Obtaining Time Derivative of Low-Frequency Signals With Improved Signal-to-Noise Ratio

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Abstract—Accurate prediction of heat flux is desired in many transient aerospace and heat treatment applications, but it is challenging since the heat flux–temperature integral relationship implicitly requires the time derivative of experimentally obtained temperature data. The temperature data collected in practical environments invariably contain noise from various sources. Predicting heat flux from transient temperature data is well known to be ill posed. High-frequency noise in the temperature data causes unbounded numerical derivatives with increasing sampling rate. However, it has theoretically been demonstrated that a stable and accurate heat flux can be predicted using the time derivative of temperature \( \frac{dT}{dt} \) even in the presence of significant white noise. This motivates this paper in developing a voltage–rate sensor interface for low-frequency applications in solid heat-conducting bodies. The present concept is to amplitude modulate the voltage data and then differentiate them at a higher frequency. The voltage–rate interface, which is used in conjunction with an existing in situ temperature sensor, can deliver real-time heating rate with improved SNR, which is verified by both simulation (Matlab and PSpice) and experiments. The SNR is also shown to improve with increasing sampling rate, which is an advantage of this interface.

Index Terms—Amplitude modulation, differentiation, heat flux measurements, ill-posed problems, signal-to-noise ratio (SNR).

I. INTRODUCTION

For heat conduction in solids, the variation of in-depth temperature data \( T \) (temperature in degrees Celsius) should only involve low frequencies owing to the nature of diffusion. Thus, the recorded high-frequency occurrences are invariably due to contamination in the received signal. Using such temperature data to predict the heat flux is ill posed since high-frequency noise is amplified and becomes highly unstable as the sample rate is increased [1]–[6]. These inverse problems are termed ‘ill posed’ since a small perturbation in the input can produce arbitrary large variations in the output. It has been shown [5] that if the data are presented in rate form \( (dT/dt) \), then the severity of the ill-posed problem is greatly reduced. However, obtaining reliable derivatives in real time has been an elusive inverse problem in itself as the process involves differentiation, which is the primary source of ill-posed problems.

The goal of this paper is to develop an interface for the existing in situ temperature sensors (e.g., thermocouples) from which the rate information can be extracted with improved SNR. This interface is placed between the actual in situ sensor and the real-time application unit. In general, sensors convert physical quantities into electrical signals. The proposed interface then converts the voltage signal into voltage rate. As most sensors deal with voltage signals, this interface can be used in a variety of applications, such as solid and fluid mechanics, pressure and seismic analysis, as well as aerospace, heat treatment, and defense and homeland security applications. By developing this interface for low-frequency real-time applications, the rate data can provide meaningful information even when the measurements are taken in a highly noisy environment.

This is an addon to the existing hardware infrastructure, so the cost to upgrade is small. Higher-order time derivatives become available through cascading. Diffusive heat transfer is the application base for this paper. Thus, this paper investigates and develops such a sensor interface based on amplitude modulation for low-frequency physics.

In addition to its application in telecommunication, amplitude modulation has also been used in signal conditioning circuitry, such as chopper-stabilized amplifier. The chopper-stabilized amplifier uses an ac carrier (chopping signal) to amplitude modulate the input signal to a higher frequency, so as to avoid amplifying low-frequency noise, and then synchronously demodulate the ac signal and low-pass filter to reestablish the dc signal [7]. Therefore, chopper-stabilized amplifiers can achieve a very low dc offset and drift. However, a square wave is used for modulation, which may produce interfering signals consisting of higher-harmonic frequencies. Subsequent aggressive filtering, with bandwidth of less than a few hundred hertz, limits the usefulness of the wider available bandwidth. Our design aims to minimize noise in both higher and lower ends of the frequency spectrum, as opposed to only low-frequency noise (mostly 1/f noise) suppression in the chopper amplifier. A sinusoidal wave is used as carrier to avoid the ripple voltage present in the chopper, so that high-frequency parasitics is minimized, and sufficient bandwidth is maintained for signal processing with fidelity.

II. CASE STUDY OF ILL-POSED DATA

The ill-posed nature of the heat conduction problem can be illustrated by considering a heat transfer experiment, where the
prediction of heat flux is calculated from the temperature in constant property, heat conductivity, and half-space geometry with trivial initial condition using the integral expression [5]

\[
q''(x, t) = \sqrt{\frac{\rho C k}{\pi}} \int_{u=0}^{t} \frac{dT(x, u)}{dt} \frac{du}{\sqrt{(t - u)}},
\]

\[x \geq 0, \quad t \geq 0 \tag{1}\]

where \(T\) represents the temperature at an arbitrary position along the \(x\)-axis, \(q''\) is the heat flux in the \(x\)-direction, \(\rho\) is the density, \(C\) is the heat capacity, and \(k\) is the thermal conductivity. Equation (1) is often used in null-point calorimetry and thin-film heat flux gauges [5]. Finding the heat flux for \(x > 0\) using noisy in-depth temperature data is unreliable as the process is ill-posed [5], [6]. The differentiation involved in (1) is the culprit or source of the ill-posed effect.

The fundamental problem with differentiators is that the gain of the signal is proportional to the signal frequency. Thus, the high-frequency components in the input signal are amplified more than the low-frequency components. If we are dealing with a signal of low frequency (\(f < 100\) Hz), as is the case with many heat transfer/diffusion applications, then the signal gain is low. In contrast, white noise with a uniform spectrum is amplified more than the signal components. This causes the SNR of the output signal to decrease. In the context of (1), it has been shown [6] that if temperature data are contaminated with white noise, then the RMS error of the measured surface heat flux grows as \(\sqrt{N}\), where \(N\) is the sample size of the temperature data. This can be problematic since it is highly desirable in many applications to have a large \(N\). As a comparison, if contaminated heating/cooling rates were available, then the RMS heat flux error decreases as \(\sqrt{\log(N)/N}\) with increasing \(N\) [2] and, hence, is well posed. To improve the SNR, one needs to increase the number of samples of the rate. Using an infinite sample signal, i.e., an analog signal instead of a digital signal, further reduces the RMS error of the heat flux.

An example of the advantages of using rate information is given in Fig. 1 [5]. Fig. 1(a) shows the simulated temperature data, and Fig. 1(b) shows the predicted heat flux using (1). The exact heat flux solution is a Gauss function for this simulation. However, as the simulated input temperature data are contaminated with noise generated from a uniform probability density...
function (i.e., white noise), the predicted heat flux, as displayed in Fig. 1(b), is highly in error. On the other hand, if \( dT/dt \) data were available per Fig. 1(c), then the predicted heat flux closely reconstructs the desired Gaussian shape shown in Fig. 1(d). The predicted heat flux using rate data appears excellent, although the noise is eight times greater in the rate data than in the temperature data. Details of this particular heat transfer study can be found in [5].

From the foregoing discussion, it is evident that there exists a need to acquire real-time values for \( dT/dt \). A voltage–rate interface that is designed to produce an accurate depiction of rates in accordance with the concept \((dT/dt = dT/dV \cdot dV/dt)\) is proposed. Here, the voltage signal is extracted by an \textit{in situ} sensor, such as a thermocouple, and would provide the necessary voltage to the proposed interface. The property \( dT/dV \) is available from the \textit{in situ} sensor’s calibration curve.

Finding a stable and reliable differentiator \((dV/dt)\) is difficult when using a noisy voltage signal. Analog filtering eliminates some of the problems caused by high-frequency noise components but does not solve the entire problem of low-frequency differentiation. This paper proposes to amplitude modulate the low-frequency voltage signal and then differentiate it at high frequencies. The SNR can substantially be increased by using this amplitude modulation differentiator. For example, if the signal has a passband of 1–100 Hz, then the signal has a 100% change in the frequency domain; but if the signal is amplitude modulated by a carrier of 100 kHz, then the signal has a 100% change in the frequency domain; but if the signal is amplitude modulated by a carrier of 100 kHz, then the variation is from 100 to 100.1 kHz, which is less than 0.1%.

The gain of the noise signal is also affected by the same ratio. Hence, differentiation at a higher frequency will improve the SNR. The method of extracting a differentiated signal after amplitude modulation and differentiation is explained in the following sections using single-tone signals.

III. EXTRACTION OF THE DIFFERENTIATED SIGNAL

A double-sideband suppressed-carrier (DSBSC) method is chosen for amplitude modulation because of its mathematical simplicity during differentiation. The DSBSC modulation [8] is performed by multiplying the modulation signal \( m(t) \) with the carrier wave \( c(t) \) as \( AM(t)_{DSBSC} = m(t) \cdot c(t) \), where time is denoted by \( t \). Differentiating the amplitude-modulated wave produces

\[
AM'(t)_{DSBSC} = m'(t) \cdot c(t) + m(t) \cdot c'(t) \tag{2}
\]

where the prime (’) denotes time differentiation. The desired component \( m'(t) \) is in the first term in (2), which is the AM of the differentiated signal with the carrier wave. Rate information can be extracted by isolating and demodulating \( m'(t)c(t) \).

The procedure for extracting the differentiated component is subsequently explained with the help of two single-tone signals. This method can also be applied to other types of passband signals. Consider a single-tone modulating signal \( m(t) = A_m \cos(\omega_m t) \) and a carrier wave \( c(t) = A_c \cos(\omega_c t) \), where \( A_m \) and \( A_c \) are the magnitudes of the modulating and carrier signals, respectively, and \( \omega_m \) and \( \omega_c \) are the modulating signal and carrier frequencies. The amplitude-modulated signal \( AM(t) \) can be written as the sum of two frequency components as

\[
AM(t) = m(t) \cdot c(t) = A_m A_c \cos(\omega_m t) \cos(\omega_c t) = \frac{A_m A_c}{2} \left[ \cos(\omega_c - \omega_m) t + \cos(\omega_c + \omega_m) t \right]. \tag{3}
\]

If we employ an active \( RC \) differentiator to differentiate this signal, then the output of such an \( RC \) differentiator is \( V_{out}(t) = -RC(d(AM(t))/dt) \), where \( R \) and \( C \) are the resistor and capacitor used during differentiation. The gain of the differentiator varies with the frequency \( \omega \). Because the AM signal has two different frequencies \( \omega_c - \omega_m \) and \( \omega_c + \omega_m \), they are consequently differently amplified. The differentiated AM signal is given as

\[
V_{out}(t) = -RC \frac{A_m A_c}{2} \times \left[ - (\omega_c - \omega_m) \sin(\omega_c - \omega_m) t \right] - (\omega_c + \omega_m) \sin(\omega_c + \omega_m) t. \tag{4}
\]

Equation (4) can be rewritten as the sum of two AM signals as

\[
V_{out}(t) = RCA_m A_c \omega_c \sin(\omega_c t) \cos(\omega_m t) + RCA_m A_c \omega_c \cos(\omega_c t) \sin(\omega_m t). \tag{5}
\]

Here, the first term has a larger magnitude than the second term as \( \omega_c \) is larger than \( \omega_m \).

According to (5), the differentiation of the AM signal results in the following two components: 1) a desired component \( \cos(\omega_c t) \sin(\omega_m t) \), i.e., \( c(t)m'(t) \), and 2) an undesired component \( \sin(\omega_c t) \cos(\omega_m t) \), i.e., \( m(t)c'(t) \). The undesired component is removed by subtracting \( m(t)c'(t) \) from the output signal of the differentiator.

The steps involved in obtaining \( m(t)c'(t) \) are as follows: Again, by differentiating the carrier with an active \( RC \) differentiator, we obtain \( DC(t) = -RC(d(\omega_c t)/dt) = RCA_c \dot{\omega}_c \sin(\omega_c t) \), which is \( -RC\dot{\omega}_c \). Then, using this signal as a carrier signal in the amplitude modulation of \( m(t) \), we obtain

\[
m(t) * DC(t) = RCA_m A_c \dot{\omega}_c \cos(\omega_m t) \sin(\omega_c t). \tag{6}
\]

This signal has the same amplitude \( RCA_m A_c \dot{\omega}_c \) as the first term in (5). Combining this component to the differentiated AM signal in (5) using a summing circuit only leaves the desired component, i.e., \( RCA_m A_c \omega_m \cos(\omega_c t) \sin(\omega_m t) \). This is a small quantity as it only depends on the gain of the differentiator at the modulating frequency \( \omega_m \), which is much lower than the carrier frequency used.

The last step to obtain the desired voltage rate \( m'(t) \) is to demodulate the output from the summing circuit \( m'(t)c(t) \).

Fig. 2 shows a block diagram of the voltage rate extraction process. The carrier signal divides into two branches: one to an amplitude modulation and later differentiation unit, and the other to a differentiator and modulator unit to generate the undesired component. The differentiators in both stages can also be used to amplify the signal to a desired magnitude and thus eliminate a further amplification stage. These signals are
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![Block diagram of the low-frequency voltage–rate sensor interface]

Fig. 2. Block diagram of the low-frequency voltage–rate sensor interface

given as inputs to a summing amplifier, where the undesired signals are cancelled out, and the output is \( m'(t)c(t) \). The output is then demodulated to obtain the desired differentiated input signal \( m'(t) \), for which a better SNR is expected.

The whole purpose of AM differentiator is to obtain an output with less noise than the direct signal differentiation at low frequencies. The SNR of the output signal is an important figure of merit in the entire process of differentiation. To verify the benefits of the proposed process, in both simulations and experiments, white noise is added to the input signal, and the SNR is measured at both input and output for comparison. The results are given in the following sections.

IV. CIRCUIT DESIGN

The components for developing the interface are low-pass filters for the input and the demodulated signal; an oscillator for generating the carrier signal; multipliers for modulating and demodulating the signal; and differentiators for differentiating the carrier and the amplitude-modulated signal.

The frequency range of signals from heat transfer sensors is typically below 100 Hz. Fig. 3 shows the difference in the spectra of the temperature and rate data used in [5]. Since the desired signal is concentrated in the low-frequency spectrum, filtering of high-frequency noise in the modulating signal will greatly improve the SNR. A low-pass filter, which has minimum jitter in the stopband, is required for this operation. To minimize the phase shift, the cutoff frequency of the filter needs to be a decade higher than the actual pole present in the system and, thus, chosen as 1 kHz. The second filter is designed for the demodulation of the differentiated signal, with cutoff frequency greater than the highest frequency in the differentiated signal. We used the data in [5] for the proof of concept; the cutoff frequency of the filter after demodulation is chosen to be above 1500 Hz. Based on these requirements, a two-pole Sallen–Key low-pass filter [9] has been implemented for both cases.

A voltage-controlled oscillator (VCO) chip EXAR XR2206 is adopted to produce the carrier wave for modulation, where oscillations from VCO are controlled using a phase-locked loop built in the chip. The integrated circuit also provides external symmetry adjustment for removing any distortion present at the input. For multipliers, an AD633 chip (Analog Devices, Inc.), which is a four-quadrant multiplier, is used for both amplitude modulation and demodulation stages. It has two high-impedance differential input pairs to minimize the loading effects. The amplitude modulator has an additional feature of dividing the output voltage by a factor of 10 so that the output does not exceed the opamp rail voltages.

The differentiators play a crucial part in the entire system. The circuit diagram of a generic differentiator is shown in Fig. 4. The transfer function for the differentiator is given by

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{j\omega R_2 C_1}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2)}.
\]

(7)

For a basic differentiator, \( C_1 \) and \( R_2 \) are sufficient, but by adding \( R_1 \) and \( C_2 \), the circuit can work as a differentiator at low frequencies and as a low-pass filter at high frequencies to further increase the SNR. The gain can be set to reach a maximum at a pole frequency \( \omega_h \). If two poles are designed to coincide at a single frequency, i.e., \( \omega_h = 1/R_1 C_1 = 1/R_2 C_2 \), then above \( \omega_h \), the slope of the gain will change from 20 to \(-20\) dB/dec, filtering out the unwanted high-frequency components. For frequencies lower than \( \omega_h \), the transfer function is approximated as \(-j\omega R_2 C_1 \). Therefore, the zero of the transfer function in the frequency domain for (7) is given as \( \omega_z = 1/R_2 C_1 \), where \( \omega_z \) is the frequency at which the gain of the differentiator is 0 dB. Between the frequencies \( \omega_z \leq \omega \leq \omega_h \), the signal is differentiated with a gain of \(-j\omega R_2 C_1 \).

The differentiators are used at two places in the circuit, i.e., for differentiating the carrier and the AM signal. The differentiation of the carrier is straightforward as it is a
single-frequency \( \omega_c \) signal. In contrast, the AM wave is the sum of two frequencies \( \omega_c - \omega_m \) and \( \omega_c + \omega_m \). To achieve a gain higher than 0 dB, the lower frequency component \( \omega_c - \omega_m \) of the AM is designed to be higher than the zero \( \omega_z \) in the frequency domain. In addition, to minimize the phase shift introduced by the poles, \( \omega_h \) needs to be greater than 10\( (\omega_c + \omega_m) \).

Another factor to consider in the design is the magnitude of the differentiated output. According to (5), the magnitude of the differentiated output is given as \( R_2 C_1 \omega_m A_m A_c \), which is clearly dependent on the gain of the differentiator at the modulating signal frequency. If the gain at the modulating frequency \( \omega_m \) is small, then the output signal may become too small to be retrieved. However, if the gain of the differentiator at \( \omega_m \) is designed to be large, then there are limitations on amplitude modulation, because the opamps used cannot deliver outputs higher than power supply voltages.

The complete schematic of the circuit is shown in Fig. 5. The modulating signal is amplified to a magnitude of 20 \( V_{p-p} \) and filtered using a low-pass filter. It is then amplitude modulated with a carrier signal of 2 \( V_{p-p} \). The undesired AM component after differentiation is cancelled out at the stage of the summer, and the output is obtained by filtering the demodulated signal.

There are several limitations in the circuit design, such as voltage offset, circuit noise, and phase errors generated by each and every element. The opamps are carefully chosen to have the least offset voltage and circuit noise. It is also required to use offset nulling circuits [10] before the summing circuit such that the signals that are summed together have zero offset. This helps in completely cancelling the undesired signal. Variable resistors are used in both offset adjustment circuit and phase shifters to adjust the signal during the experiment.

To determine the values of \( R_2 C_1 \) and the carrier frequency in the experiments, there are three major considerations.

1) The output amplitude of the differentiated signal only depends on \( RC \) and the modulating frequency \( f_m(\omega_m) \), according to (5).

2) The SNR of the output signal improves as the carrier frequency increases. However, the opamp rail voltage limits the use of a higher carrier frequency in the experiment, according to (5), since differentiation of the modulated signal cannot produce waveforms beyond the opamp’s rail-to-rail voltage.

3) The output magnitude is also limited by the values of \( RC \), since the gain from the differentiated carrier also depends on this term.

Therefore, there is a tradeoff between the magnitude of the differentiated signal at the end of the process and the SNR of the output. While the SNR increases with a higher carrier frequency...
frequency, the maximal allowed output signal goes down and may not properly be retrieved. On the other hand, if a low-carrier frequency is used, then the SNR of the output signal can be low even if the amplitude is high; as a result, the noise dominates the signal. The value of $R_2C_1$ determines the location where the magnitude of the output signal crosses 0 dB in the frequency domain. The rail-to-rail voltage limited the carrier frequency in the experiments to 10 kHz.

V. SNR

To determine the SNR of the signal, white noise is simulated and superimposed onto the voltage/temperature data in Fig. 1(a) taken from [5]. The improvement in SNR using the sensor interface is first mathematically simulated in Matlab and later in PSpice, which also takes electrical noise and distortion into consideration. The simulation results show that the SNR improves as the carrier frequency of the amplitude modulation increases. This improvement is also experimentally verified.

For the experiments, an Agilent arbitrary waveform generator (33220A) is used to supply the continuous input waveform. The input signal was taken from [5]. The output of the circuit is recorded with an Agilent MSO6012A oscilloscope and downloaded to a computer. The frequency spectrum of the continuous wave is measured using a Sleuth spectrum analyzer (Core Technology Group, Inc.).

The input voltage/temperature data have a passband of less than 100 Hz. Differentiating such low-frequency signals with traditional methods is very noisy, as shown in Fig. 6. The obtained SNR is “10.2,” which is deemed low. Further processing of the data cannot be done in real time. However, if the same signal is passed through the AM differentiator proposed in this paper, the SNR improves above 1000 in simulations and is proportional to the carrier frequency used.

Fig. 7 shows the output from the AM differentiator at a carrier frequency of 1 kHz, while Fig. 8 shows the differentiated signal at a carrier frequency of 10 kHz. It can be seen that the results closely match the theoretical prediction. PSpice results also show a similar improvement of SNR when the carrier signal frequency is increased. The simulations are performed using the model files of the actual component used in the experiment. Additional circuits, such as buffer amplifier to avoid loading and variable-gain amplifiers to obtain the desired magnitudes, are used during PSpice simulations.

The foregoing observations with respect to SNR have experimentally been verified. The oscillographs of the input and output signals of the experimental circuit for different carrier frequencies are shown in Figs. 7 and 8.
Fig. 7. AM differentiation of temperature data in Fig. 1(a) at a carrier frequency of 1 kHz.

Fig. 8. AM differentiation of temperature data in Fig. 1(a) at a carrier frequency of 10 kHz.

Fig. 9. Experimental waveforms. (a) Input and (b) output of AM differentiator.

Not only a higher carrier frequency but also an increasing sampling rate will reduce the RMS error. As the sampling rate increases, the RMS error decreases, which improves the SNR of the signal at a given carrier frequency. Fig. 11 shows the improvement in SNR with increasing carrier frequency for three different sampling rates (960 100, 1 920 100, and 3 840 100 samples/s). The SNR is larger when a continuous signal is used as an input signal instead of discrete samples. This external noise can significantly be reduced by having a prototype printed circuit board (PCB) designed for this purpose. However, the PCB design also depends on the modulating frequency. The design based on a single carrier frequency and subsequent increments in the carrier frequency necessitates a different board design. Hence, it is impractical to design a PCB board at this stage.

An exact 180° phase difference should be imposed between the undesired signals, which is the differentiated amplitude-modulated signal and the signal obtained from amplitude modulation of the differentiated carrier. This ensures the cancellation of the two undesired signals. The SNR deteriorates if the phase difference is not maintained. Precautions are taken to avoid these detrimental factors.
VI. CONCLUSION

This paper has identified an engineering gap in sensor technologies. As pointed out by prior work, using rate information is advantageous for inverse analysis. The goal is to study the feasibility of a universal solution with an interface module to convert the voltage output from sensors to voltage rate. Direct differentiation of the signal will produce an output in proportion to the signal frequency. Therefore, noise and errors prevalent in all measurements will increase relative to the signal, thus deteriorating the SNR even after painstaking smoothing/filtering. By upconverting the signal spectra with amplitude modulation, as suggested in this paper, the SNR of the derivatives will remain manageable to warrant reliable subsequent processing.

This paper has presented a prototype of the proposed strategy. The experiments were performed and compared with PSpice simulation results. The SNRs were improved in both cases, although the improvement of SNR in experiments was less than that in simulations due to external noises and opamp limitations. As the voltage signals are commonly adopted for sensor outputs, this interface module can be used with a number of sensors to extract the rate information. The success of this paper can potentially impact aerospace, energy, geophysical, and seismic sciences, health care, engineering sciences, defense, and national security applications.

REFERENCES


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