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Integrating Credit and Market Risk: A Factor Copula Based Method

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Abstract

This paper presents a factor copula model for the integration of Chinese commercial banks’ credit risk and market risk. By defining the dependence structure through a set of common factors reflecting the macro-economic situation, this model reveals the intrinsic correlation between credit risk and market risk. We derive the integration process with factor copula and generate common factors by performing a principal component analysis on 4 different macro-economic indicators that have impact on bank’s profit, namely the GDP growth, M2 growth, benchmark for loan rate, and the ratio of new loans to GDP. In the empirical study, 15 Chinese listed banks are chosen to construct the model. The results are compared with that of elliptical copulas and Archimedean copulas, we find that factor copula gives a more prudential result in risk integration.

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Keywords: factor copula; risk integration; common factor

1. Introduction

Sound risk management requires comprehensive assessments of the risks within financial firms, and adequate capital to cover the total risk. Currently, techniques for measuring specific risks are mature and precise, yet it is still unclear how to integrate different risk types to obtain the overall risk. A number of methods have been employed to integrate different types of risks [1]. The easiest approach is the simple summation with the assumption that the inter-correlation between different risks equal to one, thus it always overestimates the total risk. The most popular method in banking industry is the variance-covariance approach, it considers the diversification benefit between different risk types and gives a closed-form expression of the total risk, but the interactions between different risk types are assumed to be linear, which is incorrect in most cases [2].
Considering the shortcomings of the methods mentioned above, researchers are developing new methods to aggregate risks. Dimakos and Aas [3] suggest an approach to calculate the aggregate economic capital of a financial group by considering the pairwise interrisk correlations. Li et al. [4] propose an integrated risk measurement and optimization model based on Bayesian network. Schlottmann et al. [5] propose a completely different risk aggregation method based on multi-objective programming. Rosenberg and Li [6-7] use normal copula and t copula to integrate credit risk, market risk and operational risk. Kuritzkes et al. [8] propose a building block approach and conclude that diversification benefits are the greatest within a single risk factor, and decrease at the business line level. Grundke P. [9] assesses the accuracy of the total economic capital based on the top-down approach by means of a comprehensive simulation study where bottom-up approach serves as the data-generating process.

According to the definition of credit risk and market risk, they are supposed to be intrinsically related. In the field of credit and market risk integration, numerous meaningful results have been obtained. Jarrow and Turnbull [10] think that market and credit risk are related to each other and not separable, and they use interest rate and market index to depict the correlation between market and credit risk. Guo and Zhou [11] establish a loan pricing model by combining credit risk and market risk. Theodore et al. [12] estimate correlated market and credit risk in fixed income portfolios by simulating both the future financial environment and the credit rating of specific firms. Dimakos and Aas identified a set of risk factors that influence market and ownership risk. Alexander and Pezier [13] propose a risk factor model to aggregate market risk and credit risk. Jobst et al. [14] introduce a modeling paradigm to integrate credit risk and market risk in describing the random dynamical behavior of the underlying fixed income assets.

2. Factor copula model

In this paper, we describe the dependence structure of market and credit risk by using copula model conditional on the common factors constructed through principal component analysis. Instead of applying marginal distributions of the risk returns into our copula function directly, we can use their marginal distributions conditional on the common factors. The ways returns are interrelated with each other depend largely on the common factors.

2.1. Factor Structure

Define \( X_c, X_m \) as the credit and market risk return, respectively. In the factor copula model, we have

\[
\begin{align*}
X_c &= \rho_c Y + \sqrt{1-\rho_c^2} Z_c \\
X_m &= \rho_m Y + \sqrt{1-\rho_m^2} Z_m
\end{align*}
\]

(1)

In this equation, \( Y \) is either a single common factor or a vector of common factors. \( Y \) affects both credit and market risk returns, and can be modeled by macro-economic variables, \( \rho_c \) and \( \rho_m \) are constant parameters between -1 and +1. \( Z_c \) and \( Z_m \) are idiosyncratic risk factors that are independent from each other and also independent of \( Y \), they usually reflect the characteristics of a specific firm.

Suppose that \( Y \) and \( Z_i \ (i = c, m) \) are normally distributed, we get a Gaussian factor copula model with a closed-form joint distribution function [15]. However, in more general cases, \( Y \) and \( Z_i \) are not normally distributed, and we may be unable to derive a joint closed-form distribution function as the elliptical copulas and Archimedean copulas.
2.2. Construction of common factors

The common factors reflect external environmental effects on a commercial bank’s credit risk return and market risk return. According to previous research, macro-economic variables and market index can be used to monitor such external environmental effects. For example, the interest rate influences the interest income of a bank, as well as its credit loss due to default. On the other hand, interest rate has an impact on banks’ market return since banks are involved in the transactions of interest rate derivatives.

Based on an internal research conducted by China banking regulatory commission, a total of 12 indicators are chosen to establish the common factors, namely GDP growth, new loans/GDP, growth of investments in fixed assets, real estate price index, M2 growth, benchmark one-year lending rate, TED SPREAD, leverage ratio of 5000 industrial enterprises, trade surplus growth, government leverage ratio, fiscal deficit/GDP, exchange rate, and the CSI 300 index. Considering their relative importance and data availability, we choose GDP growth, M2 growth, benchmark one-year lending rate, new loans/GDP to construct common factors.

2.3. Basic Steps

Under the factor copula structure, credit risk return and market risk return are independent conditional on common factors. Denoting the probability distribution of \( Y \) and \( Z \) as \( F_Y \) and \( F_Z \), we derive the conditional probability of the integrated risk:

\[
P(X_x + X_m \leq r | Y = y) = P \left[ Z_x < \frac{r - (\rho_x + \rho_m) y \sqrt{1 - \rho_m^2} Z_m}{\sqrt{1 - \rho^2}} | Y = y \right]
\]

By integrating the conditional probability on common factor, we obtain unconditional probability, which presents as a multidimensional integral:

\[
P(X_x + X_m \leq r) = \int_{-\infty}^{\inf} F_y \, dY \int_{-\infty}^{\inf} F_{Zm} \, dZ_m \int_{-\infty}^{u} F_x \, dZ_c
\]

where, \( u = \frac{r - (\rho_x + \rho_m) y \sqrt{1 - \rho_m^2} Z_m}{\sqrt{1 - \rho^2}} \)

3. Empirical study

In this section, we apply the factor copula model to integrate credit and market risk of Chinese commercial banks. To construct the model, we collect quarterly data from financial statements of 15 Chinese systemically important listed banks, the data range is from 2006Q4 to 2012Q2 since new accounting standards are released at the beginning of 2007. We also have quarterly data of GDP growth, M2 growth, benchmark for loan rate, and the ratio of new loans to GDP from 2001Q1 to 2012Q3. Both the financial and economic data are from Wind database, the most comprehensive financial database in China. Below is a list of the 15 Chinese commercial banks.

Table 1. The 15 Chinese systemically important listed banks

<table>
<thead>
<tr>
<th>Names of the banks</th>
</tr>
</thead>
<tbody>
<tr>
<td>China Minsheng Bank</td>
</tr>
<tr>
<td>Bank of Beijing</td>
</tr>
</tbody>
</table>
In the factor copula framework, the correlation largely depends on common factors. To arrive at an aggregate risk distribution, we need to determine the distribution of common factors, as well as idiosyncratic factors, next we will describe our method in detail. As a benchmark, we consider the risk integration problem of a median bank established by averaging China merchant’s bank, China construction bank and bank of China.

3.1. Correspondence between risk earnings and items in income statements

The information in financial statements can be used to describe credit risk and market risk [16]. A typical income statement usually contains the following items: net interest income, fee and commission income, income on investment, profit and loss from fair value change, exchange gain or loss, asset devaluation, net non-operating income and expenditure. Credit risk and market risk can be mapped by the above items as:

\[ \text{Credit risk earnings} = \begin{cases} \text{Net interest income} \\ \text{Asset devaluation} \end{cases} \]

\[ \text{Market risk earnings} = \begin{cases} \text{Income on investment} \\ \text{Profit and loss from fair value change} \\ \text{Exchange gain or loss} \end{cases} \]

Fig. 1. The correspondence between risk earnings and income statement

3.2. Data pre-processing

Since banks in our sample differ in total assets, earnings should be converted into a return based measurement for direct comparison across banks. To solve this problem, we divide risk earnings by total assets. The resulting ratio of risk earnings to total assets, which we call return on assets (ROA), determines the risk level of a specific bank. Basing on this, we will take the next 2 steps to obtain the credit risk return and market risk return sample of the median bank.

- Step 1: Calculate the fluctuation of risk return. Let \( R_{i,j,t} \) be the earnings correspond to risk type \( j \) for the \( i^{th} \) bank, in period \( t \), let \( A_{i,t} \) be the total assets of the \( i^{th} \) bank in period \( t \), we define the risk return of risk type \( j \) for the \( i^{th} \) bank in period \( t \) as

\[ r_{i,j,t} = \frac{R_{i,j,t}}{A_{i,t}} \]  

(4)

Then we calculate the mean of risk return of type \( j \) for the \( i^{th} \) bank within period \( t \), denoted by \( r_{i,j} \), thus
the fluctuation $\Delta_{i, j, t}$ can be expressed as $\Delta_{i, j, t} = r_{i, j, t} - r_{j, t}$.

- Step 2: Calculate risk return for the median bank. The fluctuation variable $\Delta_{i, j, t}$ is time-dependent and reflects the temporary economic environment. For simplicity, we assume that $\Delta_{i, j, t} \sim iid$ for all banks. Thus the risk return sample of the median bank can be calculated as $r_{m, j, t} = r_{m, j} + \Delta_{i, j, t}$. Here, $r_{m, j}$ is the mean risk return of the median bank for risk type $j$. With the calculated risk return sample, we are able to construct the factor copula model for credit and market risk integration of the median bank.

3.3. Distribution fitting and parameter estimation

In order to realize the integration, we need to determine the distribution functions of common factors and idiosyncratic factors, as well as the value of $\rho_c$ and $\rho_m$, as illustrated in this section.

A principal analysis is performed on the chosen 4 indicators, and 2 common factors $Y_1$ and $Y_2$ are identified.

Table 2. The identified common factors

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP growth</td>
<td>-0.1003</td>
<td>0.8564</td>
</tr>
<tr>
<td>M2 growth</td>
<td>0.6334</td>
<td>0.2987</td>
</tr>
<tr>
<td>The benchmark one-year lending rate</td>
<td>-0.4443</td>
<td>0.4033</td>
</tr>
<tr>
<td>New loans/GDP</td>
<td>0.6256</td>
<td>0.1213</td>
</tr>
<tr>
<td>Explained(%)</td>
<td>50.05</td>
<td>28.41</td>
</tr>
</tbody>
</table>

For the first common factor, the latter three indicators clearly have a more significant contribution than GDP growth, while GDP growth dominates the second common factor.

As the common factors and idiosyncratic factors are independent from each other, $\rho_c$ can be calculated as $\rho_c = \text{cov}(X_1, Y_1)/\text{cov}(Y_1, Y_1)$. $\rho_{m1}, \rho_{m2}$ can be obtained similarly.

By subtracting common factors from risk return, we obtain the idiosyncratic series. For common factors and idiosyncratic factors, we examine their normality using Jarque-Bera test, Kolmogorov-Smirnov test, and Lilliefors test, and then determine the appropriate distribution functions to fit them. The results turned out that $Y_2$ and $Z_c$ are normally distributed at 5% level, while $Y_1$ and $Z_m$ are not.

Burtschell [17] proposed a mixed Gaussian distribution for the common factor, Xu [18] finds that a mixed Gaussian distribution gives more accurate results. Compared with Gaussian distribution, the mixed Gaussian distribution can better capture the fat-tail feature of financial series, and describe the dependence between risks under extreme cases more efficiently. As a result, we use mixed Gaussian distribution to fit the first common factor $Y_1$. For the market idiosyncratic factor $Z_m$, student t distribution is applied. Table 3 lists the distribution fitting results for the common factors and idiosyncratic factors.

Table 3. Distribution fitting results.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Mixed Gaussian ($Y_1$)</th>
<th>Gaussian ($Y_2$)</th>
<th>Gaussian ($Z_c$)</th>
<th>Student t ($Z_m$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>Pcomponents</td>
<td>mu</td>
<td>mu</td>
<td>mu</td>
</tr>
<tr>
<td></td>
<td>0.8046</td>
<td>-0.0367</td>
<td>5.5648</td>
<td>0.1496</td>
</tr>
<tr>
<td></td>
<td>mu</td>
<td>0.1954</td>
<td>mu</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.5286</td>
<td>0.9851</td>
<td>1.0848</td>
<td>1.7578</td>
</tr>
<tr>
<td></td>
<td>sigma</td>
<td>sigma</td>
<td>sigma</td>
<td>sigma</td>
</tr>
<tr>
<td></td>
<td>1.9851</td>
<td>1.0848</td>
<td>1.7578</td>
<td>0.1444</td>
</tr>
</tbody>
</table>
3.4. Integrate credit and market risk returns

In this section, we will integrate the credit and market risk by solving the multi-dimensional integral in equation 3. In section 2, we derive the integral expression of unconditional cumulative distribution probability for the integrated risk return, in which the correlation parameters and the distributions for the factors need to be determined. Based on the results in 3.3, we are able to solve the multi-dimensional integration numerically. Table 4 lists the results of integrated risk.

Table 4. Quantiles of integrated risk

<table>
<thead>
<tr>
<th>Confidence level (%)</th>
<th>0.10%</th>
<th>1%</th>
<th>2%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integrated risk return</td>
<td>-0.11</td>
<td>0.08</td>
<td>0.13</td>
<td>0.19</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 4 displays the quantiles of integrated risk at different confidence levels. Different confidence levels are associated with different credit ratings, for example, a AA rated bank is supposed to be able to absorb the loss at 0.03% confidence level. According to the New Basel Capital Accord, we determine the economic capital by 0.1% VaR, as shown in the first column of table 4. By multiplying the total assets of the median bank, we conclude that in the multi-dimensional integral method, capital allocated for credit risk and market risk is 64.754 billion.

3.5. Comparative analysis

Elliptical copulas and Archimedean copulas are widely used models in describing dependence of different risk types. In this section, we make a comparison between our results and those obtained through elliptical copulas and Archimedean copulas. Table 5 lists the integration results of different copulas.

Table 5. Results of different copulas

<table>
<thead>
<tr>
<th>Confidence level (%)</th>
<th>0.10%</th>
<th>1%</th>
<th>2%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor copula</td>
<td>-0.11</td>
<td>0.08</td>
<td>0.13</td>
<td>0.19</td>
<td>0.25</td>
</tr>
<tr>
<td>T-copula</td>
<td>-0.10</td>
<td>0.24</td>
<td>0.33</td>
<td>0.40</td>
<td>0.44</td>
</tr>
<tr>
<td>Normal copula</td>
<td>0.07</td>
<td>0.22</td>
<td>0.31</td>
<td>0.40</td>
<td>0.44</td>
</tr>
<tr>
<td>Frank copula</td>
<td>0.05</td>
<td>0.19</td>
<td>0.29</td>
<td>0.39</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Obviously, the factor copula gives the largest capital requirement while the normal copula gives the smallest. Additionally, the factor copula results exhibit fat-tailed features more obviously than other copulas, that is to say, our model captures the tail-dependence more efficiently. As for normal copula, it only considers the linear correlation between credit risk and market risk, which makes it incapable of modeling tail-dependence accurately. As for Frank copula and t copula, their results are between normal copula and factor copula. Since tail-dependence is of crucial importance in risk integration, the comparison indicates that factor copula is a competitive model in risk integration.
4. Conclusions

In this paper, we introduce a factor copula structure in risk integration. This model breaks each risk type into a set of common risk driver factors and idiosyncratic factors. Usually, the common factors are macro-economic variables, compared with elliptical and Archimede copulas, this model describes how the economic situation affects each risk type and how different risk types are correlated with each other through these common factors. In other words, the factor copula presents us more economic implications and the dependence structure is clearer.

Then we study the risk integration problem for a median bank obtained by averaging China Merchants Bank, China Construction Bank and Bank of China. The integration results of the factor copula model is compared with other two major copula families, the elliptical copulas and Archimede copulas, it turns out that the factor copula model gives the most prudential results when it comes to capital requirement. Further analysis indicates that factor copula excels other copulas in capturing tail-dependence, which makes it an outstanding model for risk integration.

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