Generalized Gibbs priors based positron emission tomography reconstruction

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ABSTRACT

Bayesian methods have been widely applied to the ill-posed problem of image reconstruction. Typically the prior information of the objective image is needed to produce reasonable reconstructions. In this paper, we study a novel generalized Gibbs prior (GG-Prior), which exploits the basic affinity structure information in an image. The motivation for using the GG-Prior is that it has been shown to be effective in noise suppression, while also maintaining sharp edges without oscillations. This feature makes it particularly attractive for the reconstruction of positron emission tomography (PET) where the aim is to identify the shape of objects from the background by sharp edges. We show that the standard paraboloidal surrogate coordinate ascent (PSCA) algorithm can be modified to incorporate the GG-Prior using a local linearized scheme in each iteration process. The proposed GG-Prior MAP reconstruction algorithm based on PSCA has been tested on simulated, real phantom data. Comparison studies with conventional filtered backprojection (FBP) method and Huber prior clearly demonstrate that the proposed GG-Prior performs better in lowering the noise, preserving the image edge and in higher signal to noise ratio (SNR) condition.

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1. Introduction

Due to the low counting rates and the limited acquisition time, clinical positron emission tomography (PET) data are usually significantly affected by Poisson noise. Reconstructing the PET images is essentially an ill-posed problem [1]. Up to now, many reconstruction strategies have been proposed to address this problem. Common strategies such as the maximum-likelihood expectation maximization (ML-EM) algorithms [2,3], which can be transformed into a well-posed one by controlling solutions that are most favorable, model the measured data by a better expressed system model that includes the physical characteristics of the system and the statistical characteristics of the measurement data and then seek an optimal solution for the estimation of an image. An alternative strategy is traditional analytic methods such as the filtered back projection (FBP) method. However, the ML-EM approach is still notorious for its slow convergence, and may result in “checkerboard effect” in the iteration process [4]. FBP based methods use simple processing such as subtraction and multiplication to correct for these factors which might result in increased error and noise in the measured data. A more powerful approach is to replace the ML-EM criterion with a Bayesian estimation criterion.

Bayesian estimation, or maximum a posteriori (MAP) estimation as a statistical approach for incorporating prior information through the choice of a prior distribution for a random field, has already been proved to be an effective solution to the ill-posed PET reconstruction problem [4]. Based on Bayesian theory, a generic contextual constraint can be transformed into some kinds of prior information to regularize the solution of the original ill-posed reconstruction problem. Therefore, the regularization by such prior information can be imposed on image reconstruction to suppress noise much more effectively. A common Bayesian prior is the Gibbs distribution of the form:

$$P(\lambda) = \frac{1}{Z} \exp(-\beta U(\lambda)) = \frac{1}{Z} \exp\left(-\beta \sum_j U(\lambda_j)\right),$$  

(1)

where $Z$ is a normalizing constant, $U(\lambda_j)$, as a regularization term, is prior energy function, $U(\lambda_j)$ is any function of a local group of pixel point $j$. $\beta$ is a constant that specifies the relative strength of the prior. The specific choice of prior distribution for $\lambda$ is, of course, a critical component in Bayesian reconstruction.

Usually, $U(\lambda_j)$ is chosen as a shift-invariant function that penalizes the differences between local neighboring pixels. However, since the prior is shift-invariant while the likelihood is not, the MAP image, which represents the position dependent
bias and resolution, cannot usually produce satisfactory results. In addition, images of real world may not be globally smooth and noisy edges intensities may often vary abruptly. For example, the simple quadratic membrane prior (QM-Prior) which smoothes both noise and edge details equally tends to produce an unfavorable over-smoothing effect [5]. Moreover, many edge-preserving Bayesian methods, including line-site model and discontinuity adaptively Markov random field model, can produce sharper edges by using a nonquadratic prior (NQ-Prior) [5,6]. Such edge-preserving Bayesian methods with NQ-Prior might work well when the noise level in the measured data is low and the image to be reconstructed is composed of homogeneous regions with sharp boundaries. However, in the case where the noise level is relatively significant and there is often not a clear separation of homogeneous, these methods tend to produce blocky piecewise smooth regions or so-called staircase effect. In clinical situations, such staircase effect may cause artifacts that might be misinterpreted as real objects when actually they are not. In addition, such edge-preserving Bayesian methods are often computationally costly due to the often needed annealing procedures and estimations of some built-in parameters of priors [7].

The QM-Prior can smooth the reconstructed image through an operation, similar to averaging, on pixel densities within a local neighborhood. The edge-preserving NQ-Priors rely on pixel density operation, similar to averaging, on pixel densities within a local neighborhood. The edge-preserving NQ-Priors can exploit the nonlocal information available in the objective image, not only density difference information between pixels in the local neighborhood, but also other factors such as gray level values, edge indicator, dominant direction, dominant frequency, etc.

Both the over-smoothing effect of QM-Priors and the staircase effect of NQ-Priors are the outcomes of their limited power to distinguish edge information from the noise information. And such limited power can be ascribed to the fact that the simple weighting strategy of the pixel intensity differences within a small fixed local neighborhood provides a very limited prior information. As the above traditional priors depend on the simple weighting strategy within a local neighborhood, we refer them as local Gibbs priors (LG-Priors) in this paper.

Motivated by some recent studies on the image denoising and segmentation by nonlocal averages [9–11], we propose a family of generalized Gibbs prior (GG-Prior) for PET reconstruction. The GG-Prior can exploit the nonlocal information available in the objective image, not only density difference information between individual pixels but also nonlocal connectivity and continuity information in the objective image. It has been shown that the new prior can significantly improve the reconstructed image quality of PET.

The rest of the paper is organized as follows. In Section 2, the linear Poisson statistical model is presented. Both the traditional LG-Prior model and the proposed GG-Prior model are discussed in Section 3. A binary optimal reconstruction algorithm based on the GG-Prior model for PET reconstruction are explained in details in Section 4. And in Section 5, the performance of the proposed method are demonstrated on simulated and phantom PET data. Relevant qualitative and quantitative comparisons are also illustrated. Finally, Section 6 summarizes our conclusions and draws discussions.

2. PET statistical model

We focus on the linear Poisson statistical model that has been used extensively for emission computed tomography, including PET and SPECT. Assuming usual Poisson distribution, the measurement model for emission scans is as follows [3]:

\[ y_i \sim \text{Poisson} \left( c_i \sum_{j=1}^{M} a_{ij} \lambda_j + r_i \right), \quad i = 1, 2, \ldots, N, \]  

where \( y_i \) is the number of photons counted in the \( i \)th bin, \( N \) is the number of detector pairs, \( \lambda_j \) is the activity at the \( j \)th pixel, \( M \) is the number of unknown image pixels, \( r_i \) represents the total detected random and scattered event counts for detector pair \( i \) in emission scan, and \( A = \{a_{ij}\} \) is the system matrix, \( a_{ij} \) is the geometric probability that an emission photon from pixel \( j \) is detected by the detector pair \( i \) in ideal conditions. \( c_i \) represents the incorporate calibration factors of scan time, detector efficiencies, attenuation factors and possibly dead time correction factors for the detector pair \( i \). The goal is to estimate the unknown activity distribution image \( \lambda = [\lambda_1, \lambda_2, \ldots, \lambda_M] \) based on the measurements \( y = [y_1, y_2, \ldots, y_N] ^T \) with \( A \) and \( r = [r_1, r_2, \ldots, r_N] \) being known where \(^T\) denotes the matrix transpose.

Then, the corresponding log-likelihood function can be written, ignoring constants independent of the \( \lambda \), as follows:

\[ L(y, \lambda) = \sum_{i=1}^{N} \ln (\text{det}(A\lambda_i)), \]  

where \( h_i = y_i \ln [(c_i + r_i)/c_i] - (c_i + r_i) \) and \( l_i(\lambda) = |A\lambda_i| = \sum_{j=1}^{M} a_{ij} \lambda_j \). Therefore the mathematical formulation of PET reconstruction is the following maximization problem:

\[ \max_{\lambda} L(y, \lambda). \]  

However, this problem is ill-posed and solving this alone will produce oscillatory solutions [2]. This is why prior information is needed.

3. Prior models

3.1. Traditional local Gibbs prior model

Traditionally, the value of \( U(\lambda_j) \) is commonly computed through a simply weighted sum of potential functions \( v(\cdot) \) of the differences between pixels in the local neighborhood \( \mathcal{N}_j \):

\[ U(\lambda_j) = \sum_{k \in \mathcal{N}_j} w_{kj} v(\lambda_k - \lambda_j). \]  

Generally, different choices of the potential function \( v(\cdot) \) lead to different priors. When the potential function has the form \( v(\cdot) = t^2/2 \), the prior is the conventional space-invariant QM-Prior. We can also choose the edge-preserving NQ-Prior by adopting a nonquadratic potential function \( v(\cdot) \), such as the Huber potential function:

\[ v(\cdot) = \begin{cases} t^2/2, & |t| \leq \delta \\ \delta |t| - \delta^2/2, & |t| > \delta \end{cases} \]

where \( \delta \) is the threshold parameter [48]. The weight \( w_{kj} \) in (5) takes a positive value that denotes the degree of interaction between pixels \( k \) and \( j \), and it is usually defined as the reciprocal of the distance between the pixels \( k \) and \( j \). So in a square lattice of image \( \lambda \), the two dimensional positions of the pixels \( k \) and \( j \) (where \( k \neq j \)) are denoted by \((k_x, k_y)\) and \((j_x, j_y)\), respectively. \( w_{kj} \) is defined as the reciprocal of the 2D Euclidean distance \( D_{kj} = (|k_x - j_x|^2 + |k_y - j_y|^2)^{1/2} \) between the two pixels, i.e., \( w_{kj} = 1/D_{kj} \). Such kinds of simple weight-determining methods have been widely used in Bayesian image reconstructions and restorations [7].

From above we can note that the local prior can only provide fixed LG-Prior information for image reconstruction. And, through
an averaging alike effect, the local QM-Prior is prone to producing over-smoothing effect which smooths out both noise and the important details [5]. The local edge-preserving NQ-Priors, which can be able to preserve some edge information by adjusting a nonquadratic potential function that increases less rapidly when the differences between adjacent pixels are relatively bigger, still tend to produce some unfavorable artifacts [8].

3.2. Generalized Gibbs prior model

Motivated by the nonlocal averages in the image denoising and segmentation by nonlocal averages [9,10], we propose a family of generalized Gibbs priors for Bayesian image reconstruction.

The construction of the proposed GG-Priors is as follows:

$$U(\lambda, j) = \sum_{k \in N_j} w_{kj} \phi_s(\lambda_k - \lambda_j),$$

$$w_{kj} = \exp\left(\frac{-|k-j|}{2\sigma^2}\right) \exp\left(\frac{-D(k,j)}{h^2}\right),$$

$$D(k,j) = ||\lambda(\mathcal{V}_k) - \lambda(\mathcal{V}_j)||_2^2,$$

$$\lambda(\mathcal{V}_k) = \{\lambda(l) : l \in \mathcal{V}_k\},$$

$$\lambda(\mathcal{V}_j) = \{\lambda(l) : l \in \mathcal{V}_j\},$$

where $N_j$ is a large search neighborhood set to incorporate geometrical configuration information in the image, the weight $w_{kj}$, which reflects the degree of connectivity affinity between pixels $k$ and $j$, is defined as a decreasing function of the similarity of the two neighborhoods $\mathcal{V}_k$ and $\mathcal{V}_j$ (named similarity neighborhoods) centered on pixels $k$ and $j$, respectively. The parameter $\sigma$ controls how much one wishes to penalize distance of two grid points in the weight, while $h$ controls how much one wishes to penalize similarity of the two patches. Larger $\sigma$ allows one make use of more remote information, while larger $h$ gives results with sharper features.

Weight $w_{kj}$ and similarity function $D(k,j)$ in (8) can be made as done in many styles. In our experiments, the $L_2$ distance of two cubical patches is measured. And the direct $L_2$ distance gives a good measurement of similarity. The function $\phi_s$ should satisfy (i) $\phi_s \in C^1$, and (ii) $\phi_s$ is strongly convex on any bounded interval. Examples of edge-preserving functions $\phi_s$ that satisfy the two requirements are

$$\phi_s = \frac{1}{2} |t|^2, \quad 1 < \alpha < 2,$$

$$\phi_s = \frac{1}{2} |t|^2 - \log \left(1 + \frac{|t|^2}{\alpha}ight), \quad \alpha > 0,$$

$$\phi_s = \log \left(\cosh \left(\frac{|t|}{\alpha}\right)\right), \quad \alpha > 0,$$

$$\phi_s = \sqrt{\alpha + t^2}, \quad \alpha > 0,$$

see [12]. In our experiments we only focused on PET reconstruction based on (12) with $\phi_s(t) = |t|$ and $t^2$. If we set $\phi_s(t) = t^2$, this GG-prior may be similar to the nonlocal Gibbs Prior [13].

To convey the meaning of the weight $w_{kj}$ more lucid and convenient, illustrations of the weight $w_{kj}$ computation and distributions in the proposed GG-Prior model are shown in Fig. 1. The sizes of neighborhoods $\mathcal{N}$ and $\mathcal{V}$ are fixed $31 \times 31$ and $7 \times 7$, respectively. In Fig. 1(a), the weight $w_{kj}$ of pixel $j$ as indicated by the arrow is computed through the formula of (8) with $L_2$ distance of two cubical patches $\mathcal{V}_k$ and $\mathcal{V}_j$. In Fig. 1(b), the weight distributions are illustrated for the four centered pixels as indicated by the arrows consisted of the two edge regions (as shown in the upper left and bottom left, respectively), one homogeneous objective region (as shown in the bottom right) and one straight edge line region (as shown in the upper right). Obviously, the weight $w_{kj}$ are distributed in the direction of the straight edge line when the centered pixel $j$ is in a straight alike configuration, and they are distributed in the direction of the edge line when the centered pixel $j$ is in a oblique edge. We can also notice that the prior weight for the pixel in homogeneous region are distributed in the corresponding homogeneous region.

4. Generalized Gibbs priors based binary optimal reconstruction algorithm

With the analysis of Section 3, choosing the GG-Prior, the PET image $\lambda$ based on the MAP estimation can be obtained through an iterative maximization of the cost function $\Phi(\lambda)$

$$\hat{\lambda} = \arg \max_{\lambda \in \text{supp}(\lambda)} \Phi(\lambda),$$

$$\Phi(\lambda) = U(\lambda, \lambda) - \beta U(\lambda).$$

But, (8)-(11) show that the weight $w_{kj}$ is the function of objective image $\lambda$, which makes $U(\lambda)$ neither quadratic nor convex. Thus, we propose the following binary optimal algorithm to solve the optimization.

After setting an initial estimate $\hat{\lambda}$ for the first iteration, we update weights $w_{kj}$ and the image $\lambda$ alternatively by the

*Fig. 1. Illustrations of the weight $w_{kj}$ computation and distributions in the proposed GG-Prior model. The sizes of neighborhoods $\mathcal{N}$ and $\mathcal{V}$ are fixed $31 \times 31$ and $7 \times 7$, respectively: (a) The weight $w_{kj}$ of pixel $j$ as indicated by the arrow is computed through the formula of (8) with $L_2$ distance of two cubical patches $\mathcal{V}_k$ and $\mathcal{V}_j$. (b) The weight distributions for the four centered pixels as indicated by the arrows consisted of two edge regions (as shown in the upper left and bottom left, respectively), one homogeneous objective region (as shown in the bottom right) and one straight edge line region (as shown in the upper right).*
following binary optimal strategy in each iteration phase until convergence.

- **Weight update.** Let \( \hat{\lambda} \) fixed, compute \( w_{\hat{\lambda}} \) using (8)–(11).
- **Image update.** For the second stage of the maximization, we hold \( w_{\hat{\lambda}} \) fixed at its previous estimate and maximize \( \Phi(\hat{\lambda}) \) with regard to \( \lambda \).

Because the Hessian Matrix of \( L(y, \hat{\lambda}) \) is strictly negative definite [3] and according to (8)–(11), we get

\[
0 < w_{\hat{\lambda}}(\lambda) = \exp \left( \frac{\|k-j\|^2 - 2\sigma^2}{2\sigma^2} \right) \exp \left( \frac{D(k,j)}{\lambda^2} \right) \leq 1.
\]

(18)

Considering the strictly convexity for \( \phi_a \) in (7) and the positivity for penalty parameter \( \beta \), we can reach a conclusion that, given the computed value for \( w_{\hat{\lambda}}(\hat{\lambda}) \), the second derivatives of \( \Phi(\hat{\lambda}) \) in (17) is definitely concave in each iteration. Thus based on the theory of local linearization [14], we can monotonically maximize the posterior energy function \( \Phi(\hat{\lambda}) \) using iterative algorithms such as the paraboloidal surrogate coordinate ascent (PSCA) [15]. In our study, the standard PSCA algorithm was modified to incorporate the GG-Prior using a local linearized scheme in each iteration process [14].

Implementation of the PSCA+GG-Prior algorithm with the binary optimal strategy is similar to that in [15]. Thus our PET image reconstruction can be summarized as presented in Table 1.

### 5. Experimentation

In this section, we present some experiments for PET reconstruction with simulated and real phantom data.

#### 5.1. Emission reconstruction of simulated data

In this experiment, two synthetic simulated phantom data with 128 \( \times \) 128 square pixels, simulated by Dr. Jeff Fessler’s image reconstruction toolbox [16], were used for emission reconstruction. Fig. 2 shows the two synthetic simulated phantoms. Fig. 2(a) is a Zubal sectional phantom with pixel values from 0 to 8, and the total counts amount to \( 4 \times 10^5 \). Fig. 2(b) is a Shepp-Logan head phantom with pixel values from 0 to 8, and the total counts amount to \( 3 \times 10^3 \).

The simulated sinogram data for above two phantom data were all Poisson distributed and the percentages of simulated delayed coincidences \( r_i \) factors (scatter effects are ignored) in all counts were all set to be 10%. All the \( c_i \) factors were generated using pseudo-random log-normal variates with a standard deviation of 0.3 to account for detector efficiency variations.

Generated by the ASPIRE software system [17], the two transition probability matrixes used in the reconstructions of above phantoms all corresponded to parallel strip-integral geometry with 128 radial samples and 128 angular samples distributed uniformly over 180°. The algorithms described in this paper were all implemented in Matlab 7.0 (The MathWorks, Inc.) programing environment. The code was run on a PC with Intel(R) Pentium(R) 4, 3.16 GHz, 3.00 GHz processor and 512 MB of memory.

Comparison studies with Huber prior, GG-Prior and FBP were performed. For FBP algorithm, we chose the ramp filter with cutoff frequency equal to the Nyquist frequency. Without considering how to choose the penalty parameter \( \beta \), decay parameter \( h \), \( \sigma \) and threshold parameter \( \delta \) in either algorithm, we rather studied the reconstruction results obtained by a broad range of parameter values by hand in terms of maximum of signal-to-noise ratio (SNR), which was defined by

\[
\text{SNR} = 10 \log_{10} \left( \frac{\sum_j |i(j) - \bar{X}|^2}{\sum_j |i(j) - \bar{X}_{\text{phantom}}(j)|^2} \right),
\]

(19)

where \( i(j) \), \( \bar{X} \) and \( \bar{X}_{\text{phantom}}(j) \) denote the reconstructed image, the mean of the reconstructed image \( \hat{\lambda} \) and the original true phantom image, respectively.

In order to show the performance of the choice of the neighborhood systems \( \mathcal{N} \) for the Huber-Prior and GG-Prior (set \( \psi = 7 \) as a constant) based reconstructions, we did further experiments for the Zubal phantom data using the following varying search neighborhood sizes \( \mathcal{N} : 7, 9, 11, 13, 15, 21, 31 \) for the GG-Prior, 8, 16, 32 for the Huber-Prior. In all the cases, the parameter \( \beta \) was set to be 1.2, and the values of \( h, \sigma \) for the GG-Prior and \( \delta \) for the Huber-Prior were fixed to 0.8, 1.2 and 0.2, respectively. The reconstructed FBP image was used as the initial images in all the iterations. The 150th iterated images in all the reconstructions were chosen as the results, which kept a convergent SNR value and did not change visually compared to further iterations.

In Fig. 3, the measurement SNRs are shown as a function of neighborhood size for the Zubal phantom reconstructions. It depicts an interesting phenomenon that the optimal search neighborhood size should be medium size (around 21 \( \times \) 21) for the GG-Prior optimal reconstruction. The SNRs of the Huber-Prior reconstructions with the larger neighborhoods made only a small
improvement. But the computation time increased very much with the larger neighborhoods for all the cases. Between the balance of the SNR and computation cost for each reconstruction, we should adopt the aptitude neighborhood systems for the different cases.

Fig. 4 shows the FBP reconstructions and Huber-Prior and GG-Priors based Bayesian reconstructions using the Zubal and Shepp-Logan phantoms data. For the GG-Priors based PSCA algorithm the search neighborhood \( N_j \) in (7) was set to be \( 21 \times 21 \) neighborhood and the two similarity neighborhoods \( V_k \) and \( V_j \) in (9) were both set to \( 7 \times 7 \). For the reconstruction using the Huber-Prior, the \( N_j \) was also set to be \( 8 \times 8 \). The values of h and \( \sigma \) for the GG-Prior and \( \delta \) for the Huber-Prior were fixed to 0.8, 1.2 and 0.2, respectively. The hyper-parameter \( \beta \) was set to be 1.4 in all the reconstructions. Fig. 4(a) and (e), (b) and (f), (c) and (g), (d) and (h) are the reconstructed images using the FBP with the ramp filter, the Huber-Prior, the GG-Prior with \( (\phi_a = t^2) \), and the GG-Prior with \( (\phi_a = |t|) \), respectively.

5.2. Qualitative comparisons

From Fig. 4, it is seen that FBP reconstructions suffer severely from noise effect and streak artifacts, and the reconstructions using the proposed GG-Priors exhibit excellent performance in both suppressing noise effect and preserving edges, and are almost without the unfavorable oversmoothing effect which is usually seen with the staircase effect for the Huber-Prior.

Fig. 5 displays the horizontal profiles of the Huber-prior and GG-prior \( (\phi_a = t^2) \) based two phantoms reconstructions in Fig. 4. It can be observed that the resulting images using the GG-Priors have the profiles that agree much closer to the profiles of the phantoms than others.

5.3. Quantitative comparisons

Table 2 displays SNR comparisons with regards to the reconstructed images of Zubal and Shepp-Logan phantoms in Fig. 4. It indicates that the images from the reconstructions using the proposed GG-Prior had significantly higher SNRs than others.

5.4. Computational complexity analysis

For a 2D square image of \( N \times N \) pixels, the computational complexity of the weights computation using the proposed GG-Prior is \( N^2 \times S^2 \times 7^2 \), where \( S \) denotes the size of the search neighborhood. As an example, for \( N=128 \) and \( S=21 \), we need \( 128^2 \times 21^2 \times 7^2 \approx 3.55 \times 10^9 \) operations in each iteration. Most of the computation time is in this part comparing with traditional prior models. Considering the same sizes of phantom data and system matrixes, same computational cost is needed for the reconstructions of different phantom data when same type of prior is used. So we only give the computation time costs for the Zubal phantom data using the different reconstruction method. The corresponding computation time costs for all reconstructions are presented in Table 3. It can be seen that the computation costs for the proposed GG-Priors based reconstructions are comparably very higher than other methods.

5.5. Emission reconstruction of real phantom data

In this study, a real hot-lesion phantom emission scan data was used with 192 radial samples and 192 angular samples over 180° [18]. The sinogram data had been precorrected for scatter, attenuation and geometric arc correction etc. The fifth 2D slice data of the total nine slices was used. The objective image consisted of \( 128 \times 128 \) pixels with 0.356 cm pixel resolution. The total counts amounted to \( 4.4 \times 10^6 \). All ci factors and ri factors were assumed to be ones and zeros, respectively.

![Fig. 3. SNRs as a function of the window size for Zubal phantom reconstruction based on the Huber-Prior and GG-Prior, respectively.](image)

![Fig. 4. FBP reconstructions and Huber-Prior and GG-Priors based Bayesian reconstructions in the Zubal, Shepp-Logan phantoms study. (a) and (e) are the FBP reconstructions, (b) and (f) are the Huber-Prior based reconstructions, (c) and (g) are the GG-Prior based reconstructions with \( (\phi_a = t^2) \), and (d) and (h) are the GG-Prior based reconstructions with \( (\phi_a = |t|) \).](image)
The reconstructed image from the FBP algorithm was used as the initial image in the iterations. In order to test the proposed GG-priors, we compared it with the Huber-Prior. Iteration numbers for all the priors based reconstructions were set to be 50. The proposed GG-Prior based PSCA algorithm was also used here.

Fig. 6 shows the reconstructed results from different methods with different parameter settings. For all of the priors based reconstructions, the parameters $\beta$, $\delta$, $h$ and $\sigma$ were selected in a process of trial and error by covering a range of parameter settings. Obviously, when suitable parameter values are set, the reconstructions using the proposed GG-Priors outperformed the FBP reconstruction and the Huber-Prior based reconstruction in terms of noise suppression and edge preserving.

The corresponding computation time for the different reconstructions are presented in Table 4. It can also be seen that the computation cost for the proposed GG-prior based reconstructions are comparably higher than others.

6. Conclusions and discussions

In this paper, we have proposed a novel edge-preserving generalized Gibbs prior combined with the Bayesian theory for ill-posed PET reconstruction. The goal of the proposed GG-prior is to identify the shape of objects (e.g. tumors) that are distinguished from the background by sharp edges in the reconstructed PET images. In this way, the proposed prior was built by exploiting more nonlocal similarity information in image field via a new weights computation strategy (See Section 3.2).

From Table 2, Figs. 4 and 6, we can notice that the proposed GG-prior based PSCA algorithm performs competitively compared to the conventional FBP and Bayesian reconstruction with Huber prior, and remains more efficient for low SNR and noise-corrupted images. Noise and artifacts in limited counts PET reconstruction are greatly suppressed using the presented GG-prior based PSCA reconstructions algorithm. Comparison studies with reconstruction based on the Huber prior have clearly shown the benefit of the proposed approach.

As for the proposed GG-prior and its corresponding PSCA algorithm, we would like to make the following discussions. First,
we note that, as a version of the widely-used OSL (one-step-late) iteration algorithm [4,19,20], the proposed GG-prior based PSCA algorithm can be feasibly and effectively implemented using the proposed binary optimal reconstruction strategy [14]. However, due to the fact that the weights $w_{k}$ depend on the unknown image intensity, the proposed GG-prior in (8) is nonconvex for a global optimization. As known, like many existing OSL algorithms that global convergence is an open issue, the proposed GG-prior based PSCA algorithm also suffered from the lack of strict global convergence proof. Although no guarantee of global convergence, the PSCA+GG-Prior algorithm is still effective in practice and able to converge to at least a local maximum as the joint algorithm proposed in [8]. The effect of the PSCA+GG-Prior algorithm is also testified by our practical experiments.

We can also note that another disadvantage of PSCA+GG-Prior algorithm is how to choose the values of the parameter $\mathcal{N}, \mathcal{V}, h$, and $\beta$. For the choice of the parameter $\mathcal{N}$, we have conducted in-depth study in terms of maximum of SNR (See, Section 5.1). Fig. 3 shows that the optimal $\mathcal{N}$ should be medium size (around $21 \times 21$) for the proposed GG-prior based optimal PET reconstruction at the fixed value of $\mathcal{V}$ ($7 \times 7$). In the same way, we can also further discuss the selection of the value $\mathcal{V}$ and other parameters in a process of trial and error. From the test we find the tradeoff between resolution and noise can be controlled by certain parameters (i.e., $\beta$ and $h$) and they can either choose a fixed value, or make the relaxation dependent on a number of other variables, like iteration number and current image estimate. In this paper, we briefly fixed them and did not consider methods for choosing them, but rather studied the results obtained by a broad range of parameter values by hand in terms of maximum of SNR. This scheme also seems to be a process of trial and error. Therefore, more theoretical insight in choosing the parameters is necessary, but is outside the scope of the present paper.

Next, as presented in Tables 3 and 4, the computation costs of the proposed GG-Prior based PSCA algorithm is very higher than other methods. The reason has been analyzed in Section 5.4. A practical issue to be considered in any future step, would be to develop effective methods to reduce the computational complexity of the weights computation in (8).

Finally, the proposed GG-prior can be efficiently applied in order to regularize many types of inverse problems in image processing such as single photon emission computed tomography (SPECT) [21,22], magnetic resonance imaging (MRI) [23], and low dose computed tomography (CT) [24]. In any further work, it would be interesting to further explore different applications for the proposed GG-prior in this paper.

Conflict of interest statement

None declared.

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