Optimal and Suboptimal Beamforming for Multi-Operator Two-Way Relaying with a MIMO Amplify-and-Forward Relay

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Abstract—In this work, we consider optimal and suboptimal beamforming designs in a multi-operator two-way relaying network with a multiple-input-multiple-output (MIMO) amplify-and-forward (AF) relay. Such a network is interference limited and thus, an interference nulling strategy is reasonable. We first derive the necessary condition for interference nulling and introduce a closed-form algebraic solution, i.e., the projection based separation of multiple operators (ProBaSeMO). This solution can be adjusted to satisfy various system design criteria. However, it is only suboptimal. For the design of optimal relay transmit strategies, we study two QoS design criteria for the network. The first is to minimize the relay transmit power subject to a signal-to-interference-plus-noise ratio (SINR) constraint at each user. The second is the SINR balancing technique with a relay transmit power constraint. These two problems are non-convex. However, we show that both problems can be efficiently solved using convex approximation techniques. The simulation results verify the sub-optimality of the ProBaSeMO method when compared to the optimal designs. However, the ProBaSeMO technique approaches optimality as the number of relay antennas increases and enjoys a significantly reduced computational complexity.

Index Terms—two-way relaying; MIMO; semidefinite programming, second-order cone programming.

I. INTRODUCTION

Relaying is a means of reducing the deployment cost, enhancing the network capacity and mitigating shadowing effects. Among various relaying techniques, two-way relaying uses the spectrum in an efficient manner and it can be used to facilitate the resource sharing in wireless networks. For instance, we have studied a resource sharing scenario in [1] where the spectrum as well as a MIMO AF relay (infrastructure) is shared between multiple operators to enable the communication between users of different operators. To accomplish this form of spectrum and infrastructure sharing, we have proposed the projection-based separation of multiple operators (ProBaSeMO) scheme (block diagonalization (BD) with algebraic norm-maximizing (ANOMAX) and regularized BD (RBD) with ANOMAX in [1]). It has been shown that the ProBaSeMO method can achieve a significant sharing gain in terms of sum rate in the case of two operators.

In this work, we consider a similar resource sharing scenario and deal with the optimal design of relay transmit strategies. We first derive the interference nulling condition. Furthermore, we show that our ProBaSeMO algorithm [1] fulfills this condition and can be extended to the $L$ operator case. Afterwards, we investigate two different optimal beamforming design criteria. The first design criterion is to minimize the relay transmit power subject to a SINR constraint at each user terminal (UT). The second one is to maximize the minimum SINR of the UTs in the network subject to a relay transmit power constraint, i.e., SINR balancing. The two problems are non-convex. Nevertheless, we show that both problems can be solved efficiently using established convex approximation techniques. The simulation results show that the ProBaSeMO algorithms can approach optimality when the number of antennas at the relay increases. Furthermore, it has less computational complexity compared to the convex optimization based techniques.

II. SYSTEM MODEL

We consider the system in Fig. 1. Several pairs of UTs which belong to $L$ operators communicate with each other with the help of one relay. The relay has $M_R$ antennas while each UT has a single antenna. We assume that the channel is flat fading. The channel between the $k$th user of the $ℓ$th operator and the relay is denoted by $h_k^{(ℓ)} \in \mathbb{C}^{M_R}$ ($k \in \{1,2\}$ for users, $ℓ \in \{1,\ldots,L\}$ for operators).

The two-way AF relaying protocol consists of two phases: In the first phase, all the UTs transmit their data simultaneously to the relay. The received signal vector at the relay is...
where $x_k^{(ℓ)}$ stands for mutually uncorrelated transmitted symbols with zero mean and variance $P_k^{(ℓ)}$, $∀k, ℓ$. The zero-mean circularly symmetric complex Gaussian (ZMCSGC) noise vector at the relay is denoted by $n_R ∈ C^{M_R}$ and $E\{|n_R|^2\} = σ^2_R I_{M_R}$. In the second phase, the relay forwards the signal to all UTs simultaneously. The signal transmitted by the relay can be expressed as

$$
\tilde{v} = G \cdot r.
$$

(2)

where $G ∈ C^{M_R × M_T}$ is the relay amplification matrix. The received signal $y_k^{(ℓ)}$ at the $k$th UT of the $ℓ$th operator can be written by

$$
y_k^{(ℓ)} = h_k^{(ℓ)T} \tilde{v} + n_k^{(ℓ)}
$$

(3)

where $n_k^{(ℓ)}$ denotes the ZMCSGC noise with variance $σ_k^{(ℓ)^2}$. Taking into account that the self-interference can be subtracted from the received signal $y_k^{(ℓ)}$ at the receiver, the SINR at the $k$th UT of the $ℓ$th operator can thus be written by

$$
η_k^{(ℓ)} = \frac{E\{|h_k^{(ℓ)T} G h_k^{(ℓ)T} x_k^{(ℓ)}|²\}}{E\{|h_k^{(ℓ)T} G n_R|²\} + σ_k^{(ℓ)^2}}
$$

(4)

To derive the optimal $G$, further algebraic manipulations are required. The transmit power at the relay can be expanded as

$$
E\{|v|²\} = E\{Tr\{Gr(Gr)H\}\}
$$

\[\begin{aligned}
&= Tr \left\{ G \left( \sum_{k, ℓ} P_k^{(ℓ)} h_k^{(ℓ)T} h_k^{(ℓ)H} + σ^2_R I_{M_R} \right) G^H \right\} \\
&= \sum_{k, ℓ} P_k^{(ℓ)} Tr\{G h_k^{(ℓ)H} h_k^{(ℓ)T} G^H\} + Tr\{σ^2_R GG^H\} \\
&= \sum_{k, ℓ} P_k^{(ℓ)} (G h_k^{(ℓ)H})^H G h_k^{(ℓ)} + σ^2_R g^H g \\
&= g^H C g
\end{aligned}
$$

where $\| \cdot \|$ denotes the Euclidean norm and $g = \text{vec}\{G\}$. Moreover, $C$ is a positive definite Hermitian matrix which is defined as

$$
C = \sum_{k, ℓ} P_k^{(ℓ)} (h_k^{(ℓ)T} h_k^{(ℓ)H})^T \otimes I_{M_R} + σ^2_R I_{M_R}.
$$

(6)

The fact that $\text{Tr}\{ΓC\} = \text{Tr}\{ΓΓ\}$ and $\text{vec}\{ΓX\} = (Γ^T Γ)\text{vec}\{X\}$ is used in the derivation. Following a similar procedure, the SINR $η_k^{(ℓ)}$ can be rewritten as

$$
η_k^{(ℓ)} = \frac{g^H D_k^{(ℓ)} g}{g^H (E_k^{(ℓ)} + F_k^{(ℓ)}) g + σ_k^{(ℓ)^2}}
$$

(7)

where $D_k^{(ℓ)}$, $E_k^{(ℓ)}$, and $F_k^{(ℓ)}$ are defined as

$$
D_k^{(ℓ)} = P_k^{(ℓ)} (h_{3-k}^{(ℓ)H})^H (h_{3-k}^{(ℓ)T} h_k^{(ℓ)T}),
$$

$$
E_k^{(ℓ)} = \sum_{k=1, 2} P_k^{(ℓ)} (h_k^{(ℓ)T} h_k^{(ℓ)T})^H (h_k^{(ℓ)T} h_k^{(ℓ)T}),
$$

$$
F_k^{(ℓ)} = σ^2_R (I_{M_R} \otimes (h_k^{(ℓ)T} h_k^{(ℓ)T})^T).
$$

(8)

The matrices $D_k^{(ℓ)}$ and $E_k^{(ℓ)}$ are positive semidefinite Hermitian matrices while the matrices $F_k^{(ℓ)}$ are positive definite Hermitian matrices, $∀k, ℓ$. Our goal is threefold. First, we extend the ProBaSeMO concept to the case of $L$ operators. The second goal is to find the matrix $G$ which minimizes the relay power and the third is to study the SINR balancing technique.

III. SUBOPTIMAL AND OPTIMAL BEAMFORMING DESIGN

A. Interference Nulling Condition and ProBaSeMO

To null the inter-operator interference completely, the following condition needs to be fulfilled, i.e.,

$$
g^H E_k^{(ℓ)} g = 0, ∀k, ℓ. \tag{9}
$$

Equation (9) implies that a common null space has to exist for all $E_k^{(ℓ)}$. Recalling that $E_k^{(ℓ)}$ are positive semidefinite matrices, it is clear that (9) is equivalent to

$$
g^H \left( \sum_{k, ℓ} E_k^{(ℓ)} \right) g = g^H E_a g = 0. \tag{10}
$$

Using subspace analysis, it can be shown that the rank of $E_a$ is equal to min($M_R, 2(L-1)$). Thus, to null the interference completely $M_R > 2(L-1)$ antennas are required at the relay. The ProBaSeMO algorithm is a low-complexity suboptimal relay transmit strategy which fulfills the constraint in (10). In [1] it is proposed for the two-operator case. In the following we extend the concept of ProBaSeMO to $L$ operators.

The main idea of the ProBaSeMO algorithm is to parallelize the system design of each operator, i.e., decoupling different operators and then reweighting the transmit strategy for the system of each operator separately. Towards this end, the inter-operator interference needs to be canceled. It is observed from our scenario that the inter-operator interference is created in both the first and second transmission phases. To null all the interference, we find it is useful to decompose the relay amplification matrix $G$ into

$$
G = G_0 \cdot G_T = G_0 \cdot G_T \cdot G_S \cdot G_R ∈ C^{M_R × M_T}
$$

(11)
where $G_R \in \mathbb{C}^{LM_R \times M_R}$ and $G_T \in \mathbb{C}^{M_R \times LM_R}$ are filters designed to suppress the inter-operator interference during the first and second phase, respectively. The parameter $\gamma_0 \in \mathbb{R}^+$ is chosen such that the transmit power constraint at the relay is fulfilled. Moreover, $G_S$ is block diagonal since it represents the processing performed for each operator individually. The overall transmit and receive filter matrices $G_T$ and $G_R$ can also be partitioned as

$$G_T = \begin{bmatrix} G_T^{(1)} & \ldots & G_T^{(L)} \end{bmatrix}, \quad G_R = \begin{bmatrix} G_R^{(1)^T} & \ldots & G_R^{(L)^T} \end{bmatrix}^T$$

To design $G_T$ and $G_R$, a typical interference-nulling routine such as the BD approach can be followed [1]. Now we briefly introduce how to derive $G_R^{(j)}$ using BD. Let us define the combined channel matrix $H^{(j)} \in \mathbb{C}^{M_R \times 2(L-1)}$ for all UTs except for the UTs of the $j$th operator as

$$H^{(j)} = \begin{bmatrix} H^{(1)} & \ldots & H^{(L-1)} & H^{(L+1)} & \ldots & H^{(L)} \end{bmatrix},$$

where $H^{(l)} = [h^{(l)}_1, h^{(l)}_2] \in \mathbb{C}^{M_R \times 2}$ is the users’ concatenated uplink channel matrix of the $l$th operator. Then the columns of the matrix $G_R^{(j)}$ should lie in the left null space of $H^{(j)}$ so that the signal of the $l$th operator will not cause interference to all the other operators. Let $\hat{L}^{(j)} = \text{rank}(H^{(j)})$ and define the singular value decomposition (SVD) of $H^{(j)}$ as

$$\hat{H}^{(j)} = [\hat{U}^{(j)}_1 \hat{U}^{(j)}_2 | \hat{S}^{(j)} | \hat{V}^{(j)^H}],$$

where $\hat{U}^{(j)}_1$ contains the last $(M_R - \hat{L}^{(j)})$ left singular vectors. Thus, $\hat{U}^{(j)}_1$ forms an orthogonal basis for the left null space of $H^{(j)}$ such that $\hat{U}^{(j)^T} H^{(j)} = 0$. Then a linear combination of the rows of $\hat{U}^{(j)}_1$ is the candidate for matrix $G_R^{(j)}$ and we choose

$$G_R^{(j)} = \hat{U}^{(j)}_1 \hat{U}^{(j)^H} \in \mathbb{C}^{M_R \times M_R}.$$  

Due to the reciprocity of the channel, we have $G_T^{(j)} = G_R^{(j)^T}$. Moreover, we deploy the ANOMAX algorithm in [2] to design $G_S$ in this paper.

It is also clear that $M_R > 2(L-1)M_U$ has to be fulfilled so that the left null space of $\hat{H}^{(j)}$ cannot be empty. Since the ProBaSeMO algorithm is modularized it can be easily adapted to various system design criteria, e.g., sum rate maximization, relay power minimization, SINR balancing, etc.

B. Relay Power Minimization

In this part, we look for the optimal $g$ which minimizes the transmit power at the relay subject to an SINR constraint at each UT. The optimization problem is expressed as

$$\min \ g \quad g^H C g$$

s.t. $g^H D_k^{(j)} g + \sigma_k^2 \geq \gamma_k^{(j)}, \forall k, \ell.$

(16)

Problem (16) is mathematically similar to the beamforming problems in [3] and [4] which are in general non-convex. It can be further expanded as the following equivalent problem

$$\min \ g \quad g^H C g$$

s.t. $g^H B_k^{(j)} g \geq \gamma_k^{(j)} \sigma_k^{(j)^2}, \forall k, \ell.$

(17)

where $B_k^{(j)} = D_k^{(j)} - \gamma_k^{(j)} (E_k^{(j)} + F_k^{(j)})$. Each constraint in (17) is a superlevel set of a quadratic function [5]. Such a set is convex if and only if the quadratic function is concave, i.e., $B_k^{(j)}$ is negative semi-definite, $\forall k, \ell$. It is clear that in this case the feasible set is empty since $g^H B_k^{(j)} g \leq 0, \forall k, \ell$. Hence, problem (17) may not be solvable in polynomial time, but its approximate solution can be obtained by using either the semi-definite programming (SDP) approach [3] or the iterative second-order cone programming (SOCP) approach [4]. In the sequel we will discuss the two approaches.

In general, the SDP approach which uses semidefinite relaxation technique (SDR) works as follows [3]. We introduce a new variable $X = gg^H$ and rewrite problem (17) as

$$\min \ X \quad \text{Tr}\{CX\}$$

s.t. $\text{Tr}\{B_k^{(j)} X\} \geq \gamma_k^{(j)} \sigma_k^{(j)^2}, \forall k, \ell$  

$X \succeq 0, \text{rank}\{X\} = 1$  

(18)

where $\text{Tr}\{\cdot\}$, $\succeq$, and $\text{rank}\{\cdot\}$ denote the trace of a matrix, the positive semi-definiteness, and the rank of a matrix, respectively. Dropping the rank-one constraint, problem (18) can be approximated by the following convex SDP problem which can be solved efficiently by the interior-point method [5].

$$\min \ X \quad \text{Tr}\{CX\}$$

s.t. $\text{Tr}\{B_k^{(j)} X\} \geq \gamma_k^{(j)} \sigma_k^{(j)^2}, \forall k, \ell$  

$X \succeq 0$  

(19)

Obviously, problem (19) is a relaxed version of the original problem (16), i.e., the optimal value of (19) is a lower bound of problem (16). If the optimal solution $X_{\text{opt}}$ of (19) is rank-one, it is also optimal for the original problem and the optimal $g_{\text{opt}}$ is the principle component of $X_{\text{opt}}$. Due to the relaxation, $X_{\text{opt}}$ is generally not rank-one. Although a rank-one solution of (19) always exists if the number of constraints in (19) is less or equal to three [6], our problem has always more than three constraints, i.e., at least two operators and two UTs per operator. Thus, we apply the randomization method in [3] to extract the rank-one approximation from $X_{\text{opt}}$.

Since the SDP solution is in general not optimal for our problem, it is worth applying an alternative approach which is the iterative SOCP method [4]. In the traditional SOCP method, the rank-one property of the matrix $D_k^{(j)}$ is exploited and the constraints in (16) are rewritten as

$$\sqrt{g^H (H_k^{(j)^T} H_k^{(j)T}) g + \sigma_k^{(j)^2}} \geq \gamma_k^{(j)}, \forall k, \ell.$$

(20)
If we introduce
\[ \tilde{U}_k^{(ℓ)} = \begin{bmatrix} \sigma_k^{(ℓ)} & 0 \\ 0 & (E_k^{(ℓ)} + F_k^{(ℓ)}) \end{bmatrix}, \]
\[ \tilde{g} = [1, g^\top]^\top, \quad \tilde{h}_k^{(ℓ)} = [0, (h_k^{(ℓ)} \otimes h_k^{(ℓ)})^\top]^\top, \]
(21) can be rewritten as
\[ |\tilde{g}^\top \tilde{h}_k^{(ℓ)}| \geq \sqrt{\gamma_k/P_k} \|\tilde{U}_k^{(ℓ)} \tilde{g}\|, \forall k, ℓ. \] (22)
With the conservative approximation [4]
\[ |\tilde{g}^\top \tilde{h}_k^{(ℓ)}| \geq \text{Re} \left\{ \tilde{g}^\top \tilde{h}_k^{(ℓ)} \right\} \] (23)
where Re \{\cdot\} denotes the real part, the non-convex part of the constraint (22) can be strengthened as
\[ \text{Re} \left\{ \tilde{g}^\top \tilde{h}_k^{(ℓ)} \right\} \geq \sqrt{\gamma_k/P_k} \|\tilde{U}_k^{(ℓ)} \tilde{g}\|, \forall k, ℓ. \] (24)
Introducing the auxiliary variable \( t \) and the matrix
\[ \tilde{V} = \begin{bmatrix} 0 & 0^\top \\ 0 & C \end{bmatrix}, \] (25)
problem (16) can be approximated by the following convex SOCP problem
\[
\begin{align*}
& \min_{t, \tilde{g}} \quad t \\
& \text{s.t.} \quad \|\tilde{V}^\top \tilde{g}\| \leq t, \quad \tilde{g}_k = 1 \\
& \quad \quad \quad \text{Re} \left\{ \tilde{g}^\top \tilde{h}_k^{(ℓ)} \right\} \geq \sqrt{\gamma_k/P_k} \|\tilde{U}_k^{(ℓ)} \tilde{g}\|, \forall k, ℓ.
\end{align*}
\] (26)
Since replacing (22) by (24) yields a restricted convex feasible set which is a subset of the original feasible set of problem (16), it guarantees that the optimal solution of (26) is always feasible for (16). However, the drawback of this approach is that the solution of (16) may not be optimal for (26) and it may turn the original feasible problem into an infeasible one. Thus, the performance and feasibility strongly depend on how accurately the non-convex feasible set of problem (16) is approximated. To improve the convex approximation, we apply the iterative SOCP approach which is proposed in [4].

C. SINR Balancing

In this section, we study the SINR balancing problem subject to a relay power constraint. The optimization problem can be formulated as
\[
\begin{align*}
& \max \min_{g, \forall k, ℓ} \quad \eta_k^{(ℓ)} \\
& \quad \quad \text{s.t.} \quad g^\top C g \leq P_R
\end{align*}
\] (27)
or equivalently as
\[
\begin{align*}
& \max_{g, ℓ} \quad t \\
& \quad \quad \text{s.t.} \quad g^\top C g \leq P_R, \\
& \quad \quad \quad \frac{g^\top (E_k^{(ℓ)} + F_k^{(ℓ)}) g + \sigma_k^{(ℓ)}}{g^\top (E_k^{(ℓ)} + F_k^{(ℓ)}) g + \sigma_k^{(ℓ)}} \geq t, \forall k, ℓ
\end{align*}
\] (28)
where \( P_R \) is the maximum allowable relay transmit power. Problem (27) is non-convex. Following the idea of SDR in the previous section, we introduce \( X = g g^\top \) and drop the non-convex rank-one constraint. The problem is then reformulated into
\[
\begin{align*}
& \max_{X, t} \quad t \\
& \quad \quad \text{s.t.} \quad \text{Tr} \{C X\} \leq P_R, \quad X \succeq 0 \\
& \quad \quad \quad \text{Tr} \{(D_k^{(ℓ)} - t(E_k^{(ℓ)} + F_k^{(ℓ)}) X)\} \geq t\sigma_k^{(ℓ)}, \forall k, ℓ (29)
\end{align*}
\] Problem (29) is a quasi-convex problem similar as in [7]. Hence, it can be solved using the same procedure as in [7], i.e., using a simple bisection search algorithm in which a feasibility problem is solved at each step. Due to the relaxation, the solution \( X_{\text{opt}} \) might not be feasible for the original problem. The randomization techniques [3] can still be applied to obtain the final \( g \).

IV. SIMULATION RESULTS

In this section we present simulation results only for \( L = 2 \). The simulated flat fading channels are spatially uncorrelated Rayleigh fading channels. The noise variances at all nodes are the same, i.e., \( \sigma_k^{(ℓ)^2} = \sigma_R^2, \forall k, ℓ \). All the simulation results are obtained by averaging over 1000 Monte Carlo runs. “ZF” is a channel inversion technique in [8]. “ProBaSeMO(BA)” is the algorithm derived from the framework of ProBaSeMO. “SDP” is the convex approximation using SDP and randomization technique [3] while “lower bound” is obtained from (19). “iSOCP” is the iterative SOCP technique and “BiSDR” stands for SDP with rank-one extraction plus bisection search.

Fig. 2. Relay transmit power vs. SINR constraint, SNR = 15 dB

Fig. 2 shows the relay transmit power vs. a common SINR constraint with SNR = 15 dB, i.e., the transmit power of the UTs is 15 dB above the noise power level. It can be observed that the difference of the ProBaSeMO solution to the lower bound reduces for increasing \( M_R \). Moreover, the two convex approximation techniques SDP and iterative SOCP merge with the lower bound. This implies that both approximation techniques are accurate enough for our problem.
Fig. 3. SINR balancing, $P_R = 1$ W

Fig. 3 depicts the results corresponding to the SINR balancing approach where this time, the maximized minimum SINR vs. SNR is shown. The total relay power $P_R$ is fixed to unity and thus $SNR = 1/\sigma^2$. Again the method based on convex approximation yields the best results. However, the ProBaSeMO method, which yields competitive results, requires a significantly lower computational complexity.

V. Conclusion

In this paper, we have studied the beamforming design in a multi-operator two-way relaying network with a MIMO AF relay. Two system design criteria have been chosen. First, we have minimized the transmit power at the relay subject to an SINR constraint per user. Second, we have discussed the SINR balancing problem with a relay power constraint. Both problems are generally non-convex. Thus, to solve the optimization problems, we have applied convex approximation techniques. We have introduced a sub-optimal algorithm based on the ProBaSeMO framework which can be applied in both design criteria cases. Simulation results demonstrate that the ProBaSeMO algorithm yields competitive results compared to the convex approximation techniques especially when a large number of antennas are deployed at the relay. However, it requires much less computational complexity.

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