1. Introduction

An EEG signal is the recording of neural electrical activities caused by nerve firings. Typically, EEG signals, carrying information about rhythmic activities at different frequency bandwidths of δ—delta (0.5–4 Hz), θ—theta (4–8 Hz), α—alpha (8–13 Hz), β—beta (13–30 Hz) and γ—gamma (30–50 Hz) [1–3], are recorded using electrodes placed across the scalp. EEG waveforms are characterized by three components, including shape, frequency, and amplitude. Based on those components, useful signatures/features in brain signals can be extracted by various techniques. However, EEG recordings are usually contaminated by physiological artifacts from various sources, such as eye blinking/movement, heart beating and movement of other muscle groups [4]. Such types of artifacts are mixed together with brain signals, making interpretation of EEG signals difficult [5].

Eye movement or blinks usually produce large electrical potentials, generating significant electrooculographic (EOG) artifacts in recorded EEG signals. Removal of EOG artifacts is nontrivial because those artifacts overlap in frequency and time domains with EEG signals. Fortunately, the effect of EOG artifacts on EEG signals is found most significantly in low frequency bands such as δ, θ and α [6]. Eye blinking generates spike-like shaped signal waveforms with their peaks reaching up to 800 μV and occurs in a very short period of 200–400 ms [7]. Meanwhile, artifacts generated by eye movement are square-shaped, smaller in amplitude but last longer in time and concentrate in lower frequency bands [8].

In recent years, there has been an increasing interest in applying various techniques to remove ocular artifacts from EEG signals [4,5,8–13,16–22,44]. The methods for removing EOG artifacts based on regression have been widely studied [4,9,11,12,39,44]. Regression methods often assume that the scalp potential is a linear combination of brain and ocular potentials. By subtracting propagated EOG from EEG recordings, EEG signals can be recovered [11]. Regression can also be done in frequency domain based on the concept that subtraction in the frequency domain is equivalent to filtering in the time domain. By eliminating spectral estimates of EOG from EEG recordings, it is possible to recover the non-contaminated EEG [11]. Both types of regression methods are off-line and rely on EOG recordings, which are however, not always available [4,9,17].

Berg and Scherg [13,38] proposed a principle component analysis (PCA) based technique for removing eye movement artifacts. This method assumes that each EEG channel recording is simultaneously generated by multiple sources across the scalp. By decomposing multiple channel EEG data into principle components using PCA, the artificial sources can be identified and
removed. Their experiments showed that the PCA based method outperformed regression based models. However, PCA methods usually failed to completely separate artifacts from cerebral activities [14], and the orthogonal assumption for data components, which is always required while using PCA, is hardly satisfied [8]. Independent component analysis (ICA), which was originally developed for blind source separation (BSS) problems, has been used as an alternative method for EEG artifact removal [4,15–17]. ICA usually requires a large amount of data and visual inspection to eliminate noisy independent components, making the method time-consuming and not suitable for real-time applications.

Recently, wavelet analysis has been used as an effective tool for measuring and manipulating non-stationary signals such as EEG. Wavelet-based methods, especially the wavelet thresholding techniques, have received significant attentions for EEG artifact removal [17–22]. For this class of methods, wavelet coefficients at low-frequency sub-bands are corrected by some thresholding functions before signal reconstruction. As an online artifact removal method, the most important advantage of using this method for EEG correction is that it does not rely on either the reference EEG signal or visual inspection. However, its performance is not consistent because the method is sensitive to the selections of wavelet basis and thresholding functions. Thus, an online method which can remove EOG artifact effectively is still desirable.

This paper proposes a novel, robust, and efficient Wavelet Neural Network (WNN) technique to remove EOG artifacts by combining the approximation capabilities of both wavelet and neural network methods. In WNN, EOG recordings are not required once the NN being trained and the WNN algorithm can perform artifact correction in a single channel data. The method (1) decomposes the contaminated EEG signals to a set of wavelet coefficients, (2) passes the coefficients located in low frequency wavelet sub-bands through a trained artificial neural network (ANN) for correction and (3) reconstructs a clean version of EEG signals based the corrected coefficients. We applied the method to EEG data contaminated by EOG artifacts and compared the results with those obtained by other state-of-the-art methods including ICA and a wavelet thresholding method.

The rest of the paper is organized as follows. Section 2 reviews the related work and the motivation of this paper. Section 3 details the proposed technique. Section 4 describes the datasets used in this paper and the experiment design. Experimental results are presented in Section 5. We provide discussions in Section 6 and conclude this paper in Section 7.

2. Related work

2.1. EEG model

Cerebral signals, recorded by an EEG recording system, result from neural firing activities. On the other hand, EOG artifacts are non-cerebral activities spreading over the entire recording scalp and contaminating the EEG electrode recordings. For that reason, an EEG recording can be represented as a superposition of a true EEG signal and some portions of the artifacts. When an EOG artifact presents, it is assumed that the model for the contaminated EEG signal is in the following form [17],

\[ E_{\text{rec}}(t) = E_{\text{true}}(t) + k \times EOG(t) \]  

where \( E_{\text{rec}}(t) \) is the recorded contaminated EEG, \( E_{\text{true}}(t) \) denotes the true EEG signal, \( EOG(t) \) represents the original potential changes caused by ocular activities and \( k \) symbolizes the propagated factor and varies between 0 and 1 depending on the location of the recording electrode. Hence, \( k \times EOG(t) \) represents the propagatedocular artifact from the eye to the recording site, which directly adulterates the brain signals. Estimating \( E_{\text{true}}(t) \) from observed \( E_{\text{rec}}(t) \) is non-trivial and is equivalent to minimizing the effect of ocular artifacts. Similar to other artifact removal techniques, the goal of the proposed wavelet neural network technique is to recover \( E_{\text{true}}(t) \) from \( E_{\text{rec}}(t) \).

As a random signal, a true EEG signal owns the noise-like (flat) power spectrum. In some cases when a subject performs specific tasks, the biological neural system introduces activities at particular frequencies making the power spectrum deflated. As a major artifactual type, once mixed with \( E_{\text{true}}(t) \), the ocular artifact \( k \times EOG(t) \) causes proliferation in low frequencies and generates spike-like shape data segments across time domain. These properties are utilized by both wavelet thresholding [17] and the proposed WNN technique for artifact removal.

2.2. Wavelet transform and its application to EOG Artifact removal

2.2.1. Wavelet transform

The wavelet transform [23–26] is a transform in which a set of basis functions, known as wavelets, are well localized both in time and frequency domains. Wavelets can be constructed from a single function \( \psi(t) \), named mother wavelet or analyzing wavelet, by means of translation and dilation,

\[ \psi_{a, b}(t) = \frac{1}{\sqrt{a}} \psi \left( \frac{t - b}{a} \right) \]  

Continuous wavelet transform (CWT) of a signal \( x(t) \), defined as the correlation between the wavelet and the signal itself, can be implemented by the following formula,

\[ W(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) \psi^* \left( \frac{t - b}{a} \right) dt \]  

where \( \psi^*(t) \) denotes the complex conjugate of \( \psi(t) \). The above Eq. (3) indicates that the wavelet is passed through the analyzed signal and yields a set of coefficients representing the image of the analyzed signal at different scales in time and frequency domains. The scale parameter \( a \) plays a crucial role in wavelet transform. While value of \( a \) changes from high to low, the wavelet is expanded and becomes less sharper in frequency domain. Accordingly, the low frequency terms can be analyzed with a less sharper time resolution, which is a useful property especially in analyzing transient waveforms such as EEG corrupted with ocular artifacts, where transients occur at low frequency.

Wavelet transform results in a time-scale decomposition in which scales are basically related to frequency [27]. The highest scale corresponds to the sharpest frequencies represented in the signal (less or equal to half of the sampling rate), and bandwidth of this scale ranges from a half to a quarter of the sampling rate. While that bandwidth is reduced by two, the number of coefficients at lower resolution decreases approximately by a factor of two compared to that of the higher resolution next to it. A proper selection of coefficients from different scales may be used to compress or represent original/corrected signals by using the inverse formula of Eq. (3). Discrete wavelet transform (DWT) is the discretized version of wavelet transform applied to discrete time series, in which parameters \( a \) and \( b \) in Eqs. (2) and (3) can be represented as \( a = 2^i \) and \( \tau_j = 2^j \), where \( i \) and \( j \) are positive integers. Selection of \( i \) and \( j \) determines properties of mother wavelet function \( \psi(t) = 2^{-i/2} \psi(2^{i/2} t - j) \), which constitutes an orthonormal basis of Hilbert space, consisting of finite–energy signals [28]. DWT can be implemented with a simple recursive filtering scheme providing a highly efficient wavelet representation of the
original signal. At each stage of that filtering process, the signal is passed through and convolves with a pair of low- and high-pass filters, \(g\) and \(h\), respectively, as in Eqs. (4) and (5). Signal reconstruction can be realized by using an inverse filtering operation as shown in Fig. 1.

\[
y_{\text{low}}[n] = \sum_{k=-\infty}^{\infty} x[k]g[2n-k] \tag{4}
\]

\[
y_{\text{high}}[n] = \sum_{k=-\infty}^{\infty} x[k]h[2n-k] \tag{5}
\]

The wavelet transform and wavelet reconstruction can be illustrated at Fig. 1(a) and (b). The (forward) wavelet transform is implemented in the following process: the original signal is first passed through a high pass filter \((H_0)\) and a low pass filter \((G_0)\), which are designed based on the properties of wavelet basis function, and then downsampled by two. After that, a high frequency coefficient series and a low frequency coefficient series, called detail and approximation, respectively, are obtained. The approximation is then continuously used as the input of the next level wavelet transform. Finally, there would be one approximation and detail at each level are upsampled by two and passed through and convolves with a pair of low- and high-pass filters, \(G_1\) and \(H_1\) respectively, as in Eqs. (4) and (5). Signal reconstruction can be realized by using an inverse filtering operation as shown in Fig. 1.

The core idea of the wavelet thresholding method is to use a Butterworth lowpass filter to smooth the EEG signal before further processing. The optimal value of threshold can be adjusted based on the SURE risk estimate along with a soft-like thresholding function. In this paper, a time-scale adaptive algorithm based on the SURE risk estimate along with a soft-like thresholding function was used for EEG ocular artifact removal due to its effectiveness illustrated in [17].

In this method, the wavelet coefficients of EEG in specific wavelet sub-bands are corrected as follows,

\[
w^c_k(w,t) = \begin{cases} 
W + t - \frac{w}{2^{k+1}} , & w < -t \\
\frac{1}{2^{k+1} \pi^2} W^{2k+1} , & |w| \leq t \\
W - t + \frac{w}{2^{k+1}} , & w > t
\end{cases} \tag{7}
\]

where \(k\) is a positive number and \(w^c_k\) represents the corrected version of a wavelet coefficients \(w\) using the thresholding value \(t\). The optimal value of \(t\) can be adjusted based on the SURE risk using the following adaptive steps

\[
t(i+1) = t(i) - \nabla t(i) \tag{8}
\]

where the adjustment of threshold at step \(i\) is defined by

\[
\nabla t(i) = \alpha(i) \frac{\partial R(t)}{\partial t} \tag{9}
\]
Joint approximate diagonalization of eigen-matrices (JADE) \[49\] performed on spatial components. Finally, ICA determines estimated version \( \mathbf{u} \) of original independent sources \( \mathbf{s} \) using the following linear transformation \( \mathbf{u} = W \mathbf{x} \), where the rows in the output data matrix represent time courses of activation of the ICA components \[4\].

Several algorithms have been proposed to implement ICA such as information maximization (Infomax) \[31\], Fixed-point ICA \[48\], Joint approximate diagonalization of eigen-matrices (JADE) \[49\] and the second-order blind identification (SOBI) \[50\]. In this research, Infomax, an unsupervised neural network learning algorithm capable of decomposing mixtures into independent components based on the linear relationship \( \mathbf{x} = \mathbf{A} \mathbf{s} \), where no knowledge is available about the sources or the mixing matrix. ICA identifies an unmixing matrix, \( \mathbf{W} \), which decomposes the mixed data into a sum of temporally independent and spatially fixed components (noted that ICA can also be performed on spatial components). Finally, ICA determines estimated version \( \mathbf{u} \) of original independent sources \( \mathbf{s} \) using the following linear transformation \( \mathbf{u} = W \mathbf{x} \), where the rows in the output data matrix represent time courses of activation of the ICA components \[4\].

2.3. Independent component analysis

2.3.1. Background of ICA

Independent component analysis (ICA) was first proposed by Herault and Jutten in 1986 \[4,14\] for the blind source separation (BSS) problem. ICA aims to recover independent source signals \( \mathbf{s} = (s_1, s_2, \ldots, s_d) \), from recorded mixtures \( \mathbf{x} = (x_1, x_2, \ldots, x_d) \) mixed by an unknown, full-rank matrix \( \mathbf{A} \). The basic problem of ICA is to estimate the mixing matrix \( \mathbf{A} \) or equivalently, the original independent sources \( \mathbf{s} \) based on the linear relationship \( \mathbf{x} = \mathbf{A} \mathbf{s} \), while no knowledge is available about the sources or the mixing matrix. ICA identifies an unmixing matrix, \( \mathbf{W} \), which decomposes the mixed data into a sum of temporally independent and spatially fixed components (noted that ICA can also be performed on spatial components). Finally, ICA determines estimated version \( \mathbf{u} \) of original independent sources \( \mathbf{s} \) using the following linear transformation \( \mathbf{u} = W \mathbf{x} \), where the rows in the output data matrix represent time courses of activation of the ICA components \[4\].

Several algorithms have been proposed to implement ICA such as information maximization (Infomax) \[31\], Fixed-point ICA \[48\], Joint approximate diagonalization of eigen-matrices (JADE) \[49\] and the second-order blind identification (SOBI) \[50\]. In this research, Infomax, an unsupervised neural network learning algorithm capable of decomposing mixtures into independent components based on mutual information maximization between network input and output \[4,31\], was used to perform EEG artifact removal.

2.3.2. ICA in EEG Artifact removal

EEG artifact removal using ICA can be realized based on several assumptions \[4,16\]: (1) neural electrical recording signals are stationary, (2) the number of sources generating EEG is the same as the number of recording channels, (3) the sources are independent, (4) the sources distributions must be non-Gaussian and (5) the mixing matrix is squared and invertible. Keeping those assumptions in mind, the following procedure can be used to eliminate EEG artifacts. By performing the ICA technique, we first estimate the unmixing matrix \( \mathbf{W} \), which is then utilized to recover the estimated independent components \( \mathbf{s} \) from original mixtures \( \mathbf{x} \) using the linear relationship \( \mathbf{s} = \mathbf{W} \mathbf{x} \). The independent components (ICs) are then categorized into artifactual and non-artifactual components. The artifactual components are then replaced by zeros and \( \mathbf{s} \) becomes \( \mathbf{\hat{s}} \). Multiplying the inverse of unmixing matrix with the new ‘clean’ set of ICs, \( \mathbf{\tilde{x}} = \mathbf{W}^{-1} \mathbf{s} \), it is possible to recover the clean, corrected EEG.

2.3.3. Information maximization approach (Infomax): an ICA algorithm

In \[31\], Bell and Sejnowski developed a neural network based algorithm named Infomax that blindly separates mixtures \( \mathbf{x} \) into a set of independent sources \( \mathbf{s} \), using the information maximization principle. The core idea of Infomax algorithm is to maximize the joint entropy, \( H[\mathbf{g}(\mathbf{s})] \) where \( \mathbf{g} \) is a sigmoid function, by using a stochastic gradient ascent approach \[43,31\]. The goal is to minimize the mutual information between the independent components \( s_i \). Infomax updates weights of the ICA unmixing matrix \( \mathbf{W} \) by using the gradient of the entropy defined as

\[
\Delta \mathbf{W} = \frac{\partial H(\mathbf{y})}{\partial \mathbf{W}} = \mathbf{E} \left[ \frac{\partial \ln \mathbf{J}}{\partial \mathbf{W}} \right]
\]

where \( y_i = g(u_i) = 1/(1 + e^{-u_i}) \), \( E \) denotes the expected value, \( \mathbf{y} = [g(u_1), \ldots, g(u_n)] \) and \( \mathbf{J} \) is the absolute value of the determinant of the Jacobian matrix:

\[
J = \det \left[ \frac{\partial y_i}{\partial u_j} \right]
\]

where \( i,j \) get values between 1 and size of \( \mathbf{W} \). The learning rule defined by Eq. (12) can be rewritten in a succinct way as

\[
\Delta \mathbf{W} = [\mathbf{W}^{T}]^{-1} + \mathbf{\tilde{y}} \mathbf{x}^T
\]

where \( \mathbf{\tilde{y}} = [\tilde{y}_1, \ldots, \tilde{y}_n]^T \), which has each element defined by

\[
\tilde{y}_i = \frac{\partial y_i}{\partial u_i} \frac{\partial u_i}{\partial \mathbf{w}} = \frac{\partial \ln y_i}{\partial \mathbf{w}}
\]

3. Proposed method

Up to this point, we have presented general theories of both wavelet transform and ICA. Also, the ideas behind using wavelet transform, or more specifically, a wavelet thresholding method using the adaptive SURE risk thresholding function, and ICA with the Infomax as a representative, were summarized. ICA is effective in multi-type artifact removal and should be a good benchmark method for educational purpose in EEG-related research. However, as mentioned previously, ICA is time-consuming due to its computational complexity. Being a batch algorithm, ICA needs to be performed on the whole data set with at least an adequate number of data points. Instead of using the entire data set, wavelet thresholding can be performed for EOG artifact removal on a single channel data. However, the selection of wavelet is sensitive to the time-frequency properties of EEG waveforms. Specifically, experiments conducted during this work showed that using only one specific wavelet or even various wavelets from one particular mother wavelet for different EEG data segments usually gives unacceptable results. In addition, the thresholding function needs to be selected with care. Obviously, both wavelet thresholding and ICA are not desirable methods for real-time processing applications. In this paper, a solution is being sought which is applicable to an online EOG artifact removal task. Recognizing the universal approximation of both neural network and wavelet-based methods, the idea of combining them should be very attractive.

In this paper, a novel algorithm, Wavelet Neural Network (WNN), for EEG artifact removal is presented. In this method, the WNN is trained with simulated data resembling the time-spectral properties of raw EEG signals. The trained WNN is then used to correct contaminated data. In both testing and training processes, the original signal is first decomposed by the wavelet transform into different frequency coefficients. Coefficients from low-frequency sub-bands are then interpolated to maintain same lengths. The trained artificial neural network is fed with such interpolated inputs to yield the corrected coefficients. Finally, the corrected coefficients are downsampled and reconstructed to obtain artifact-free EEG signals. The structure of the proposed wavelet neural network is illustrated in Fig. 2.

The core idea of the method, decomposing the signal with wavelet transform and using an ANN to correct them, can be
viewed in a more succinct (and perhaps more precise) way. That is by combining the time/frequency property of wavelet and the universal approximation capability of neural network, we would be able to keep useful information related to cognitive activities while eliminate artifacts in EEG.

3.1. EEG data simulation

It has been summarized in [1] that a desirable EEG model should (1) give a better understanding of brain functionalities, (2) provide testing tools for novel EEG-related methods, and (3) be useful for medical education and medicare training. There have been several EEG simulating models [35–37] proposed. The model utilized in [1] was originally designed for medical research in pharmacodynamics, the psychological science of effects caused by drug on humans [1,33]. This EEG simulation model is used in this paper for the purpose of training and validating the proposed WNN method as well as other algorithms such as wavelet thresholding on EOG artifact removal [34].

EEG signals can be simulated based on three assumptions [1], (1) segments of the spontaneous EEGs can be described as linearly filtered Gaussian noise, (2) non-stationary components in the spontaneous EEG can be simulated by changing the characteristics of that filtering process and (3) the spectral property of the simulated EEG data resembles that of actual signal. As shown in Fig.3, a set of Gaussian noises (GN) were generated and then filtered by a number of lowpass and bandpass filters with different cut-off frequencies (Table 1) that are similar to the spectral property of EEG frequency bands. The filtered signals are multiplied with different gains ($g_i$, $i = 1, ..., 6$) corresponding to its EEG rhythmic properties. Those EEG simulated rhythmic signals are then summed up. That synthetic clean simulated signal is multiplied with an overall gain $g_0$ to get an amplitude corresponding to that of actual EEG. After that, transients visually recognized such as blinking and eye movements, collected from real raw EEG signals were filtered by lowpass filters and added to contaminate the simulated data. In the original model, simulated EEG signal is obtained by multiplying the summation of linearly filtered Gaussian noises and transients with an overall gain. The model in Fig. 3 slightly differs from that one. The summation of those linearly filtered Gaussian noises is multiplied with an overall gain to form clean simulated EEG signal before being added transients, which are EOG artifacts extracted from real EEG data.

3.2. Neural Network for EEG Artifact removal

3.2.1. Training

In the training process, we first generate clean EEG signals using the EEG simulator. The non-contaminated EEG signals are contaminated by adding real filtered-extracted artifacts then (transients). Other than the added real artifacts, those two EEG signals are identical. Note that in the EEG model described earlier in Eq. (1), clean and contaminated signals can be considered as $\text{EEG}_{true}(t)$ and $\text{EEG}_{rec}(t)$, respectively. The network training problem is equivalent to educating an ANN how to recover $\text{EEG}_{true}(t)$ from $\text{EEG}_{rec}(t)$. Both $\text{EEG}_{true}(t)$ and $\text{EEG}_{rec}(t)$ are decomposed using the wavelet transformation. The approximation and detail coefficients in several low-frequency sub-bands are interpolated to make them the same lengths. These interpolated coefficient series are then combined as a training dataset for training the ANN. A diagram of the presented process is given in Fig. 4.

3.2.2. Testing and Artifact removal

In the testing and real data artifact removal processes, early are the same as described in the training process. $\text{EEG}_{rec}(t)$ is decomposed by wavelet transform. Coefficients in different sub-bands are interpolated to make them the same lengths. The testing data are then passed through the trained ANN for modification and the modified coefficients are then down-sampled to the original lengths and returned to original wavelet coefficient series for wavelet reconstruction. Fig. 5 illustrates the testing/correcting procedure, where $\text{EEG}_{true}(t)$ is the estimated true EEG signal.
3.2.3. Neural Network training algorithm

Output weight optimization backpropagation (OWO–BP) algorithm [40,43], as presented below, is used to train a fully-connected multi-layer perceptron (MLP) neural network (Fig. 6). Experimental results show that one hidden layer neural network structure is good enough for EOG artifact removal issue. Sigmoid function is used as activation function for the single hidden layer. The number of inputs equals to the number of output and is the number of wavelet sub-bands in which wavelet coefficients will be modified by the ANN.

As described carefully in [45,46], OWO–BP is a supervised learning technique combining BP method and OWO scheme to adapt a feed-forward neural network (Fig. 6). The BP algorithm updates the hidden weights \( w_{hi} \) which connect the input units \( x_p \) to the hidden units \( net_{H_p} \). Meanwhile, the OWO method is capable of solving linear equations for the output weights \( w_o \), which connect the hidden unit activations \( O_p \) with the output units \( y_p \). The hidden unit activation is calculated from hidden unit output using a sigmoid function. OWO also computes the which can be initialized to determine the step size of the learning procedure.

The standard training error for the \( i \)th output can be written as,

\[
E(i) = \frac{1}{N_p} \sum_{p=1}^{N_p} \left( y_p(i) - \hat{y}_p(i) \right)^2
\]

where \( y_p(i) \) and \( \hat{y}_p(i) \) are the \( i \)th actual and corresponding desired outputs and \( N_p \) is the number of training patterns. The actual output is computed as

\[
y_p(i) = \sum_{j=1}^{N_o} w_{oi}(j) \hat{y}_p(j)
\]

where \( w_{oi}(j) \) is the output weight from \( j \)th basis function unit to the \( i \)th output unit and \( N_o \) is the number of basis functions.

The basis functions are defined as

\[
x_p(j), \quad 1 \leq j \leq N
\]

\[
O_p(j=N+1), \quad N+2 \leq j \leq N+N_h + 1
\]

\[
1, \quad j = N+1
\]

where \( N_h \) is the number of hidden units. As the activation of hidden units, \( O_p \) is defined as

\[
O_p = f(\text{net}_{H_p})
\]

where the function \( f \) is called activation function which is differentiable. The Sigmoid function \( f(x) = 1/(1+e^{-x}) \) is used in this paper.

Consequently, the first derivative of \( E(i) \) with respect to \( w_{oi}(j) \) is given by

\[
\frac{dE(i)}{d w_{oi}(j)} = -2 \left[ \rho(i,j) - \sum_{j=1}^{N_h} w_{i,j} \alpha(i,j) \right]
\]

where \( N_o = N+N_h + 1 \), \( \alpha(i,j) \) and \( \rho(i,j) \) are the auto-correlation and cross-correlation matrices, respectively, defined as

\[
\alpha(i,j) = \sum_{p=1}^{N_o} \hat{y}_p(i) \hat{y}_p(j)
\]

\[
\rho(i,j) = \sum_{p=1}^{N_o} \hat{y}_p(i) \hat{y}_p(j)
\]

The optimized weight \( w_{oi}(j) \) is obtained by setting \( dE(i)/d w_{oi}(j) \) to zero, or equivalently

\[
\sum_{j=1}^{N_h} w_{oi}(j) \alpha(i,j) = \rho(i,j), \quad 1 \leq i \leq M,
\]

where, \( M \) is the number of outputs. Solving Eq. (23) is equivalent to minimizing \( E(i) \) [43].

The next step is to update hidden weights \( w_{hi}(j) \) using the BP method based on the steepest descent rule,

\[
w_h(j,i) = w_h(j,i) + \delta_1 \left( \frac{\partial E}{\partial \text{net}_{H_p}(j,i)} \right)
\]

In Eq. (24), \( \partial E/\partial \text{net}_{H_p}(j,i) \) is determined as

\[
\frac{\partial E}{\partial \text{net}_{H_p}(j,i)} = x_p(i) w_{oi}(j)
\]

where

\[
\delta_{ph}(j) = J \left( \text{net}_{H_p} \right) \sum_{k=1}^{N_{out}} \left[ (\hat{y}_p(i) - y_p(i)) w_o(k,j) \right]
\]

and \( \delta \) is an invisible learning rate, which is calculated as

\[
\delta_1 = z_1 \left( E(t-1)/\sum_{i=1}^{N_o} \sum_{j=1}^{N_{out}} \left( \frac{\partial E}{\partial \text{net}_{H_p}(j,i)} \right)^2 \right)
\]

where \( t \) is the iteration number, and \( z_1 \) is a visible learning rate that can be initialized to determine the step size of the learning procedure.

3.3. Performance metrics

Three metrics are used to assess the performances of artifact removal algorithms in this paper, including power spectral density (PSD), root mean square error (RMSE) and frequency correlation.

PSD is a popular metric used to show information about the frequency content of signals. PSD can be estimated by nonparametric methods such as Bartlett, Welch, Blackman and Tukey or parametric methods such as Yule-Walker, Burg, etc. [47]. In this
paper, PSD of EEG signals is computed by the Welch method, which is developed based on the idea of averaging periodogram spectrum estimates over overlapping data segments.

Calculation of the correlation in frequency domain before and after artifact removal is equivalent to the correlation of the signals in time domain before and after filtering \([17,41]\). The frequency correlation between \(\hat{x}\) and \(\hat{y}\) is computed as,

\[
c = \frac{1}{2} \sum_{n=1}^{N_w} \left( \hat{x}^\ast \hat{y} + \hat{y}^\ast \hat{x} \right) \sqrt{\sum_{n=1}^{N_w} \hat{x}^2 \sum_{n=1}^{N_w} \hat{y}^2}
\]

where \(w_1\) and \(w_2\) are the lower and upper limits of the interested power spectrum region to be calculated, \(c\) is the correlation value that will be assigned to the frequency of \((w_1 + w_2)/2\). If \(\hat{x}\) and \(\hat{y}\) are identical, \(c\) gets 1, otherwise, \(c\) obtains a value between 0 and 1. In this paper, the ‘window size’, \(w_1 – w_2\), is selected as 2, due to its computational efficiency.

The value of RMSE shows the difference between the corrected and original non-contaminated signal. This value is proportional to the accuracy of the method used for correction and defined as

\[
RMSE = \sqrt{\frac{1}{N_v} \sum_{t=1}^{N_v} (EEG_{true}(t) – EEG_{true}(t))^2}
\]

where \(N_v\) is the length of the contaminated EEG segment.
4. Experiments

In this section, we show experimental results obtained using the proposed algorithm and compare them with those from other algorithms.

4.1. Datasets

The method was validated on two datasets, one was recorded in a driving test [34] and another was for a visual selection task experiment (for more information, see http://sccn.ucsd.edu/~arno/fam2data/publicly_available_EEG_data.html).

4.1.1. Driving test dataset

This dataset was collected when participants were performing a driving test [34]. The EEG information was collected by a 128-channel recording system at the sampling rate of 1000 Hz along with other information including description of the task, system dynamics related information, performance measures, physiological signals (ECG, respiration, etc.), and eye tracking. The workload was also analyzed according to the driving conditions (city-driving, stopped, highway passing, etc.). Due to the recording conditions, eye movements and blinks activities happened at high frequencies making the data, especially at frontal recording channels, highly contaminated by ocular artifacts.

4.1.2. Visual selection task dataset

This dataset was recorded by a 32-channel recording system at sampling rate of 128 Hz during the course of 238.3125 s while the subject participated in a selective attention task, where the subject was asked to attend to circles flashed in a random order at one of five displayed locations [4]. Even the design of the task helps restrict the eye movements and blinks, but generally, the data are still highly contaminated by the ocular artifacts, which are dominant at EEG recorded from the electrodes F3, Fz, F4, etc. located in the frontal sites of the subject head.

4.2. Experimental settings

For each dataset, three artifact removal methods were implemented for comparison: the ICA method, the wavelet thresholding algorithm and the proposed WNN technique. For each algorithm, PSD and frequency correlation were computed before and after artifact removal to illustrate the effectiveness of each algorithm. MSE/RMSE was used for method accuracy comparison.

<table>
<thead>
<tr>
<th>Method</th>
<th>RMSE Training</th>
<th>RMSE Testing</th>
</tr>
</thead>
<tbody>
<tr>
<td>WNN</td>
<td>11.2389</td>
<td>12.2473</td>
</tr>
<tr>
<td>Wavelet thresholding</td>
<td>16.1941</td>
<td>16.4942</td>
</tr>
</tbody>
</table>

Table 2: RMSE of signals before and after correction for driving test dataset.

Fig. 10. Frequency correlation between (a) contaminated and wavelet thresholding corrected simulated signals and (b) clean and wavelet thresholding corrected simulated signals (c) contaminated and WNN corrected simulated signals and (d) clean and WNN corrected simulated signals, all for testing.
between WNN and wavelet thresholding. For the proposed method, an EEG signal was first simulated to train an ANN and the trained WNN was tested on each of the data sets. Wavelet thresholding method was implemented by following the instructions in [17] and for ICA, EEGLAB software [42] downloaded from http://sccn.ucsd.edu/eeglab/downloadtoolbox.html was used.

5. Results

5.1. Results for the driving dataset

5.1.1. Results for simulated data

For the proposed WNN algorithm, two simulated EEG segments with a length of 5 s are shown in Fig. 7(a) and (b). The segments were created by the simulation model described in Section 3.1 with a sampling rate of 1000 Hz. The artifacts were taken from the driving test data set and added to the simulated data segments. Data in Fig. 7(a) were then used to train the neural network in the proposed WNN algorithm. The trained WNN model was applied to a testing data segment (Fig. 7(b)). The simulated signal was decomposed up to 8 levels with wavelet Coif3 from Coiflet wavelet family, which is constructed from orthonormal bases of compactly supported wavelets with high symmetry, regularity and vanishing moments [52]. The three lowest frequency sub-band coefficients, in which most of the EOG artifacts’ coefficients reside, were then corrected by the trained 3-5-3 (3 input units, 5 hidden units and 3 output units) ANN and used for wavelet reconstruction. Number of hidden units was determined experimently based on the RMSE from the simulated testing EEG dataset. The corrected testing EEG signal is shown in Fig. 8. Fig. 9 shows PSD of the contaminated, corrected and clean EEG signals for training and testing. Fig. 10 shows frequency correlations among those simulated signals. As mentioned previously, ICA needs to be performed on multiple channels and is not applicable to the single channel simulated datasets.

Table 2 shows the difference between clean and decontaminated signals by WNN and wavelet thresholding via the metric of MSE/RMSE.

5.1.2. Results on real data

Due to the recording conditions, signals from some channels with bad-connections to the scalp were deteriorated and not usable. Data for those channels (25, 47, 46, 112, 125 and 126) were removed from the original dataset. The resized dataset contains data from 122 channels, each consists of 60 s. Three methods were applied, WNN, ICA and wavelet thresholding, on this dataset.

For WNN technique, EEG signals were decomposed to 8 levels with wavelet “Coif3”. Then, three low frequency sub-band coefficients were corrected by a ANN with the structure of 3-5-3, which was determined for the simulation data. Those corrected coefficients were then used to reconstruct decontaminated signals.

For Infomax ICA, it took a computer, equipped with Intel(R) Core(TM) 2 CPU 6400 @ 2.13 GHz and RAM 2.00 GB, 27 min with 382 steps to yield the unmixing matrix. After that, the independent components (ICs) were obtained by multiplying the unmixing matrix with the original mixtures. After visually inspection, artifactual ICs: 1–7, 10, 11, 13, 14, 16–20, 23, 27, 29, 33, 40, 44, 45, 49 and 57 were made zero. Corrected signal was obtained by multiplying that inverse version of unmixing matrix with that ‘new’ IC set.

The wavelet thresholding method was used to adaptively correct four low frequency sub-band (0–1.95, 1.95–3.9, 3.9–7.8 and 7.8–15.6 Hz) coefficients, which were obtained after contaminated EEG data segment was decomposed by wavelet transform up to 9 stages. For specific data segments, corrections were repeated a number of times with various wavelets and at different stages.
levels of decomposition in order to make the corrected data most acceptable by visual inspection. Wavelets from either Coiflet or Daubechies family could be selected because experiments show that they could extract features of artifacts efficiently.

Fig. 11 shows a segment with three spike artifacts and its corrected versions made by various methods in time domain. The results given by three methods demonstrate their abilities on artifact removal. Fig. 12 shows PSD plots for one sample artifact removed segment in the driving test data by the three algorithms. Fig. 13 shows frequency correlations between contaminated and corrected segments.

5.2. Results for the visual selection task dataset

5.2.1. Results on simulated data

For WNN training and testing, simulated data segments of length 30 s, equivalently 3840 samples, were generated at the sampling rate of 128 Hz. Then simulated data were contaminated with artifacts taken from real data at channels number 1 and 3, or FP1 and FP3 equivalently. After being decomposed by wavelet transform with wavelet Coif3 to six levels, coefficients at low frequency sub-bands: 0–2, 2–4, 4–8 and 8–16 Hz, where EOG artifacts mainly occur, were passed through a ANN with the structure of 4-6-4 (4 input units, 6 hidden units and 4 output units) for training purposes. Number of hidden units was set experimentally and dataset-orientedly.

Fig. 14(a) shows visual appearance of simulated EEG data (30 s), which is contaminated by numerous artifacts, before and after they have been corrected. The zoom-in result displayed in Fig. 14(b) indicates that WNN just focuses on removing artifacts across contaminated data segments; meanwhile, non-contaminated information is well-preserved. Fig. 15 shows PSD of clean, contaminated, wavelet-thresholding-corrected and WNN-corrected simulated signals used for both training and testing of the ANN. The PSD of simulated EEG signal corrected by WNN and that of the clean simulated EEG signals are almost overlapped. That is promising because it demonstrates that WNN performed very well in frequency domain. Fig. 16 shows the frequency correlation of contaminated and clean simulated signals with their corrected versions by using WNN and wavelet thresholding.

RMSE, as displayed in Table 3, shows the difference between the original non-contaminated and corrected signals made by WNN and wavelet thresholding for both training and testing data.

5.2.2. Results for real data

A real EEG data segment of 30 s, or 3840 samples, was decomposed with wavelet “Coif3” to six levels. Then, coefficients from four low frequency sub-bands were interpolated and fed to the trained WNN model with a structure of 4-6-4. The WNN

![Fig. 13. Frequency correlation between contaminated and decontaminated EEG, (a) by ICA, (b) by wavelet thresholding and (c) by WNN.](image-url)
outputs corrected coefficients, which were down-sampled and then used to reconstruct corrected EEG signals.

ICA algorithm was realized by using the same method as in the driving test experiment. Almost three minutes with 275 steps were needed to obtain the unmixing matrix of size $32 \times 32$. By careful visual inspection combined with IC topographic mapping method, which was realized by using the recording system descriptions given along with dataset, ICs numbered 1, 2, 4–7 and 22 were identified as artifact and were set as zero. After multiplying new zero-valued and other non-zero-valued ICs with the mixing matrix, corrected EEG data were obtained.

Wavelet thresholding method was implemented with wavelet Coif4 with 6 levels of composition. The number of decomposition levels and wavelet basis function were determined based on performance of the method. We should note here that wavelet basis function is not necessary to be the same one selected for WNN correction. Coefficients at four low frequency sub-bands were corrected by an adaptive thresholding function.

Fig. 14(a)–(c) shows visual appearance of a real EEG data segment (30 s), which was contaminated by numerous artifacts, before and after correction. Specifically, artifacts were removed efficiently by WNN (Fig. 17(b)–(d)). Fig. 18 shows the PSD of signal before and after correction by all three methods. The frequency correlation was shown in Fig. 19. WNN and wavelet thresholding correct the contaminated signal in the low frequency range. Meanwhile, ICA performed corrections on the entire useful EEG frequency range.

6. Discussion

It is observed that the WNN algorithm can remove ocular artifacts efficiently while keeping cerebral background information. Similar to wavelet thresholding, WNN only needs one single-channeled data to perform correction. This is more convenient compared to the ICA method, which performs artifact removal on the whole dataset. Furthermore, the proposed method has proved to be an effective and stable artifact removal algorithm through repeated experiments on various datasets. In contrast, we found that wavelet thresholding fails to produce consistent results.

Low frequency components from contaminated EEG signals, as shown in PSD plots, were reduced significantly compared to corrected signals. Recall that ocular artifacts appear largely in the range of low frequency, the method attenuated those artifacts
efficiently. Frequency correlations between corrected and contaminated EEG signals taken from both real and simulated data confirm that slow waves, corresponding to EOG artifacts, were reduced significantly. Meanwhile, cerebral information in other frequency ranges were well-preserved.

As shown in Fig. 8 and more clearly in Fig. 9, WNN sometimes overcorrects EEG signals at low frequency range. It might be explained as follow, EOG artifacts extracted from raw EEG signals after a low-filtering process still contain some portions of low frequency cerebral information. This cerebral information will be considered as artifacts during training and thus will confuse the WNN. However, a complete separation of low frequency cerebral information from artifacts is not simple and is currently under investigation, which may lead to future publications. Fortunately, as it could be seen in Figs. 15, 16, 19, performance degradation is not severe for other cases.

For ICA method, corrections are made on the entire frequency range rather than just focus on low frequency components. Correlations between corrected and contaminated EEG signals in low frequency region are the best among the three algorithms, indicating that ICA is an effective algorithm for removing EOG artifacts.

RMSE of correction on simulated data made by WNN is smaller than that made by wavelet thresholding, which implies the WNN is more accurate.

ICA is capable of removing multiple types of EEG artifacts. However, a large resource of computing power is needed in order to perform ICA for artifact removal. It also requires either an automatic or a manual step to determine which independent components are artifactual ones. Both mentioned points lead to conclusion that an online implementation of ICA is not convenient. Meanwhile, as mentioned previously, once the network being trained, EOG recordings are not needed for artifact correction in WNN. In addition, processing time needed for each data segment correction made by WNN can be realized online with limited computing resource by using a sliding window across the contaminated signals.

In the experiments on WNN correction, wavelet transform and reconstruction were performed with basis function
“Coiflet 3” due to its excellent capability of localizing EOG artifact. For each data set, the number of wavelet stages can be determined based on sampling rate without reference to any other specific characteristics of contaminated EEG segments. Note that number of samples needed for each data segment should be determined based on experimental results. If this number is too large, the network might face problem with complexity. On the other hand, if it is too small, the network might not have enough data, thus lacks of information, to perform correction properly. As a rule of thumb, a neural network with one hidden layer is good enough for many approximation issues. In our research, the number of network training iterations is set at 200. This number can be changed to improve training accuracy. Last but not least, OWO–BP was utilized in this paper but many other neural network learning schemes can be applied for network training.

7. Conclusions

A wavelet neural network was proposed for EOG artifact removal task. The algorithm combines the approximating
capability of both neural network and wavelet transform to locate and eliminate artifacts. Experimental results on the driving and visual task selection datasets show that WNN can effectively remove artifact and achieve better results than wavelet thresholding. WNN is also computationally efficient and more convenient than ICA making it possible an automatic online algorithm.

Acknowledgement

This project is funded by the NASA (Contract No: NNX10CB27C). We thank Dr. Alan T. Pope, our COTR, for his comments and suggestions as we performed this research. We also thank Dr. Kara Latorella for her comments and feedback during various discussions over the course of this project.

References

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