Dynamic Modeling of a Mobile Humanoid Robot

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Abstract—This paper first presents a modeling method for a mobile humanoid robot whose upper human-like body is mounted on a mobile platform supported by three wheels (two driving wheels and one caster wheel). The upper body connects the platform by two-DOF joint and the wheeled platform moves on the ground subject to nonholonomic constraints. Then based on Lie groups and screw theory, the kinematics and dynamics of the nonholonomic mobile humanoid robot are developed by simplifying the model into five parts and the dynamic equations of the entire system are formulated from the aspect of energy.

Index Terms—Dynamics, Mobile Humanoid Robot, Nonholonomic Constraints, Screw Theory.

I. INTRODUCTION

In recent years, many studies about humanoid robots have been mainly focused on realizing human-like actions such as walking, jumping, dancing, going up or down stairs and carrying things. The humanoid robots are divided into the biped humanoid robots (BHR) and the mobile humanoid robots (MHR) classified by the structures, and a detailed survey has been made on the current mobile humanoid robots in our another related work [1]. Almost all the MHRs have the similar designs of the upper bodies mounted on the mobile bases, and their advantages and disadvantages of the MHRs are discussed and compared with the BHRs in [1].

Since the MHR is able to perform manipulation tasks in a much larger workspace than a fixed-base manipulator, it is necessary to understand how to properly and effectively coordinate the motions of the mobile platform and the upper manipulators because of the dynamic interactions between them. Based on these considerations, this paper provides a systemic method for the kinematics and dynamic modelling of a typical MHR based on screw theory [2].

Some related works can be found on studying the dynamics of mobile robots, which are highlighted in the follows. In [3], the impedance method is applied to control MR Helper and the control algorithm is implemented in the experimental system for handling an object in cooperation with the robot and the human. The dynamics modeling of a nonholonomic mobile manipulator is studied, which consists of one multi-DOF serial manipulator and a conventional wheeled mobile platform, the recursive velocity and acceleration of each component of the system are derived in the global reference frame, the dynamic equations of the mobile manipulator systems are established by utilizing forward recursive formulation for open-loop multi-body systems [4]. A dynamic model is established for the mobile manipulator, which explicitly takes into account the dynamic interactions between the manipulator and the mobile platform, then designs a nonlinear feedback controller for the mobile manipulator that is capable of fully compensating the dynamic interaction [5].

The dynamic modeling and tracking control of a nonholonomic wheeled mobile manipulator with two robotic arms are studied and a tracking controller with fully dynamic compensation ability is designed for the mobile manipulator system via Lyapunov stability theory [6]. The dynamics and various control methods are investigated on the mobile manipulator [7].

In this paper, a mobile humanoid robot is designed and is composed of two 7-DOF arms, the upper body and the mobile platform, which is supported by two driving wheels and one caster wheel. In Section II, the configuration of the mobile humanoid robot is shown and many definitions are given out. In Section III, the kinematics of different parts are developed. The dynamic equations of the entire system are formulated using Lagrangian equations in Section IV. Finally, the conclusion remarks are obtained in the last section.
II. ARCHITECTURE OF THE MOBILE HUMANOID ROBOT

The configuration of the mobile humanoid robot is the upper human-like body mounted on the mobile platform, which has two 7-DOF arms as shown in Fig. 1. The detailed description can be found in our related work [8].

In order to realize easy modeling, the whole upper body system can be divided into five subsystems (mobile platform, waist, head and dual arms) and five kinematic chains are introduced as shown in Fig. 2.

- **The basic kinematic chain**: From the global frame $G$ to the frame of the platform $C$;
- **The main serial kinematic chain**: From the frame of the platform $C$ to the frame of the shoulder $M$;
- **The chain of the head**: From the frame of the shoulder $M$ to the frame of the head $H$;
- **The chain of the left arm**: From the frame of the shoulder $M$ to the frame of the left arm $i$th joint $L_i$;
- **The chain of the right arm**: From the frame of the shoulder $M$ to the frame of the right arm $i$th joint $R_i$;

Two assumptions are adopted in the modeling of the mobile humanoid robot system.

- Two 7-DOF arms are installed laterally and symmetrically.
- There are no slipping and no sideways between the wheels and the floor.
- Every link of the robot is rigid.

The notations shown in Fig. 2 will be used in the derivation of the kinematics and dynamics of the MHR, and their detailed information can be found in our work [8].

III. KINEMATIC ANALYSIS OF THE SYSTEM

For the kinematics of the mobile humanoid robot, another work had been discussed [8], but different method is used here. The work [9] undertakes the dynamic modeling of omni-direction mobile manipulator based on the screw theory, Lie groups, reciprocal product of twist and wrench and Joudain principle. Following the work [9], we build up the modeling of a mobile humanoid robot with screw theory introduced in [2].

The rotating angles of the two driving wheels are chosen as the generalized coordinates because the nonholonomic constrained mobile platform owns only 2-DOF under planar motion assumption. Certainly, all are done with the assumptions of the no-slipping and no-sideway motions.

A. Kinematics of the platform

The pose matrix of the body fixed frame $C$ relative to the global frame $G$ can be written as

$$ R_c = \begin{bmatrix} c\varphi & -s\varphi & 0 \\ s\varphi & c\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1) $$

where $s\varphi = \sin \varphi, c\varphi = \cos \varphi$, the same denotations will be used to other following angles.

The spatial velocity of the frame $C$ can be known

$$ \dot{V}_c = g^{-1} \dot{R}_c = \begin{bmatrix} 0 & -\dot{\varphi} & \dot{v}_1^T \\ \dot{\varphi} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, V_c = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad (2) $$

The centers of mass (CM) of two driving wheels expressed in the frame $C$ have the vectors as shown in Fig. 2, $p_{o1}^C = [-l, -a, -h]^T, p_{o2}^C = [l, -a, -h]^T$.

The transformation of points and vectors by rigid transformations has a simple representation in terms of matrices and vectors in $\mathbb{R}^4$ [2], we append 1 to the homogeneous coordinates of a point $p$ to yield a vector and append 0 to the vector $v$ in $\mathbb{R}^4$ as following,

$$ \tilde{p} = \begin{bmatrix} p^T \\ 1 \end{bmatrix}, \tilde{v} = \begin{bmatrix} v^T \\ 0 \end{bmatrix} \quad (3) $$

Thus the centers of two driving wheels in the frame $C$ have the velocity form,

$$ V_{oi} = g\dot{V}_c \tilde{p}_{oi}^C, \ (i = 1, 2), \quad (4) $$

Fig. 2. The skeleton of the mobile humanoid robot
Let $v_y$ be the vector which align with the $y$ axis of frame $C$ in the same direction and $v_x$ be the vector which align with the $x$ axis of frame $C$ in the same direction, $v_y = [0 \ 1 \ 0]^T$, $v_x = [1 \ 0 \ 0]^T$. The instantaneous tangential velocity of the driving wheel is $r \theta_i$ and the velocity in the direction of $o_1o_2$ is zero due to the nonholonomic constraints.

$$ (g^*v_y)^T g^*V_c \hat{p}_{i0} = v_{i0}^y = -r \theta_i, $$

$$ (g^*v_x)^T g^*V_c \hat{p}_{i0} = v_{i0}^x = 0, \quad (i = 1, 2) \quad (5) $$

where $\theta_i$ means only rotation was considered and $g^T g^* = I$.

Let $\hat{\theta}_i = [\hat{\theta}_1 \ \hat{\theta}_2]$ combining Eq. 2, 3 and 5, the velocity of the platform CM $V_C$ in the body-fixed frame can be expressed as

$$ V_c = \begin{bmatrix} v_c \\
\omega_c 
\end{bmatrix}_{\times 1} = P_p \begin{bmatrix} \hat{\theta}_1 \\
\hat{\theta}_2 
\end{bmatrix} = P_p \hat{\theta}_a \quad (6) $$

where $P_p \in R^{6 \times 2}$, the spatial form is $P_p = \frac{1}{2}$$

So differentiating Eq. 6 the acceleration of the platform CM in the body-fixed frame will have the following forms

$$ \ddot{V}_c = \begin{bmatrix} \dot{v}_c \\
\dot{\omega}_c 
\end{bmatrix} = P_p \ddot{\theta}_a \quad (7) $$

B. Kinematics of the driving wheels

The pose matrix of the body fixed frame $e_i (i = 1, 2)$ of driving wheels relative to the body fixed frame $C$ can be written as

$$ R_{e_i} = \begin{bmatrix} 1 & 0 & 0 \\
0 & c \theta_i & -s \theta_i \\
0 & s \theta_i & c \theta_i \n\end{bmatrix}, \quad (8) $$

The CMs of the driving wheels have the position vectors $p_{oi}$ in the frame $C$ as following,

$$ p_{oi} = \begin{bmatrix} -l \\
-a \\
-h \n\end{bmatrix}, \quad p_{ao} = \begin{bmatrix} l \\
-a \\
-h \n\end{bmatrix} \quad (9) $$

Let $g_{ce_i}$ denote the configuration matrix of the $i$–th wheel’s frame relative to the frame $C$ and it can be expressed as

$$ g_{ce_i} = \begin{bmatrix} R_{e_i} & p_{oi} \\
0 & 1 \n\end{bmatrix} = e^{\hat{e}_i o \theta_i} g_{ce}(0), \quad (10) $$

where $\xi_{e_i} \in R^6$ is the twist for the screw motion of the wheel’s frame relative to the body-fixed frame, $g_{ce}(0)$ is the initial configuration whose $X_e, Y_e, Z_e$ are aligned to the frame $C$ respectively, $g_{ce}(0) = \begin{bmatrix} I & p_0 \\
0 & 1 \n\end{bmatrix}$.

The body twist of the wheel frame $e_i$ relative to the frame $C$ in the body-fixed reference frame becomes

$$ V_{ce} = (g^*_{ce} \xi_{ce})^\nu = Ad_{e_i(0)} \xi_{ce} \hat{\theta}_i = B_d \ddot{\theta}_i \quad (11) $$

where $Ad_{e_i}$ denotes the adjoint transformation of the configuration $g_{ce},$

$$ Ad_{e_i(0)} = \begin{bmatrix} I^T & -I^T \hat{p}_{oi} \\
0 & I^T \n\end{bmatrix}, \quad \hat{p}_{oi} = \begin{bmatrix} 0 & h & -a \\
h & 0 & l \\
a & -l & 0 \n\end{bmatrix} \quad (12) $$

Then the CM velocities $V_{ei}$ of the frame $e_i$ will be known through the transformation of body velocities between the frame $C$ and $e_i$ [2],

$$ V_{ei} = Ad_{e_i} V_c + V_{ce} \quad (13) $$

Substituting Eq. 11 and 13 into Eq. 12, the CM velocities $V_{ei}$ will be

$$ V_{ei} = \begin{bmatrix} v_{ei} \\
\omega_{ei} \n\end{bmatrix} = Ad_{e_i} P_p \ddot{\theta}_a + B_d \ddot{\theta}_a \quad (14) $$

where $P_{di} \in R^{6 \times 2},$

$$ P_{d1} = \begin{bmatrix} 0 & 0 & 0 \\
-r c \theta_1 & 0 & 0 \\
0 & rs \theta_1 & 0 \n\end{bmatrix}, \quad P_{d2} = \begin{bmatrix} 0 & 0 & 0 \\
r c \theta_2 & 0 & 0 \\
0 & rs \theta_2 & 0 \n\end{bmatrix} $$

The CM accelerations of the frame $e_i$ will be derived by differentiating Eq. 14, and the detailed forms are neglected here.

$$ V_{ei} = \begin{bmatrix} \dot{v}_{ei} \\
\dot{\omega}_{ei} \n\end{bmatrix} = \ddot{P}_p \ddot{\theta}_a + P_d \ddot{\theta}_a \quad (15) $$

C. Kinematics of the bracket

It is easy to obtain the pose matrix of the body fixed frame $e_b$ of the bracket relative to the body fixed frame $C$ as

$$ R_{eb} = \begin{bmatrix} c \psi & -s \psi & 0 \\
s \psi & c \psi & 0 \\
0 & 0 & 1 \n\end{bmatrix} \quad (16) $$

The CM of the bracket frame has the position vectors $p_{ob}$ in the frame $C$ as following,

$$ p_{ob} = \begin{bmatrix} 0 \\
0 \\
-h_b \n\end{bmatrix} \quad (17) $$

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With the similar method in the above section, the CM velocities \( \dot{V}_{eb} \) in the frame \( eb \) will be known through the transformation of body velocities,

\[
\dot{V}_{eb} = Ad_{g^{e_b}_{c_b}} V_e + V_{ce_b} \tag{18}
\]

where the parameters \( g_{ce_b}, \xi_{ce_b} \in \mathbb{R}^6 \), \( g_{ce_b}(0) \) and \( Ad_{g^{e_b}_{c_b}} \) of the bracket have the same denotations as the above section and the detailed definitions are neglected here.

\[
V_{ce_b} = (g^{e_b}_{c_b})^{-1} \dot{V}_{ce_b} = Ad_{g^{e_b}_{c_b}(0)} \dot{\xi}_{ce_b} \dot{\psi} = B_b \dot{\psi} \tag{19}
\]

where

\[
B_b = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T
\]

The CM velocities of the bracket frame \( eb \) relative to the frame \( C \) in the body-fixed reference frame is written as

\[
V_{eb} = \begin{bmatrix} \dot{V}_{eb} \\ \omega_{eb} \end{bmatrix} = Ad_{g^{e_b}_{c_b}} P_b \dot{\theta}_a + B_b \dot{\psi} = P_b \dot{\theta}_a' \tag{20}
\]

where \( \dot{\theta}_a' = [\dot{\theta}_1 \ \dot{\theta}_2 \ \dot{\psi}]^T \).

With the definitions given below: \( \alpha = \frac{\theta_2 - \theta_1}{2} \), \( \delta = \frac{\theta_2 + \theta_1}{2} \), \( \rho = \frac{\dot{\psi}}{2} \),

\[
P_b = \begin{bmatrix} -r(\alpha \psi + (s \psi)/2) & r(\alpha \psi - (s \psi)/2) & 0 \\ r(\alpha \psi - (s \psi)/2) & -r(\alpha \psi + (s \psi)/2) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \delta \rho & -\delta \rho & 1 \end{bmatrix}
\]

Differentiating Eq. 20, the CM accelerations of the bracket frame \( eb \) will be derived by

\[
\ddot{V}_{eb} = \begin{bmatrix} \ddot{V}_{eb} \\ \omega_{eb} \end{bmatrix} = P_b \ddot{\theta}_a' + P_b \ddot{\psi} \tag{21}
\]

**D. Kinematics of the caster wheel**

Similarly, \( R_{e_b e_a} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c \theta_3 & -s \theta_3 \\ 0 & s \theta_3 & c \theta_3 \end{bmatrix} \), through the transformation of body velocities between the frame \( eb \) and the frame \( e_3 \), the body velocities \( V_{e_3} \) of the frame \( e_3 \) will be obtained easily,

\[
V_{e_3} = Ad_{g^{e_3}_{e_b}} V_{eb} + V_{ge_3} \tag{22}
\]

where \( V_{ge_3} = B_3 \dot{\theta}_3 \), the CM of the castor wheel frame has the position vectors \( p_{ge} \) in the frame \( e_b \) as \( p_{ge} = [0 \ d \ -h_3]^T \), and the parameters \( g_{e_3 e_b}, \xi_{e_3 e_b} \in \mathbb{R}^6 \), \( g_{e_3 e_b}(0) \), \( Ad_{g^{e_3}_{e_b}} \) of the castor wheel frame can be got easily as above,

\[
B_3 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix}^T
\]

So, combining the Eq. 20 and 22, the CM velocities of the castor wheel frame \( e_3 \) relative to the frame \( C \) in the body-fixed reference frame is written as

\[
V_{e_3} = \begin{bmatrix} \dot{V}_{e_3} \\ \omega_{e_3} \end{bmatrix} = Ad_{g^{e_3}_{e_b}} V_{eb} + B_3 \dot{\theta}_3 = P_c \dot{\theta}_3 = P_c \dot{\theta}_a' \tag{23}
\]

where \( \dot{\theta}_a' = [\dot{\theta}_1 \ \dot{\theta}_2 \ \dot{\psi} \ \dot{\theta}_3]^T \), \( P_c \in \mathbb{R}^{6 \times 4} \),

\[
P_c = \begin{bmatrix} -r(\alpha \psi + (s \psi)/2 + \delta) & r(\alpha \psi - (s \psi)/2 + \delta) & -d & 0 \\ r(\alpha \psi - (s \psi)/2) & -r(\alpha \psi + (s \psi)/2) & 0 & 0 \\ -r(\alpha \psi - (s \psi)/2) & r(\alpha \psi + (s \psi)/2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \delta \rho \dot{\theta}_3 & -\delta \rho \dot{\theta}_3 & \theta_3 & 0 \\ \theta_3 \dot{\rho} \theta_3 & -\theta_3 \dot{\rho} \theta_3 & c \theta_3 & 0 \end{bmatrix}
\]

Differentiating Eq. 23, the CM acceleration \( \ddot{V}_{e_3} \) of the castor wheel frame will be obtained,

\[
\ddot{V}_{e_3} = \begin{bmatrix} \ddot{V}_{e_3} \\ \omega_{e_3} \end{bmatrix} = P_c \ddot{\theta}_a' + P_c \ddot{\psi} \tag{24}
\]

**E. Kinematics of the waist, head and dual arms**

With the introduction of easy modeling and five kinematic chains in previous section, it will be convenient to deduce the kinematics of the waist, head and dual arms.

As shown in Fig. 3, \( G, C, M, H, L, R \) represent respectively the global frame, the frame of platform, shoulder, the end-effector of head, left arm and right arm, and they are consistent with Fig. 2. The frame \( C \) to the frame \( M \) is taken as the main serial kinematic chain and the frame \( M \) will be the common frame shared by the kinematic chains of the head and dual arms.

Three kinematics loops are formulated:

- **loop a**: it contains the chain \( C \), chain \( M \) and chain \( L \).
- **loop b**: it contains the chain \( C \), chain \( M \) and chain \( R \).
- **loop c**: it contains the chain \( C \), chain \( M \) and chain \( H \).

The kinematics of any link in the chain or loop can be got by the transformation from the reference frame. In this paper, as the above section, the body velocities and accelerations of the links expressed in the body-fixed frame are derived through the transformations relative to the frame \( C \). It is a straight-forward but not the only method. With
the introduction of kinematic loop, they can also be obtained relative to other frames like L or R.

For the arbitrarily chosen link of the left arm in the loop a, the configuration of the frame $L_{i}$ relative to the frame $C$ has the form

$$g_{c,i} = e^{\xi_{m_{1}} q_{m_{1}}} e^{\xi_{m_{2}} q_{m_{2}}} \cdots e^{\xi_{m_{i}} q_{m_{i}}} g_{c,l}(0), \quad (i \leq 7)$$

(25)

where $\xi_{m_{1}}, \xi_{m_{2}} \in R^{6}$ are the twists for the screw motion of the waist joints, $\xi_{m_{i}} q_{m_{i}}$ is the twist of the $i_{th}$ link frame in the left arm, $g_{c,l}(0)$ is the initial configuration of the frame $L_{i}$ relative to the frame $C$ as shown in Fig. 2.

The body velocity of the $i_{th}$ link frame relative to frame $C$ is given by

$$V_{c,i} = J_{c,l}(q_{i})\dot{q}_{i}$$

(26)

where $q_{i} = [q_{m_{1}}, q_{m_{2}}, q_{l_{1}}, \ldots, q_{l_{i}}]^T$, $J_{c,l}$ is the body Jacobian corresponding to $g_{c,l}$ [2]. $J_{c,l}$ has the form

$$J_{c,l}(q_{i}) = [\xi_{m_{1}}, \xi_{m_{2}}, \xi_{l_{1}}, \ldots, \xi_{l_{i}}, 0, \ldots, 0]$$

(27)

where $\xi_{l_{i}}$ is the $j_{th}$ instantaneous joint twist relative to the $i_{th}$ link frame,

$$\xi_{l_{j}} = Ad_{\xi_{l_{j}}(0)}^{-1}\partial\xi_{l_{j}}(\theta_{l_{j}}) \quad (j \leq i)$$

So the CM velocity of $i_{th}$ link frame can be written as

$$V_{i} = \left[ v_{i} \quad \omega_{i} \right] = Ad_{\xi_{i}} V_{c} + V_{c,i}$$

(28)

Combining Eq. 6, 25, 26, 27 together, the CM velocity of $i_{th}$ link frame will be

$$V_{i} = Ad_{g_{c,i}} P_{i} \dot{\theta}_{a} + J_{c,i}(q_{i})\dot{q}_{i} = P_{i} \dot{\theta}_{a} + J_{c,i}(q_{i})\dot{q}_{i}$$

(29)

where $P_{i} \in R^{6 \times 2}$, and $P_{i} = Ad_{g_{c,i}} P_{p}$

Differentiating Eq. 29, the CM acceleration of $i_{th}$ link frame will be written as

$$\ddot{V}_{i} = \left[ \ddot{v}_{i} \quad \dot{\omega}_{i} \right] = P_{i} \ddot{\theta}_{a} + P_{i} \dot{\theta}_{a} + J_{c,i}(q_{i})\ddot{q}_{i} + J_{c,i}(q_{i})\dot{q}_{i}$$

(30)

Similarly for any link of the right arm in the loop b or any link of the head in the loop c or the link of the waist in any loop, their kinematics can be derived with the above method. The kinematics equations about them are not described here.

In order to formulate the dynamics and to provide the basis for further simulations of the whole robot, the flowchart of kinematics solution is given out in Fig. 4.

IV. DYNAMICS OF THE MHR

The whole mobile humanoid robot can be divided into five main parts: the mobile platform, waist, left arm, right arm and head. The mobile platform is composed of two driving wheels, the caster wheel, bracket and the platform, which all do not affect the potential energy of the whole robot. So the dynamics are analyzed here from the aspect of energy (both kinetic energy and potential energy).

There are 23 parts or links in this robot and their mass denotations $m_{p}, m_{w_{1}}, m_{w_{2}}, m_{w_{3}}, m_{b}, m_{m}, m_{l_{1}}, m_{l_{2}}, m_{h}$ represent respectively the platform, driving wheel 1, driving wheel 2, caster wheel, bracket, $i_{th}$ link of waist, $i_{th}$ link of left arm, $i_{th}$ link of right arm and $i_{th}$ link of head.

The Lagrangian for the robot in terms of the joint angles and joint velocities is of the form

$$L(q, \dot{q}) = \frac{1}{2} \dot{q}^{T} M(q) \dot{q} - V(q)$$

(31)

where $q \in R^{n}$ are the joint angles,

$$q = [\theta_{1}, \theta_{2}, \theta_{3}, \psi, \varphi, q_{m_{1}}, q_{m_{2}}, \ldots, q_{l_{1}}, q_{l_{2}}, \ldots, q_{m_{1}}, q_{h_{1}}, q_{h_{2}}]^{T}$$

$M(q)$ is the inertia matrix and $V(q)$ is the potential energy due to gravity.

Through the substitution and transformation [2], the dynamic equation can be rewritten as

$$M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + N(q, \dot{q}) = \tau$$

(32)

where $M(q)$ is the manipulator inertial matrix, $C(q, \dot{q})$ is the Coriolis matrix for the manipulator, the vector $C(q, \dot{q}) \dot{q}$ gives the Coriolis and centrifugal force terms in the equations of motion, $N(q, \dot{q})$ includes gravity terms and other forces which act at the joints and $\tau$ is the vector of actuator torques.

For the first term of Eq. 32, it has the form

$$M(q) = \begin{bmatrix} M_{11} & M_{12} & \cdots & M_{123} \\ \vdots & \vdots & \cdots & \vdots \\ M_{23,1} & \cdots & M_{23,23} \end{bmatrix} = \sum_{i=1}^{n} J_{i}^{T}(q)M_{i}, J_{i}(q)$$

(33)
where

\[
M_i = \begin{bmatrix}
  m_i & 0 & 0 \\
  0 & m_i & 0 \\
  0 & 0 & m_i \\
  0 & 0 & 0 \\
  I_{xi} & 0 & 0 \\
  0 & I_{yi} & 0 \\
  0 & 0 & I_{zi}
\end{bmatrix}
\]

\(I_{xi}, I_{yi}, I_{zi}\) are the moments of inertia about the \(x\), \(y\) and \(z\) axes of the \(i^{th}\) link frame.

For an arbitrarily chosen link in the whole body, the joint twist is given by \(\xi_i = [-\omega_i \times p_i]/\omega_i\), so \(J_i\) can be easily obtained through Eq. 27.

The second term in Eq. 32 can be computed directly from the inertia matrix by

\[
C_{ij}(q, \dot{q}) = \sum_{k=1}^{2} \Gamma_{ij,k} \dot{q}_k = \frac{1}{2} \sum_{k=1}^{2} (\frac{\partial M_{ij}}{\partial q_k} + \frac{\partial M_{ik}}{\partial q_j} - \frac{\partial M_{jk}}{\partial q_i}) \dot{q}_k
\]

The third term in Eq. 32 is the gravitational forces and it can be written as

\[
N(q, \dot{q}) = \frac{\partial V(q)}{\partial q}
\]

Not that the gravitational forces on the mobile platform will be neglected because of the planar motion, so only the gravitational force on the upper body are taken into account.

\[
V(q') = \sum_{i=1}^{2} m_i g h_m(q') + \sum_{i=1}^{7} m_i g h_i(q') + \sum_{i=1}^{7} m_i g h_i(q')
\]

\[+ \sum_{i=1}^{2} m_i g h_h_i(q')\]

where \(q' = [q_{m1}, q_{m2}, q_{l1}, \ldots, q_{l7}, q_{h1}, \ldots, q_{h2}, q_{h3}, q_{h4}]^T\), \(h\) is the height of the cm for the \(i^{th}\) link and it can be found using the forward kinematics map by Eq. 25.

Substituting Eq. 33, 34, 37 into Eq. 32, the dynamics equation of the mobile humanoid robot can be obtained.

For further simulation, the procedures of dynamics solution for the whole system are given out in the flowchart as shown in Fig. 5.

V. CONCLUSION

This paper studies the kinematics and dynamics of the mobile humanoid robot. The robot is a little bit complicated, however the modeling can be simplified under reasoning assumptions with kinematic chains and loops. Based on Lie groups and screw theory, the Lagrangian equation is used to establish a dynamics model which is derived from the aspect of energy(both kinetic and potential energy). The procedure of kinematics and dynamics are also presented. Moreover, the dynamic modeling of this paper will provide an important basis for our future works, such as motion planning, tip-over stability, intelligent control and so on.

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