Capacity Analysis for Dimmable Visible Light Communications

Jun-Bo Wang*, Qing-Song Hu†, Jiangzhou Wang‡, Ming Chen*, Yu-Hua Huang‡, Jin-Yuan Wang*
*Southeast University, China; †Nanjing University of Aeronautics and Astronautics, China; ‡University of Kent, UK;
jbwang@seu.edu.cn, qingsonglin@nuaa.edu.cn, j.z.wang@kent.ac.uk, chenming@seu.deu.cn, hyuhua2k@163.com, jinyuan798@seu.edu.cn

Abstract—This paper aims to derive the upper and lower bounds for the channel capacity of dimmable visible light communications (VLC) systems. Because the information is modulated into the instantaneous optical intensity of light emitting diodes (LEDs), the transmitted optical intensity signal is represented by a nonnegative input, which is corrupted by an additive white Gaussian noise. Considering the illumination support and LED device ability in VLC systems, the transmitted optical intensity signal must satisfy the illumination and the allowed peak optical intensity constraints. With these constraints, a lower bound on the channel capacity is derived through the maximum mutual information between the channel input and output, which is determined by the source entropy maximization. By applying the dual expression of capacity, an upper bound on channel capacity is derived. Both the upper and lower bounds are presented as the closed forms. The numerical results show that the presented bounds are very tight. Moreover, the peak optical intensity constraints will result in the loss of channel capacity. However, such capacity loss is so small that it can be negligible when the allowed peak optical intensity is twice of the nominal optical intensity of LED devices.

I. INTRODUCTION

Compared with incandescent or fluorescent lamp, white light emitting diodes (LEDs) have been emerging as a prominent technology for next generation illumination infrastructure due to their advantages [1], such as long lifetime, high efficiency, cost effectiveness, small size and low power consumption [2]. Hence, LEDs have been widely used in traffic application, flat panel displays, illumination application, and ubiquitous indicator light [3]. To further make full use of LED illumination at minimal expense, visible light communication (VLC) using white LEDs have attracted considerable interests in next-generation short-range wireless communications because of the advantages of worldwide availability [4], high security [5], immunity to radio frequency interference [6], and spatial reuse of the modulation bandwidth in adjacent communication cells [7].

In VLC, the intensity modulation and direct detection is employed, and some constraints are widely introduced in the published literature. First, information is modulated as the instantaneous optical intensity, so the optical signals are usually restricted to be a nonnegative real variable. Second, due to the luminous ability of LED device, the intensity of an optical signal should be less than an allowed threshold. Third, to protect the human’s eyes, the illumination requirements should be satisfied while transmitting data. Therefore, VLC usually offer very high signal-power-to-noise power ratio (SNR) at the receivers [8] [9], meanwhile the light brightness can be adjusted according to the users requirements but doesn’t allow very huge changes with time. With some or all of these constraints, many researches have been carried out in terms of modulation [10], coding [11], multiple input multiple output (MIMO) [12], channel modeling [13], transceiver design [14], light layout [15] [16] and so on. However, only a few papers have been published for the channel capacity of VLC in the open literature. A numerical result of the capacity for inverse source coding (ISC) for VLC was presented in [17], but no closed-form expressions have been given. In our previous work [18] [19], the upper and lower bounds on the channel capacity have been derived in the VLC systems. However, the peak optical intensity constraint of the LED has not been considered.

Different from [18] [19], this paper further considers the peak optical intensity constraint of LED and aims to analyze the channel capacity of dimmable VLC systems. Using entropy power inequality (EPI) and the entropy maximizing theorem [20], a lower bound is derived and a closed-form optimal intensity distribution of the transmitted optical signal is obtained. Then, using the dual expression of capacity [21], an upper bound expression is derived. It should be noted that both the upper and lower bounds are simple closed-form expressions.

The rest of this paper is organized as follows. System model is described in Section II. Upper and lower bounds on channel capacity are derived in Sections III and IV, respectively. Numerical results are presented in Section V before conclusions are drawn in Section VI.

II. SYSTEM MODEL

Consider a typical VLC link which transmits data over a line-of-sight (LOS) path by modulating the optical sources. Due to the cost consideration, noncoherent optical LED light sources are used as transmitters and inexpensive optical intensity detectors are used at the receiver. Moreover, only the optical intensities of the LED lights are modulated to transmit data. In this paper, optical intensity pulse amplitude modulation (PAM) is considered due to its popularity and simplicity in implementation. The optical channel for VLC can be modeled as

$$Y = HX + Z$$  \hspace{1cm} (1)
where $X$ is the transmit optical intensity signal, $Y$ is the received signal, the constant $H$ represents all losses and optoelectronic conversion factors, and $Z$ models both thermal noise and ambient-light-induced shot noise. Because $H$ scales the SNR only, without loss of generality, the value of $H$ is set to one to simplify the derivations as in [22] [23]. Therefore, the equality (1) can be simplified as

$$Y = X + Z$$

(2)

where $Z$ is independent of the signal and is modeled as zero-mean, variance-$\sigma^2$ Gaussian random variable.

Because the information is modulated as the instantaneous optical intensity, all signals are restricted to be nonnegative, i.e.,

$$X \geq 0.$$  

(3)

For practical VLC systems, the optical intensity is constrained by the luminous ability of LED device, i.e,

$$X \leq A$$  

(4)

where $A$ is the allowed peak optical intensity. Considering the illumination function of VLC systems, the average optical intensity cannot change with time and can be adjusted according to the user requirements. Mathematically, the illumination constraint can be described as

$$E\{X\} = \xi P$$

(5)

where $\xi$ is the dimming target and $P$ is the nominal optical intensity of LED devices. $\xi P$ is the target illumination intensity. Obviously, the nominal optical intensity is limited by the allowed peak optical intensity, i.e.,

$$P \leq A.$$  

(6)

Considering that users can arbitrarily adjust the light brightness without violating the constraint of the nominal optical intensity, the dimming target must satisfy

$$0 < \xi \leq 1.$$  

(7)

As special cases, $\xi = 1$ is also called as the full illumination target and $\xi = 0.5$ is the half illumination target.

III. LOWER BOUND ON CHANNEL CAPACITY

According to information theory, given by (7.1) and (8.48) in [20], a lower bound on the capacity of the optical intensity channel can be found by computing the mutual information between the channel input and output, $I(X;Y)$, for any input probability density function (PDF) $f_X(x)$, i.e.,

$$C \geq I(X;Y) = h(Y) - h(Y|X) = h(X+Z) - h(Z)$$

(8)

where $h(\cdot)$ denotes the differential entropy. To obtain a tight bound, the choice of $f_X(x)$ should yield a mutual information that is reasonably close to capacity. Such a choice is difficult to determine and might make the evaluation of $I(X;Y)$ intractable. In this paper, the EPI is used to derive (8) as

$$C \geq h(X+Z) - h(Z) \geq \frac{1}{2}\log_2 \left( \frac{1}{2\pi e\sigma^2} \right)$$

(9)

From (9), a lower bound is computed by searching the maxentropic source PDF $f_X^*(x)$ to make this lower bound as tight as possible. Mathematically, the maxentropic source PDF $f_X^*(x)$ can be obtained by solving the following optimization problem:

$$\max_{f_X(x)} h(x) = -\int_0^A f_X(x) \log_2 f_X(x) \ dx$$

(10)

subject to

$$\int_0^A f_X(x) \ dx = 1$$

(11)

and

$$\int_0^A xf_X(x) \ dx = \xi P.$$  

(12)

The optimization problem (10) can be solved by applying the method of Lagrange multipliers [24] [25]. For facilitating the following description, let $\alpha = \xi P/A$. Obviously, $\alpha \in (0, \frac{A}{2}]$. If $\alpha = 0.5$, the optimal solution to the optimization problem (10) can be expressed as

$$f_X^*(x) = \begin{cases} \frac{1}{A}, & x \in [0, A] \\ 0, & \text{otherwise} \end{cases}$$

(13)

When $\alpha \neq 0.5$ and $\alpha \in (0, P/A]$, the optimal solution to the optimization problem (10) can be expressed as

$$f_X^*(x) = \begin{cases} \frac{\ln 2}{1-2^{-bA}}2^{-bx}, & x \in [0, A] \\ 0, & \text{otherwise} \end{cases}$$

(14)

where $b$ is the solution to the following equation

$$\alpha = \frac{1}{bA\ln 2} - \frac{1}{2^{2bA} - 1}.$$  

(15)

It should be noted that $b$ in (15) can be easily obtained numerically [26].

A. the case of $\alpha \neq 0.5$

If $\alpha \neq 0.5$ and $\alpha \in (0, P/A]$, the differential entropy of signal source can be derived as

$$h(X) = -\int_0^A f_X^*(x) \log_2 f_X^*(x) \ dx$$

$$= -\int_0^A \frac{\ln 2}{1-2^{-bA}}2^{-bx}\log_2 \left( \frac{\ln 2}{1-2^{-bA}}2^{-bx} \right) \ dx$$

$$= \ln 2 \frac{2b\alpha P}{b\ln 2} \left( 1 - 2^{-bA} \right)$$

(16)
and the lower bound for channel capacity is derived as
\[
C \geq \frac{1}{2} \log_2 \left( 1 + \frac{2^{2h(X)} - 1}{2\pi \sigma^2} \right)
\]
(17)

B. the case of \( \alpha = 0.5 \)

Similarly, when \( \alpha = 0.5 \), the differential entropy of signal source can be derived as follows
\[
h(X) = -\int_A f_X(x) \log_2 f_X(x) \, dx
\]
(18)
and the lower bound of the channel capacity can be obtained as
\[
C \geq \frac{1}{2} \log_2 \left( 1 + \frac{2^{2h(X)} - 1}{2\pi \sigma^2} \right)
\]
(19)

IV. UPPER BOUND ON CHANNEL CAPACITY

From the Section 2.3, equality (3.7), in [27], it can be found that the mutual information, relative entropy and conditional entropy have the following relationship
\[
I(X;Y) = D(f_{Y|X}(y|x) \| f_Y(y))
\]
(20)
where \( f_Y(y) \) is the PDF of the channel output \( Y \), \( f_{Y|X}(y|x) \) is the conditional PDF of output \( Y \) given the input \( X \), and \( D(\cdot \| \cdot) \) denotes relative entropy, defined as
\[
D(f_{Y|X}(y|x) \| f_Y(y)) = \int \log_2 \frac{f_{Y|X}(y|x)}{f_Y(y)} f_{Y|X}(y|x) \, dy
\]
(21)
From Theorem 2.6.3 in [20], it can be known that
\[
D(f_{Y|X}(y|x) \| f_Y(y)) \geq 0.
\]
(22)
Using (20) and (22), the dual expression of capacity is given by [28]
\[
I(X;Y) \leq \int \int f_{Y|X}(y|x) \, dy \, f_X(x) \, dx
\]
(23)
where \( E_{f_X(x)}[\cdot] \) is the expected value and
\[
D(f_{Y|X}(y|x) \| f_Y(y)) = -\int \log_2 f_Y(y) f_{Y|X}(y|x) - \frac{1}{2} \log_2 (2\pi \sigma^2)
\]
(24)
From (23), the upper bound for channel capacity can be derived as [21]
\[
C = \max_{f_X(x)} I(X;Y)
\]
\[
\leq \sup_{f_X(x)} E_{f_X(x)}[D(f_{Y|X}(y|x) \| f_Y(y))]
\]
(25)
i.e.,
\[
C \leq \sup_{f_X(x)} E_{f_X(x)} \int -\log_2 f_Y(y) f_{Y|X}(y|x) \, dy
\]
(26)
It can be known from equation (26) that arbitrary output distribution leads to an upper bound on the channel capacity. Since \( X \) and \( Z \) are independent random variables, it can be known from [29] that the probability density function of \( Y \) is the integral convolution of \( f_X(x) \) and \( f_Z(z) \), i.e.,
\[
f_Y(y) = \int f_X(x) f_Z(y-x) \, dx.
\]
(27)
Although substituting (13) and (14) into (27), the output distribution can be determined accordingly, the derived output distribution is very complicated and cannot be used to derive a closed-form upper bound. Therefore, a tractable and suitable output distribution is desired to get a tight closed-form upper bound [21]. In practical VLC systems, typical illumination scenarios offer very high SNR [9]. The output signal can be approximated to the input signal, i.e.,
\[
Y \cong X.
\]
(28)
Therefore, the output PDF can be approximated as the input PDF.

A. the case of \( \alpha \neq 0.5 \)

If \( \alpha \neq 0.5 \) and \( \alpha \in (0, P/A) \), the output distribution can be obtained as
\[
f_Y(y) \cong \frac{(b \ln 2) e^{-(b \ln 2)y}}{1 - 2b/A}, \quad y \in [0, A].
\]
(29)
Substituting (29) to (26), the upper bound on channel capacity can be further derived as follows:
\[
C \leq \sup_{f_X(x)} E_{f_X(x)} \int -\log_2 f_Y(y) f_{Y|X}(y|x) \, dy
\]
\[
\leq \frac{1}{2} \log_2 (2\pi \sigma^2)
\]
(20)
\[
= \sup_{f_X(x)} E_{f_X(x)} \left[ \int \log_2 \frac{1}{\sqrt{2\pi}e^{-\frac{(y-x)^2}{2\sigma^2}}} \, dy \right]
\]
\[
= \sup_{f_X(x)} E_{f_X(x)} \left[ \log_2 \left( \frac{1}{\sqrt{2\pi}e^{-\frac{(b \ln 2)^2}{2\sigma^2}}} \right) \right]
\]
\[
= -\frac{1}{2} \log_2 (2\pi \sigma^2)
\]
(21)
where \( Q(\cdot) \) is the tail probability of the standard normal distribution, defined as
\[
Q(x) = \int_x^{+\infty} \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{t^2}{2}} \, dt, \quad \forall x \in \mathbb{R}.
\]
(31)
Since $0 \leq X \leq A$, one obtains
\[ e^{-\frac{A^2}{2\sigma^2}} - e^{-\frac{(A-X)^2}{2\sigma^2}} \leq 1 - e^{-\frac{x^2}{2\sigma^2}} \] (32)
and
\[ \frac{b}{\sqrt{2\pi} \sigma} \left( e^{-\frac{(A-x)^2}{2\sigma^2}} - e^{-\frac{x^2}{2\sigma^2}} \right) \leq \frac{|b|}{\sqrt{2\pi} \sigma} \left( 1 - e^{-\frac{x^2}{2\sigma^2}} \right). \] (33)

Using (33), (30) can be further derived as
\[ C \leq -\log_2 \left( \frac{b \ln 2}{1 - 2^{-bA}} \right) \left[ Q \left( -\frac{\xi P}{\sigma} \right) - Q \left( \frac{A - \xi P}{\sigma} \right) \right] \\
+ \frac{|b|}{\sqrt{2\pi} \sigma} \left( 1 - e^{-\frac{x^2}{2\sigma^2}} \right) - \frac{1}{2} \log_2 \left( 2\pi e \sigma^2 \right) \\
+ b\xi P \left[ Q \left( -\frac{\xi P}{\sigma} \right) - Q \left( \frac{A - \xi P}{\sigma} \right) \right]. \] (34)

B. the case of $\alpha = 0.5$

If $\alpha = 0.5$, the PDF of the input $X$ is uniform distribution. In this case, the output PDF is approximated as
\[ f_Y(y) \approx \frac{1}{A}, \quad y \in [0, A]. \] (35)

The upper bound on channel capacity is derived as
\[ C \leq \sup_{f_X(x)} \mathbb{E} f_X(x) \left[ -\int_{-\infty}^{\infty} \log_2 f_Y(y) \, df_Y(y \mid x) \right] \\
- \frac{1}{2} \log_2 \left( 2\pi e \sigma^2 \right) \\
= \sup_{f_X(x)} \mathbb{E} f_X(x) \left[ -\int_0^A \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(y-x)^2}{2\sigma^2}} \log_2 \frac{1}{A^4} \, dy \right] \\
- \frac{1}{2} \log_2 \left( 2\pi e \sigma^2 \right) \\
= (\log_2 A) \left[ -Q \left( -\frac{\xi P}{\sigma} \right) - Q \left( \frac{A - \xi P}{\sigma} \right) \right] \\
- \frac{1}{2} \log_2 \left( 2\pi e \sigma^2 \right) \\
= \log_2 \frac{A^4}{\sqrt{2\pi} \sigma^2}. \] (36)

where $\delta$ is constant determined by
\[ \delta = Q \left( -\frac{\xi P}{\sigma} \right) - Q \left( \frac{A - \xi P}{\sigma} \right). \] (37)

V. NUMERICAL RESULTS

This section first plots the derived upper and lower bounds of dimmable VLC. It should be noted that the capacity bounds without peak optical intensity constraints were presented in [18]. To facilitate the following descriptions, the nominal optical intensity is normalized by the background noise power.

Fig. 1 shows the capacity bounds with 50% dimming target versus $P$. It is shown that the channel capacity increases with the nominal optical intensity power $P$ and the gap between the upper and lower bounds is very little. By comparing the three pairs of capacity curves, it can be seen that the peak optical intensity constraints of LED devices will result in a loss of channel capacity of VLC. Moreover, such capacity loss will decrease with the increase of the allowed peak optical intensity $A$. Especially, when $A = 2P$, the difference between the capacity curves with and without the peak optical intensity constraints is so small that it can be negligible.

Fig. 2 shows the capacity bounds versus dimming target with two peak optical intensity constraints $A = P$ and $A = 2P$, respectively. For comparison, the capacity bounds without peak optical intensity constraints are also plotted. It can be seen from the figure that the gap between the derived upper and lower bounds is very little. Moreover, the capacity loss due to the peak optical intensity constraints increases with the increase of the dimming target $\xi$. However, when $A = 2P$, such capacity loss can be ignored.

VI. CONCLUSION

This paper studied the upper and lower bounds on the channel capacity of dimmable VLC with the illumination and peak optical intensity constraints. The lower bound is derived by searching maxentropic source distribution. Then
using the dual expression of capacity, the upper bound on channel capacity is derived. Both the upper and lower bounds are presented with the closed-form expressions. The numerical results show that the gap between the derived upper and lower bounds is very small. And the peak optical intensity constraint of the LED devices will result in channel capacity loss in VLC. However, when the allowed peak optical intensity is set to be twice of the nominal optical intensity of LED devices, such capacity loss is so small that it can be ignored. Although simple PAM is considered in the paper, the analytic approach can be applied to the VLC system when other advanced modulation schemes are considered [30][31].

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