The block diagram of the detection for the three-user case is shown in Fig. 5, where the half-rate repetition code is considered. The detection algorithm for three-user case can be straightforwardly generalized from that of the two-user case. For user \( A \), the transmission of each information bit is helped by user \( B \) and user \( C \), alternately. Each symbol subject to the same information bit is detected by using IC-LMMSE algorithm. Then, the LLRs of the symbols subjected to that information bit are summed up, yielding the LLR for that information bit. It is apparent that the dimension of detection for each symbol remains two for the three-user case.

**IV. SIMULATIONS**

In this section, we present the performance of the proposed scheme. In all the simulations, we employ 4-QAM modulation and half-rate repetition coding for all the users. The frame length \( L \) is 64 bits per frame. The threshold \( \Phi \) in (4) is set to 10dB and the power allocation factor \( \Gamma \) is set to \( \sqrt{0.15} \) [5].
Perform ance of the proposed scheme where the inner-user channel SNR is the same as user-destination SNR.

The FER performance of the proposed scheme is plotted in Fig. 6 where the inner-user channels and the user-destination channels are of the same SNRs\(^3\). The dashed curves are the FEP lower bounds which are numerically evaluated from (30). The solid curves are the FERs obtained by using Monte-Carlo simulations. It is clear that the proposed scheme is able to provide a larger diversity gain as the number of users increases. At FER=10^-3, the three-user scheme is about 2.4dB better than the two-user scheme since it has a larger diversity gain. It also shows that the lower bound is reasonably tight to the simulated performance.

In Fig. 7, we show the performance of the scheme where the SNR of inner-user channels is 5dB better than that of user-destination channels. At FER=10^-3, the three-user scheme is about 3.1dB better than the two-user scheme in this scenario. As the SNR of inner-user channels increases, we observe a larger performance improvement of the three-user scheme over the two-user scheme. Since the power allocation factor for three-user application of the scheme in [6][7] is not available, we did not compare our scheme with that in [6][7] in this paper.

Now, we give a brief comment on the complexity of this scheme. Compared to the scheme proposed in [6][7], in which the dimension of detection is of \(K\), the detection of our scheme is of a dimension smaller than \(K\). For the optimal maximum a posteriori probability (MAP) detection which is of an exponential complexity, the complexity reduction would be significant as the number of users \(K\) becomes larger. In addition, the power allocation for our scheme is much easier than that for [6][7].

V. CONCLUSIONS AND FUTURE WORKS

In this paper, we proposed a new cooperative transmit diversity scheme based on superposition modulation. Compared to the previously proposed schemes, full diversity is achieved by employing cooperative spatial-interleaving. We presented the IC-LMMSE detection for the scheme and derived a performance lower bound. Simulation showed that full diversity gain can be achieved and the performance bound is reasonably tight to the simulated FER. Moreover, the complexity of the scheme is smaller than the formerly proposed scheme in [6] and [7].

In the future, we will derive an upper bound on the performance of this scheme, where the assumption that the previous coded bits are correctly recovered is no longer held. Furthermore, the power allocation profile has yet to be optimized. Moreover, the performance analysis will be extended to the scheme with a better error-control code.

REFERENCES


\(^3\)The horizontal axis is the average SNR per bit at the destination.