Abstract

This paper explores potential improvements to the trust modelling of agents in multi-agent systems when a social network of advisors is employed as part of the trust modelling, and in particular, examines means of optimizing the number of advisors that should be maintained in the social network. We propose three such improvements, two directly relating to the limit of advisor network size by either setting a maximum size for a buyer’s advisor network or setting a minimum trustworthiness threshold for agents to be accepted into that advisor network, and a third which uses an advisor referral system in combination with one of the first two network-limiting techniques. We provide experimental results in defence of our approach for two distinct trust modelling systems, and show how these optimizations can improve, sometimes significantly, the accuracy of different trust models (in the context of electronic marketplaces). We believe that the proposed techniques will be very useful for trust researchers seeking to improve the accuracy of their own trust models by setting the size and composition of advisor networks.

Keywords: Trust Modeling, Social Network of Advisors, Referral, Buyer and Seller Agents, Multi-Agent Systems, Electronic marketplaces

1. Introduction

Several frameworks have been offered in the literature for modelling the trust that exists between agents when one agent leverages a social network of advisors as part of the trust modelling (Jøsang et al., 2007), including TRAVOS (Teacy et al., 2006), as well as a newer, multi-stage model referred to as the Personalized Trust Model (PTM) (Zhang and Cohen, 2008; Zhang, 2009). While “trust” may often have different meanings from one domain to the next (McKnight and Chervany, 2001), for the purposes of this paper we make use of the definition of trust as an agent’s belief that some other agent will carry out the tasks that it says it will perform (Zhang, 2009). One setting in which this trust modelling has

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been employed is that of electronic marketplaces: a buyer agent has requested, or is considering making a request to, a seller agent to carry out some task, based in part on past interactions between the seller and the buyer’s advisors, as reported to the buyer.

Although these and other models (e.g. (Jøsang and Ismail, 2002) and (Vogiatzis et al., 2010)) have shown very promising results in accurately modelling the trust of agents, there are certainly opportunities for further optimization, both with respect to the accuracy of the model, and perhaps also the complexity of the calculations. More specifically, trust modelling using social networks of advisors has traditionally proposed methods for limiting the number of peers that are consulted, to retain only the most trustworthy advisors. In general, however, these methods do not typically offer a principled methodology to set the values that serve to restrict the size of the social network that is consulted. We posit that by optimizing the size of the advisor network, we can eliminate outlier data that has little positive contribution to the model, while retaining a sufficient supply of advisors that have had experience interacting with the agents under consideration.

With this kind of presumption, in this paper, we propose an approach for determining the appropriate size and composition of social networks of advisors. More specifically, we first examine two methods that limit the size of a requesting agent’s advisor network. We consider selecting a maximum number (or maximum proportion) of advisors (or max_nbors) out of the total advisor population, or only selecting advisors that have achieved some trustworthiness threshold. Through empirical studies, we show that by using these methods - with appropriately-chosen parameters - to limit the size of the advisor network, we will obtain an overall more accurate measure of the trustworthiness of individual agents that the requesting agent is considering. We thus also look at how to best determine the parameters to use for a given scenario, in order to ensure the more accurate trust modelling results. We further show that these results should not be specific to any single model or scenario.

We also discuss the augmentation of one of these two techniques with the advisor referral system, in which advisors which have had an insufficient amount of experience with a particular agent will be replaced by other agents in the advisor network with a higher level of experience. We show that careful application of our proposed advisor referral system will result in further improvements in the accuracy of trust modelling, particularly when the advisor network is initially very small, such as when a small maximum number of advisors or a high trustworthiness threshold is used. As with the earlier methods, we also examine how to determine the most appropriate parameters when using advisor referrals. The paper presented here extends the first presentation of this model in (Gorner et al., 2011).

This work situates well within the context that Jøsang and Golbeck (2009) outline as the central concern for future research in multiagent trust modeling. They claim “... literature specifically focusing on the robustness of TRS is... limited...”; one way in which trust models can fail to be robust is in relying on a set of advisors, the size of which is set artificially, and thus may be inappropriate.
The methods we outline in this paper seek to address this concern.

The rest of the paper is organized as follows. In Section 2, we first provide a brief description of the two distinct trust models to provide context to our proposals and to the experiments which follow. We then describe our proposed approach of limiting advisor network size and using advisor referrals in Section 3. In Section 4, we provide various experimental results demonstrating the effectiveness of our approach. We then discuss in Section 5 how our research contrasts with that used in collaborative filtering recommender systems and with other approaches for setting network size. Finally, we conclude our current work and propose some directions for future research in Section 6.

2. Trust Models

One significant trust model that serves as an appropriate context for our research is the Personalized Trust Model (PTM) developed by Zhang and Cohen (2008). We also examine how our approach will improve other trust models, mainly focusing on TRAVOS (Teacy et al., 2006). In this section, we summarize these models and then use them as the backdrop for subsequent experimentation in order to demonstrate the value of our particular approach.

2.1. Personalized Trust Model

Buyer agents regularly interact with seller agents to purchase desired goods or services. Following each transaction with a seller agent, a buyer assigns a rating to that transaction, specifying whether its experience was positive (1) or negative (0), that is whether the transaction was satisfactory to the buyer or not, and submits this rating to a centralized database server. In PTM, buyers first model the trustworthiness of advisors, and then model the trustworthiness of sellers by considering their own experience with the sellers as well as the advisors' ratings weighted by the calculated advisor trustworthiness.

More specifically, a buyer agent, denoted by $b$, may wish to model the trustworthiness of all the sellers in the system, in order to determine which sellers to purchase from in the future. To do so in PTM, it first constructs a measure of private reputation $R_{pri}(a)$ of each advisor $a$, based on that advisor’s ratings for sellers that $b$ has previously dealt with, and representing an estimation of the probability that $a$ will give fair ratings to $b$, using the beta family of probability density functions. This measure is known as “private” reputation, since this evaluation makes use of the buyer’s own experiences. In a similar fashion, the buyer then calculates the public reputation $R_{pub}(a)$ of an advisor $a$, or the

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1This work was later extended for the context of the semantic web (Sensoy et al., 2009) and to integrate incentives for honesty (Zhang and Cohen, 2007). This is therefore quite a rich example of a trust model.

2This work has been a popular competitor for other trust modeling researchers including Regan (Regan et al., 2006) and Kerr (Kerr and Cohen, 2009) and is thus a worthwhile representative system. It was also compared to PTM (the other example system in this paper) in (Zhang, 2011).
probability that an advisor will provide “consistent” ratings, based on the
advisor’s ratings and other ratings for the sellers rated by the advisor. A rating is
consistent if it is the same as the majority of ratings provided for that seller by
all other buyers up to the moment that this rating is submitted.

Based on the total number of advisor \( a \)’s ratings for sellers that \( b \) has previ-
ously dealt with, \( N_{all} \), and other pre-selected parameters specifying the max-
imum acceptable error \( \epsilon \) and minimum acceptable level of confidence \( \gamma \), the
reliability of the private reputation value, \( w \), is derived. The weight \( w \) is then
used in the calculation of the overall trustworthiness of \( a \). As can be seen, a
more reliable private reputation will have a greater effect on the overall result:

\[
N_{min} = \frac{1}{2\epsilon^2} \ln \frac{1 - \gamma}{2}
\]

\[
w = \begin{cases} 
  \frac{N_{all}}{N_{min}} & \text{if } N_{all} < N_{min}; \\
  1 & \text{otherwise.}
\end{cases}
\]

\[
Tr(\alpha) = wR_{pri}(\alpha) + (1 - w)R_{pub}(\alpha)
\]

Once this value has been calculated for each advisor, a similar approach can
be taken for the trustworthiness of a given seller \( s \). First the buyer \( b \) calculates
her private reputation of \( s \) (\( R_{pri}(s) \)), or the probability that \( s \) will provide
good service, based on \( b \)’s past experiences with \( s \). Then the buyer derives the public
reputation of the seller (\( R_{pub}(s) \)), the probability that the seller will provide
good service given all advisors’ past experiences with \( s \), taking into account
\( b \)’s own model of trustworthiness of each advisor. Each advisor rating used
is discounted based on \( b \)’s previously calculated trust value for the applicable
advisor. Finally the overall trustworthiness of the seller \( s \) may be calculated, by
weighting the public and private reputations of the seller in a similar fashion to
those of advisors as indicated in Equation 3.

2.2. TRAVOS

TRAVOS has some similarities to PTM - both take a probabilistic approach
to the modelling of trust, using beta probability density functions (pdfs) - mak-
ing it a good comparator for the results we will obtain for PTM.

Under the TRAVOS approach, it is assumed that a truster agent, \( a_{tr} \), will
not generally have complete information about a trustee agent, \( a_{te} \), in order to
definitively state the probability that \( a_{te} \) will fulfill its obligations to \( a_{tr} \). At
most, we can calculate the level of trust \( \tau_{a_{tr}, a_{te}} \) as an expected value of this
probability based on the set of interaction outcomes of the past interactions
between the agents. A separate metric is then determined such that \( a_{tr} \) may
measure its confidence in this trust value, \( \tau_{a_{tr}, a_{te}} \). If the confidence is not
sufficiently high, the advice of third parties may be considered as well. This
would be performed by asking other agents to report the number of successful
and unsuccessful interactions that each has had with \( a_{te} \), the aggregate of which
are then computed and used in the calculation of \( \tau_{a_{tr}, a_{te}} \).

However, mindful of the possibility that some other agents may report un-
truthfully, TRAVOS also incorporates a mechanism to filter out the reports by
agents which have low reputations. The first stage is to estimate the probability that an agent \(a_{op}'s\) reported opinion about the trustee \(a_{te}\), denoted \(\hat{R}_{a_{op},a_{te}}\), is accurate. This is performed by (i) constructing additional beta distributions, \(D^r\) derived from \(\hat{R}_{a_{op},a_{te}}\), and \(D^o\) which is based on the outcomes of the previous interactions for which \(a_{op}\) provided to \(a_{tr}\) an opinion similar to \(\hat{R}_{a_{op},a_{te}}\) about \(a_{te}\) or some other trustee, and (ii) finding their respective expected values, \(E^r\) and \(E^o\). The range of possible values for both \(E^r\) and \(E^o\), i.e. \([0, 1]\), is then divided into several disjoint intervals (bins) of equal size to determine (in essence) whether \(E^r\) and \(E^o\) are located in the same bin, via the calculation of an accuracy value denoted as \(\rho_{a_{tr},a_{op}}\).

Finally, TRAVOS attempts to reduce the effect of unreliable opinions through an approach that discounts high values of parameters, unless the probability of a rater’s opinion being accurate is very high. This is performed through the construction of another beta distribution, \(\tilde{D}\), based on (i) the accuracy value \(\rho_{a_{tr},a_{op}}\), and (ii) the expected values and standard deviations of the uniform distribution and of the distribution(s) of the unreliable opinion(s) that are sought to be removed, \(D^r\).

We can identify three important distinctions between PTM and TRAVOS: PTM uses both private and public knowledge regarding all sellers, whereas TRAVOS uses only the private knowledge regarding some selected sellers. The method used by TRAVOS to aggregate ratings provided by certain advisors is more complex, reducing the effect of ratings from less trustworthy advisors using a method of filtering. TRAVOS reasons about the specific seller being considered when determining how much to trust an advisor. By contrast, in PTM, advisor reputation is calculated independently of any specific seller.

### 3. Improving Trust Modelling Accuracy

In this section, we describe the three modifications to the original trust models that would optimize the size and composition of advisor network to yield more accurate models.

#### 3.1. Limiting Advisor Network Size

The first two modifications relate to the limit of advisor network size. One modification sets a trustworthiness threshold, such that only those advisors that the buyer trusts at or above the given threshold would be included in the advisor network. More specifically, we choose some threshold \(L (0 \leq L \leq 1)\) which represents the minimum advisor trustworthiness value \(Tr(a)\) required for an agent to be included in the advisor network. We then define the set \(A_{L,b} = \{a_1, a_2, \ldots, a_k\}\) consisting of all advisors for which \(Tr(a) \geq L\) for a particular buyer \(b\). We then use the subset \(A_{L,b,s}\), consisting of the advisors in \(A_{L,b}\) that have provided ratings for the seller \(s\), in place of the previously-defined set \(\{a_1, \ldots, a_k\}\), the set of all advisors that have provided ratings for \(s\). The buyer also models the other advisors that are outside of the advisor network, for the purpose of exploration.
Another modification sets a maximum number of advisors (or $\text{max}_{\text{nbors}}$, also known as $k$-nearest neighbours or $k$NN) to be included in each buyer’s advisor network, and taking those with the highest trustworthiness. For a particular buyer $b$, after having calculated the personalized trustworthiness of each advisor for $b$ as per the first part of the PTM, we sort the list of all $n$ advisors from greatest trustworthiness value to least, in the set $\{ a_1, a_2, \ldots, a_n \}$. We choose some maximum number of advisors for each buyer, $\text{max}_{\text{nbors}} \leq n$, and then truncate this set to the set $A_b = \{ a_1, a_2, \ldots, a_{\text{max}_{\text{nbors}}} \}$. We thus obtain the set of $\text{max}_{\text{nbors}}$ advisors that have been calculated to be the most trustworthy for $b$. Again, the subset of $A_b$ that has provided ratings for the seller $s$ is used in place of the larger set $\{ a_1, \ldots, a_k \}$.

3.2. Using Advisor Referrals

We also wish to consider the possibility of combining one or both of the above methods with the advisor-referral technique suggested in (Yu and Singh, 2000). We diverge somewhat from the original trust models insofar as both PTM and TRAVOS models do not require us to query each advisor for a recommendation. Rather, the buyer has access to each advisor’s ratings for a given seller $s$ via a central server, and uses this data to determine the public (or network) reputation for the seller.

We thus consider that advisors can “advise” by allowing buyers to make use of each advisor’s own private reputation for a certain seller. In this case, an advisor “referral” system could be implemented using a variant of the measure used to weight private reputation in the PTM model. This would work as follows: For each advisor $a_j$ in the advisor network of $b$, that is, the set $\{ a_1, a_2, \ldots, a_k \}$, $b$ checks whether advisor $a_j$ is an acceptable advisor for the seller $s$. This will be the case if $N_{a_j}^{\text{all}} \geq N_{\text{min}}$, where $N_{a_j}^{\text{all}}$ is the number of ratings provided by an advisor $a_j$ for $s$, and $N_{\text{min}}$ is some minimum number of ratings (which may be calculated using Equation 1).

If $a_j$ is not an acceptable advisor (that is, if $N_{a_j}^{\text{all}} < N_{\text{min}}$), the algorithm will query $a_j$’s advisor network, sorted from most trustworthy to least trustworthy from the perspective of $a_j$, in order to determine, in a similar fashion, which (if any) of these advisors meet the criteria to be a suitable advisor for $s$. The first such advisor encountered that is itself not either (a) already in the set of acceptable advisors; or (b) in $A_b$ — since this would imply that the recommended advisor would be added in any event at a later stage — will be accepted.

If none of the advisors of $a_j$ meet the above criteria, this step would be repeated at each subsequent level of the network — that is, the advisors of each member of the set of advisors just considered — until an acceptable, unduplicated advisor was identified.

Once the full set of acceptable advisors has been determined, the “network” reputation would be calculated as in PTM and TRAVOS, using the advisor trustworthiness values held by the buyer $b$. This, of course, assumes that the seller $s$ has had sufficient past interactions with the various advisors in the network such that there are at least $k$ buyers that have each had at least $N_{\text{min}}$. 
Algorithm 1 Referring Advisors to Buyer b for Trustworthiness of Seller s

1: $A_b = \{a_1, a_2, \ldots, a_k\}$; {advisors in b’s advisor network}
2: $A_s = \{\}$; {set of advisors that are suitable for providing advice regarding seller s}
3: $N_{RE}$ = minimum number of ratings for a to be a suitable advisor regarding s;
4: $\text{maxnetlevel} = \lceil \log_k(|B|) \rceil$; {the maximum number of search iterations}
5: for $j = 1$ to $k$ do
6:   $N_{all} = \text{total number of ratings provided by } a_j$ for s;
7:   if $N_{all} \geq N_{RE}$ then
8:     append $a_j$ to $A_s$;
9:   else
10:      $\text{netlevel} = 2$; {no. of connections between b and the advisors being searched}
11:      $a_x = \text{null}$; {the desired suitable advisor in place of $a_j$}
12:      $A_c$ = the set of advisors for $a_j$ sorted from most to least trustworthy {as per $a_j$};
13:      while $a_x = \text{null}$ and $\text{netlevel} \leq \text{maxnetlevel}$ do
14:         $A_n = \{\}$; {the set of advisors to be considered in the next round, if necessary}
15:            for all $a_c$ in $A_c$ do
16:               $N_{all} = \text{total number of ratings provided by } a_c$ for s;
17:                  if $N_{all} \geq N_{RE}$ and $a_c \not\in A_b$ and $a_c \not\in A_s$ then
18:                     $a_x = a_c$;
19:                     break;
20:                  else
21:                     add the set of advisors for $a_c$ to $A_n$;
22:                  end if
23:            end for
24:         $\text{netlevel} + +$;
25:         $A_c = A_n$
26:      end while
27:      if $a_x \neq \text{null}$ then
28:         append $a_x$ to $A_s$;
29:      end if
30:   end if
31: end for

interactions with s, which is not guaranteed. If only a smaller number of acceptable advisors can be found, the system will simply use this reduced set to determine the network reputation.

To ensure broad coverage of the network while preventing infinite recursion, we limit the number of network “levels” calculated to at most $\lceil \log_k(|B|) \rceil$, where $B$ is the set of all buyers (advisors) in the system. However, we note that practically, in a large scale system, the number of levels may need to be smaller in order for this algorithm to be computationally efficient; we will leave such a decision for later work. We summarize this mechanism in pseudo-code format as Algorithm 1.

4. Experimentation

In our experiments below, three primary hypotheses are explored: i) that an empirical method can be designed which effectively reveals to practitioners how best to set the values of $\text{maxnbors}$ or threshold, when using an advisor-based trust modeling system; ii) that using a principled approach as in i) can yield important gains in trust modeling accuracy; iii) that using referrals may help
further improve the trust modeling accuracy. We return to discuss these three hypotheses in Section 4.4.

We now adopt the simulation approach taken in (Zhang and Cohen, 2008) to verify the effectiveness of our proposals. Specifically we simulate an environment consisting of one buyer, 80 advisors, and 100 sellers, where the sellers are evenly divided into ten groups, each having a probability of dishonesty between zero and 0.9. Here, a seller having the probability of dishonesty \( P \in [0, 0.9] \) will be dishonest \( P \times 100 \) percent of the time, and be honest \( (1 - P) \times 100 \) percent of the time. The buyer and the advisors each randomly selects and rates one seller for each of the 80 days of the simulation, such that no seller is rated more than once by a single buyer or advisor. Finally, given these ratings, the buyer calculates the trustworthiness values corresponding to each of the sellers. These tests are performed for two values of the percentage of lying (dishonest) advisors (or LA), specifically 30% and 60%, and repeated a total of ten times for each possible combination, to show that regardless of whether one is an environment with a lot of lying advisors or relatively few, we can calibrate how to set the parameter values, so our methods are generally robust.

In our experiments, we implement both PTM and TRAVOS to model the trustworthiness of sellers. The parameter settings for these two trust models are according to the experimental settings in (Zhang and Cohen, 2008) to produce the best results in the experiments. More specifically, for PTM, we set \( \gamma = 0.6 \) and \( \epsilon = 0.4 \). For TRAVOS, we set the number of bins to 2.

The effectiveness of a trust model in the experiments is defined as the accuracy of reflecting the trustworthiness of sellers. It will be measured as the average absolute difference between the actual trustworthiness of the sellers and the results for the trust model.

4.1. Effectiveness on PTM

In this section, we first verify that applying our approach for limiting the size of advisor network to the PTM system can improve the effectiveness of the trust model. We then verify that advisor referrals can improve the effectiveness of PTM when the size of the advisor network is very limited.

4.1.1. Limiting Advisor Network Size

We first evaluate the performance of PTM after applying thresholding and \( \text{max_nbors} \) for limiting advisor network size introduced in Section 3.1. The results are shown in Figure 1 for scenarios where 30% of advisors are lying, and Figure 2 for scenarios where the percentage of lying advisors is 60%. In each of these graphs, the \( x \)-axis represents the predetermined probability of dishonesty for each category of sellers. The \( y \)-axis represents the average (mean) of the trust values, averaged over all repetitions and all of the ten sellers in each category. Our ideal trust model would be represented by a slope of -1, such that (for example) the sellers that are dishonest 40% of the time are assigned a trust value of 0.6; we thus seek one or more variants that produce trust values very close to these “ideal” values. We see that, in general, using thresholding
and max_nbors does yield a trust model that tracks well with the ideal trust values, for both the 30% and 60% lying advisors (LA) cases. However, the error – the difference between the actual trust values and the ideal figures – varies depending on the parameters used, particularly the specific value of the threshold or max_nbors.

Figure 1a compares the performance of an unrestricted advisor network against several possible max_nbors parameters, for the case where 30% of the advisors are dishonest. All of the variants using max_nbors provided better results compared to not using max_nbors or thresholding at all; that is, the plotted results all came closer to the “ideal” slope than for the unrestricted network. However, as expected, the best results came not from choosing extreme parameters (i.e. a very high or very low value for max_nbors), but rather from somewhere in the middle: the results that best matched the ideal trust model came for max_nbors = 40 (that is, the top 50% of the 80 advisors in the network). This therefore provides a definitive proposal for how to set the size of social network, determined empirically.

A similar comparison for different threshold values is shown in Figure 1b. Here the results differed slightly: using a threshold between 0.5 and 0.7 improved the results compared to a network without thresholding, with a threshold of 0.55 showing the greatest improvement. Once more, this offers a specific value for designers to employ for thresholding, in contrast to having these values being arbitrarily chosen, as has been the standard procedure in trust models, to date.

We note that for thresholds of 0.8 and 0.9, the accuracy was reduced dramatically, as shown by their near-horizontal and horizontal graphs, respectively, on this figure. This is because very few advisors, except the buyer itself, would have a trust value above such a high threshold. The 0.9 threshold graph demonstrates the worst case where no suitable advisors could be found, in which case the trust model will default to assigning the seller a trust value of 0.5.

Figures 2a and 2b cover a separate but very similar set of simulations for the 60% lying advisors case. As can be seen by looking at the “No MaxNbors / No Threshold” graphs, which are very far from the ideal-case graph shown in both figures, not applying max_nbors or thresholding at all will result in poor accuracy for the trust model. However, the accuracy can be significantly improved by applying either max_nbors or thresholding, with our experiments indicating 0.55 as the best threshold value, and max_nbors = 30 as the best among the tested max_nbors options.

A summary of the simulation results for both 30% and 60% lying advisors is provided in Figure 3. In these figures we also provide results for a similar set of simulations when the fraction of lying advisors is increased to 90%. This perspective may indeed provide a more intuitive comparison of the results com-

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3 The coordinate values of points in Figure 1a are presented in Table A.1 in Appendix A.
4 The coordinate values of points in Figure 1b are presented in Table A.2 in Appendix A.
5 The coordinate values of the points on each line in Figures 2a and 2b are presented in Tables A.3 and A.4 in Appendix A, respectively.
Figure 1: Verification testing for the modifications using 30% lying advisors.

Compared to those in Figures 1 and 2. In both cases, each graph represents one of the tested percentages of lying advisors (either 30% or 60%). The $x$-axis indicates the parameter chosen, if any, for $\text{maxNbors}$ (in Figure 3a) or the
Figure 2: Verification testing for the modifications using 60% lying advisors. 

(a) Comparison of max_nbors approaches (60% LA)

(b) Comparison of thresholding approaches (60% LA)

trustworthiness threshold (in Figure 3b). The y-axis shows the mean absolute error associated with that particular simulation – in other words, the average absolute difference between the “ideal” trust model as discussed above (error
Figure 3: (a) Mean absolute error of PTM using $\max_nbors$; (b) Mean absolute error of PTM using thresholding.

= 0), and the actual results for the variant measured. For example, a seller that was assigned a 0.3 probability of dishonesty would ideally be trusted with value 0.7; if the simulation generated instead a trust value of 0.65 for that seller, there would be an error of 0.05 with respect to that seller. The case where no $\max_nbors$ value is used – that is, all 80 advisors are included regardless – is represented by the far right of the graphs in Figure 3a. In Figure 3b, the equivalent case where no thresholding is applied is represented by a threshold of zero, at the far left of the graph.

From Figure 3a, we can also see the $\max_nbors$ approach may be affected by the percentage of lying advisors. Specifically, setting $\max_nbors = 40$ when 60% of the advisors are dishonest yields significantly worse performance than when 30% of the advisors are lying. On the other hand, using $\max_nbors = 30$ yields similar performance results when either 30% or 60% of advisors are lying. If 90% of advisors are lying, however, $\max_nbors = 30$ yields poor accuracy, whereas the best performance is found by setting $\max_nbors = 10$. This result suggests that when more advisors are lying, it is better to set a smaller value for $\max_nbors$. However, from Figure 3b we see that the thresholding approach is not heavily affected by the percentage of lying advisors. Even though there is a significant reduction in accuracy when moving from 60% to 90% lying advisors, the general shape of the graph (and therefore the best choices for the threshold) is largely unchanged. This is somewhat expected since thresholding allows only the trustworthy advisors to be included in buyers’ networks.

4.1.2. Using Advisor Referrals

We now repeat our simulations to test the use of advisor referrals, using a small subset of the best-performing cases using advisor referrals. Specifically we chose $\max_nbors = 40$ and $\text{threshold} = 0.55$ based on the results in Section 4.1.1, as well as $\max_nbors = 15$, which did not perform significantly worse compared to setting a maximum size of 40, in order to test whether the smaller size might (by itself) cause any differences in later simulations. Referrals were
applied with the required minimum number of experiences ($N_{RE}$) set to 1, the most that would have been available in this scenario since we had prohibited each advisor from interacting with each seller more than once.

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(a) 30\% lying advisors

(b) 60\% lying advisors

Figure 4: Comparison of approaches with and without referrals.
The results of these simulations, as compared to the equivalent versions without advisor referrals, are shown in Figures 4a and 4b. These graphs show that using our specific referral mechanism results in accuracy about as good as, if not better than, that of the corresponding variant without referrals, in almost all cases, indicating that applying referrals should not significantly reduce the accuracy of the trust model. This is not entirely surprising for these scenarios, since in each case the size of the advisor network has already been optimized to provide the best results, with a sufficient number of users having expertise with most, if not all, sellers.

Figure 5: Mean average error of compared trust models with/without referrals.

A summary view is provided as Figure 5, consisting of a scatterplot showing how each of the variants tested performed, for both the 30% and 60% lying advisor cases, with or without referrals (when possible). As in the summary figures used in Section 4.1.1, the y-axis represents the mean absolute error (MAE) of each of the variants in calculating the trustworthiness of sellers as compared to the “ideal” trust model discussed previously. Note that in some cases, two or more data points (icons) overlap, indicating that the MAE for those simulations were approximately equal (the order in which they overlap is not meaningful). Again, as the data points corresponding to the referral-based variants are almost always at about the same level as those of their non-referral equivalents, we can conclude that adding referrals will generally yield about the same accuracy as if referrals are not used.

However, we believe that referrals could be useful when using a smaller advisor network, i.e. a smaller value of max_nbors or a higher trustworthiness threshold. In this scenario, it is less likely that the advisors within the network will have a sufficient level of experience – if any experience – in dealing with each and every seller in the system. Here, then, we would expect referred advisors

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6 The coordinate values of the points on each line in Figures 4a and 4b are presented in Tables A.5 and A.6 in Appendix A, respectively.

7 Specifically, simulations were not conducted for referrals where neither max_nbors or thresholding is used, since referrals are of no effect unless the advisor network has already been limited.
to be more useful in regards to filling in the gaps in experience. With this in mind, we proceeded to a modified version of the above evaluation that would allow for a greater role for referrals. The parameters and test conditions were the same, except that we reduced the number of sellers to 40, and increased the number of simulation days to 120. We also adopted pure random selection for the sellers, such that buyers would rate each seller a variable number of times (on average three), whereas previously they could rate each seller at most once.

![Figure 6: Comparison of trust models (max_nbors = 2, 30% lying advisors (LA))](image)

Simulations were then performed in this environment using several variations incorporating max_nbors or thresholding, as well as referrals. A subset of these simulations, using max_nbors = 2 where 30% of the advisors are dishonest, are shown in Figure 6\(^8\). The independent variable in this case is the use of referrals and, if referrals are used, the amount of experience required from the referred advisors (\(N_{RE}\)), which is varied from one to ten. It is clear, based on comparing the closeness of each of the variations’ graphs to the “ideal” graph, that while setting a maximum size for the advisor network yields a more accurate trust model, adding referrals in this case results in a further non-trivial improvement. Furthermore, increasing the \(N_{RE}\) value – the experience with the seller that each referred advisor must have – serves to improve the model further; an experience level of four – just above the expected average noted above – comes closest to matching the best-case scenario.

\(^8\)The coordinate values of points in Figure 6 are presented in Table A.7 in Appendix A.
Additional simulation results are shown in summary view as Figures 7a and 7b, which are for 30% and 60% lying advisors, respectively. Each graph represents a single possible value for \textit{max}_\textit{nbors}. The positions on the x-axis represent the \( N_{RE} \) value used for referrals, if any, except for the position at the far left which indicates the results if neither \textit{max}_\textit{nbors} nor thresholding is used. The y-axis indicates the mean absolute error for each of the variants measured, which is the same as outlined for the summary views in the previous subsection.

For a smaller network such as \textit{max}_\textit{nbors} = 2, the improvement is pronounced, as indicated by the reduced error for the \textit{max}_\textit{nbors} = 2 graph if the experience level is set to \( N_{RE} = 3 \) or \( N_{RE} = 4 \). For example, while using \textit{max}_\textit{nbors} = 2 without referrals in the 60% lying advisors had an MAE of 0.081, allowing for referrals with an \( N_{RE} \) value of 4 led to a significantly smaller MAE of 0.054. Although this does not overcome the benefits of using a larger \textit{max}_\textit{nbors} value – for example, setting \textit{max}_\textit{nbors} = 5 without using referrals resulted in an MAE of 0.034 – these results are still much closer in terms of accuracy.

Figures 7c and 7d show a similar set of simulations for various possible threshold levels, again showing how different combinations of thresholds (graphs) and referral experience levels (x-axis) affect the mean error of each trust model variant (y-axis). These figures show that, as with \textit{max}_\textit{nbors}, adding referrals with an appropriately-chosen experience level in combination with thresholding can reduce the error in the trust model for higher thresholds.\(^9\)

We conclude that our evaluation indicates that referrals can serve to improve the accuracy of the trust model if the size of the advisor network is very limited, such as if there is a very low maximum number of advisors, or a very high trustworthiness threshold.

4.2. Effectiveness on TRAVOS

To demonstrate the effectiveness of our optimizations on TRAVOS, we perform similar sets of experiments to those performed in Section 4.1. The results of these experiments are shown in Figure 8. Each figure shows two graphs, indicating how each model performs for both of the tested levels of lying advisors; as with the graphs shown previously, the data points map the applicable \textit{max}_\textit{nbors} or threshold parameter on the x-axis to the mean absolute error (MAE) of the trust model under that scenario on the y-axis.

These figures indicate mixed results with regards to the effect of applying these modifications to TRAVOS. Consider that an unrestricted network (as represented by the far right of the \textit{max}_\textit{nbors} figure, or the far left of the thresholding figure, within Figure 8) will yield a mean absolute error value between 0.15 and 0.25. Recalling our discussion of the meaning of an MAE of

\(^9\)Note that this is only up to a point. If the threshold is set too high, so that some buyers end up having advisor networks of size 1 or 0 (as appears to be the case for using a threshold of 0.8), then referrals do not add value.
Figure 7: Evaluation of effects of referrals on small advisor networks for PTM.
In the previous section, it should be clear that these MAE values indicate relatively low accuracy. In comparison, most of the models incorporating either max_nbors or a threshold will have a smaller error value, and thus improved accuracy over an unrestricted network.

However, the progression is not entirely consistent. For example, in examining Figure 8, the graphs representing the TRAVOS model have a zig-zag shape, with the MAE increasing and decreasing at various points. One possible theory is that TRAVOS itself supports inaccuracies under certain circumstances when allowing its networks to be less restrictive: it actually adds advisors into the network which reduce the trust modeling error, despite the fact that these agents should be modeled as less trustworthy than those included in a more restricted network. Another theory for the performance, based on our examination of the advisor trust values produced by our simulations is that these values sometimes cluster into small ranges – for example, several trust values of approximately 0.42 may be generated, and several more with value of approximately 0.5, but none between 0.43 and 0.49. The implication is that a relatively minor change in the threshold value may serve to eliminate a number of advisors with the same level of trust, in turn affecting the overall trust model of sellers, possibly
The results therefore reinforce the value of our proposed approach, to set an effective value for $\text{max}_\text{nbors}$ or thresholding, shown here through experimental methods. To this end, we are able to say that in general, TRAVOS works best in this scenario when $\text{max}_\text{nbors}$ is set to 20, or when a threshold of 0.5 is set.

We next look at examining the effect of advisor referrals using TRAVOS. Again, as with our work in the previous section, this is performed using a modified version of the above scenario; in this case the number of sellers is reduced to 40, and each buyer or advisor may submit 120 seller ratings, with no limit on the number of times each seller could be chosen.

The results for these tests are shown in Figures 9 and 10. As with the earlier figures, these are summary graphs which indicate the mean absolute error obtained for various combinations of minimum referral experience (RE) and $\text{max}_\text{nbors}$ / threshold parameters; each series represents a different $\text{max}_\text{nbors}$ or threshold value, while the $x$-axis indicates the corresponding RE value. Like the results for PTM (see Figure 7), these graphs show that for low values of $\text{max}_\text{nbors}$, where the advisor network size is very small, using referrals will provide a reduction in error (that is, a trust model with improved accuracy).
When using thresholding, similar reductions in error were observed by adding advisor referrals to networks using high threshold values (and hence having a small size). However, reductions in error were also occasionally seen for larger networks (those produced by using smaller thresholds): such improvements were rarely seen when applying referrals to large networks using PTM.

The high error that is seen in PTM when applying larger threshold values is generally due to the buyer having modelled very few, if any, of the advisors with such a high threshold, which leads in turn to insufficient information to model the trust of sellers (and assigning the default trust value of 0.5). In comparison, during our simulations, TRAVOS would assign high trust values, on the order of 0.8 or 0.9, to advisors more frequently, potentially because that model uses a more fine-grained model of advisor trust based on the advisor, the buyer, and the seller under consideration (whereas PTM calculates an overall value based only on the buyer and advisor). Accordingly, setting a high threshold would not affect the amount of information available to TRAVOS in the same way that it would PTM, leading to the more accurate results in this case.
4.3. Summary of Experimental Results

We have seen that either using trustworthiness thresholding or setting a maximum number of advisors will provide an improvement to the accuracy of the trust model. We have also seen that, in cases where the size of the advisor network is very small, using referrals may help to further improve the accuracy of this model. In all cases, the parameters to be used should be modestly sized – allowing a reasonable number of advisors to be used, without including a large number of advisors that contribute little to the calculations of the trust model.

With the presentation of this array of graphical results, the value of employing our empirical approach for setting max_nbors or thresholding becomes apparent: allowing these variables to be selected arbitrarily by system designers yields the weaker trust modeling performance plotted in the parts of the graphs that do not respect the proposed parameter values a practitioner is directed to, from our results. As such, our methods are in fact demonstrating that employing a principled approach not only results in better overall trust modeling accuracy but also serves to avoid the loss of accuracy that might result, if an unprincipled approach were followed.

With each graph, we have discussed what the plots suggest for the values a practitioner should use when restricting advisor network size, depending on the trust model that is used. This then confirms our first hypothesis: that an empirical method can effectively provide system designers with some valuable direction for setting their size of advisor network. The steps that a designer should follow in order to reveal the desired insights into setting parameter values have been clarified throughout this paper. We summarize these as follows:

- Run a simulation of the model with a single “buyer” agent, and a sufficient number of both advisors and sellers such that the number of interactions between either the buyer or a single advisor on the one hand, and a single seller on the other hand, will be insignificant compared to the total number of interactions. In other words, the numbers must be sufficiently large to ensure that any outliers that may exist in the data have an insignificant effect on the overall results.

- Run this simulation such that the buyer and each advisor has at least one experience with a sufficient number of sellers that any outliers will have little effect.

- Regardless of the number of sellers, they should be divided into multiple disjoint trustworthiness categories, each containing the same number of sellers. Each category is assigned a different probability that the seller will act dishonestly, which is then assigned to the sellers.

- In regards to the honesty of advisors, the simulations should be run in two sets: one where advisors are mostly reporting honestly when rating sellers, and one where advisors are mostly lying about their experiences.

- At this point, depending on which techniques are desired to be used, several options for the appropriate parameters - either max_nbors or the trust
threshold, and optionally the minimum $N_{RE}$ should be identified for the simulations.

- The simulation for each option being considered should be repeated several times (at least five to ten) and then averaged, to help negate the effect of any outliers that appear. Once the simulations are complete, find the mean absolute error - the average of the absolute differences between the expected and experimental trust values for all of the sellers - for each simulation. The option yielding the lowest mean absolute error will then indicate the optimal parameter(s).

5. Related Work

Our usage of the $max_nbors$ and thresholding approaches in this paper was inspired in large part by the evaluation of design parameters for collaborative filtering (CF) recommender systems in (Herlocker et al., 2002). That work evaluated $max_nbors$ and thresholding, two methods that had already been discussed in the CF literature (Resnick et al., 1994; Sharanand and Maes, 1995), with regards to the correlation between two agents in a recommender system. Like our work here, that work found that $max_nbors$ was effective in improving the accuracy of the recommendations, so long as the value chosen was large enough to include a sufficient number of experienced agents, and small enough to exclude those agents that contribute little to the evaluation. Unlike our findings for trust, however, the authors claim that for such a system, a $max_nbors$ value between 20 and 50 should suffice for most “real-world” scenarios, without regard to population size. Moreover, they do not find any benefit of using thresholding in the correlation case. Thus it seems clear that there are strong distinctions between collaborative filtering and trust modeling, at least in regards to how to select the best size for a social network in each application.

A number of other researchers have proposed methods of incorporating collaborative filtering techniques into trust modeling or vice versa. In most cases these have taken the form of using the basic collaborative filtering model, substituting trust as the metric as opposed to similarity (Massa and Avesani, 2004, 2007). Some later work (Lathia et al., 2008) takes this a step further by proposing a variant of $max_nbors$ (also known as $k$NN), called $k$-nearest recommenders, or $kNR$, which dynamically selects the best $k$ neighbours that are able to provide information about a particular desired item. However, modeling a seller poses more challenges. For example, an advisor may lie about particular sellers while providing truthful reporting about other sellers, simply because the advisor is a friend or competitor of those particular sellers. This is precisely why TRAVOS models the trustworthiness of advisors regarding to some specific sellers (Teacy et al., 2006), and PTM employs the notion of time windows when modeling the trustworthiness of advisors and sellers (Zhang, 2009).

Yu and Singh have also explored work related to the usage of agents in trust and reputation systems, being the first to offer the suggestion of an advisor referral mechanism for reputation management (Yu and Singh, 2000). In later
work exploring the variance of the number of agents used in trust modelling (Yu and Singh, 2003), they showed that in varying the number of “witnesses” (agents) used in generating a trust model, the prediction accuracy improves slightly under otherwise identical conditions. However, due to the nature of the model, the number of agents to consider was limited to the range of 1 to 6 – much smaller than the max_nbors values we considered; moreover, their model used a large number of simulation cycles.

We also note that the referral techniques discussed above have some similarity to the Repage system presented in (Sabater et al., 2006) – a system that is more directly related to trust modelling. Similar to certain of the referral systems discussed above (Yu and Singh, 2003), Repage combines the requesting agent’s own evaluation, or image, of the target agent with the reputation of that agent, i.e. the requesting agent’s belief about the consensus evaluation regarding the target. The latter component is derived in large part from third-party agents, known as informers, which can transmit their own reported images of the target; units known as detectors are then responsible for determining which information will be most useful in evaluating the reputation of the target. Thus, the set of informers need not be static, much as the referral mechanism we introduced in Section 3.2 seeks to find the advisors that are most experienced with a given target agent. Based on this earlier work, it stands to reason that a similar advisor referral method could be used in combination with the limiting techniques discussed in the previous section in order to yield an overall smaller advisor network size. However, we also explored the effects of our more principled approach to applying network limiting techniques – specifically, varying the size of the advisor network – alongside advisor referrals, whereas the size of the network and other parameters were kept constant in some of the earlier works, specifically (Yu and Singh, 2003; Yolum and Singh, 2005).

6. Conclusion and Future Work

In this paper, we have outlined three potential improvements to trust modelling – trustworthiness thresholding, maximum number of advisors, and advisor referrals – all of which aim to improve the accuracy of the recommendations for trustworthy agents derived from a buyer’s advisors. These three improvements can be used with different trust modelling methods, specifically the Personalized Trust Model and the TRAVOS model, as demonstrated in our study. We have also demonstrated that our proposed approach is sufficiently robust that it can be applied to offer improvements. The positive results outlined above suggest that other researchers should be able to adopt these methods when seeking to improve their own trust models. Towards this end, we have also clarified the experimental framework which can be used to derive appropriate parameter values. Thus, the value of our work lies in not merely providing improvements to the use of these techniques within trust modelling, but also in setting out a more comprehensive procedure for carefully applying those techniques. This procedure will ultimately demonstrate how to best determine an appropriate size for
a social network of this type. As the accuracy of trust models gets optimized, users will have increased confidence when interacting with other agents.

A related topic which would be promising to explore in the future is improving the performance of the trust models when using our proposed methods. Given that these methods will in many cases substantially reduce the size of the advisor network used to produce the trust model of sellers, some performance optimization of these methods could help to improve the overall performance of the trust model. In particular, if a performance or memory gain could be developed to favour very small advisor networks while using referrals, this improvement might make up for the slight difference in accuracy compared to using larger networks.

With regards to our referral mechanism, we noted earlier that there is a limit on the number of levels of advisors through which this algorithm will search when looking for an acceptable replacement advisor. Presently this is set arbitrarily, based on a prediction of how many levels will be needed to search all nodes. In future work, we might examine whether varying this limit might produce improved results for referrals. In our referral mechanism, we also made the simplifying assumption that buyers would continue to use their own previously-calculated trust models to represent the trustworthiness they should hold in each referred advisor. This method was chosen as it seemed to be the most efficient means of setting the trust of each referred advisor, given that these trust values had already been calculated, whereas using another method might require additional time-consuming calculations. As noted earlier, this method provides some positive results. However, there is room for future expansion. Currently we do not take into account the referring agents’ opinions of the referred agents. However, trust propagation, or defining an agent’s trust in another agent as a function of all of the connections between them, has been studied extensively in (Jøsang and Pope, 2005; Hang et al., 2009; Wang and Singh, 2006; Hang and Singh, 2010), and might provide an even more accurate trust model, especially in a larger network. For future work, we will also explore this direction.

Considering other qualities of advisors in selecting the optimal subset is another valuable direction for future research. For example, the proposals of Cormier and Tran (2009) introduce parameters of agent lifetime and total transaction cost of an agent. Rosaci (2012) discusses distinct dimensions that can be considered, such as competence, honesty and reliability. It would be interesting to integrate these into our approach for determining the ideal set of advisors, and thus also enriching our proposals for advisor referrals.

Work has also been done on the measure of information gain obtained as more agents are introduced into a trust or reputation system, which may be an additional factor to consider when determining how large the size of the advisor network should be (Sierra and Debenham, 2007). Another possibility would be to consider a combination of thresholding and size of neighbours, for example using a threshold but only up to a maximum number of neighbours. We could then examine experimentally how best to set these parameters.

Another avenue for future exploration is considering dynamically setting
collaborator lists, beyond having a fixed threshold or a size limit. Some first steps for this idea are explored in the context of Intrusion Detection Networks in our work with Fung (Fung et al., 2010). In this model, a greedy approach seeks to always bring in advisors that lead to the overall lowest cost. Since these models also show when the size of acquaintance list is prohibitive relative to the cost, they offer competing metrics for limiting network size. We would need to explore how to cast these algorithms in a more general light, beyond this specific application area, but then we could consider this as a competing approach for limiting advisor networks, leading to more detailed comparisons. Finally, it would be useful to consider how the domain under consideration might affect the choice of both the trust model and the specific methods or parameters used to optimize it (Koster et al., 2010). While we have focused on electronic marketplaces, other models are used in different domains – as in modelling the trust between agents collaborating on a health-related challenge – and the usefulness of our proposed methods may vary from one domain to the next.

References


Appendix A. Coordinate Values of Points in Figures 1, 2, 4 and 6

Tables A.1 to A.7 present the detailed coordinate values of the points on each line in Figures 1a, 1b, 2a, 2b, 4a, 4b and 6, respectively.

Table A.1: Coordinate Values of Points in Figure 1a

<table>
<thead>
<tr>
<th>Prob. of Seller Dishonesty</th>
<th>MaxNbors</th>
<th>Ideal Case</th>
</tr>
</thead>
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<td>15  20  30</td>
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Table A.2: Coordinate Values of Points in Figure 1b

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### Table A.3: Coordinate Values of Points in Figure 2a

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<td>0.12</td>
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Table A.6: Coordinate Values of Points in Figure 4b

<table>
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<tr>
<th>Seller Dishonesty</th>
<th>MN=15</th>
<th>MN=40</th>
<th>Thr=0.55</th>
<th>Ideal Case</th>
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<td>without referrals</td>
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<td></td>
</tr>
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<td>0.94</td>
<td>0.92</td>
<td>0.94</td>
</tr>
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<td>0.86</td>
<td>0.84</td>
<td>0.75</td>
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<td>0.67</td>
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</tr>
<tr>
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<td>0.60</td>
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Table A.7: Coordinate Values of Points in Figure 6

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<th>No Refs</th>
<th>With Referrals and RE =</th>
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<th>Worst Case</th>
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