Iterative (Turbo) Joint Channel Estimation and Signal Detection for Quadrature OFDMA Systems

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Abstract—Quadrature OFDMA (Q-OFDMA) systems have been recently proposed to reduce the complexity and peak-to-average power ratio (PAPR), and improve carrier frequency offset (CFO) robustness for OFDMA systems. However, Q-OFDMA receiver obtains frequency diversity at the cost of noise enhancement. This paper proposes an iterative (turbo) joint channel estimation and signal detection technique for Q-OFDMA systems to mitigate the noise enhancement effect and improve the BER performance. In the proposed scheme, the channel estimation technique makes use of both training symbols and soft coded data information to suppress the inter-symbol interference (ISI) caused by channel estimation errors in Q-OFDMA systems. Simulation results show that performance improvement can be achieved with the proposed algorithms.

I. INTRODUCTION

Orthogonal frequency division multiple access (OFDMA) is endorsed by leading standards such as ETSI DVB-RCT and IEEE802.16, it is also a potential candidate of next generation (4G) high speed mobile wireless communications. However, to support a number of users’ access, the number of subcarriers, \( N \), in OFDMA systems is usually very large, which provides flexibility and high spectrum efficiency, at the expense of high complexity, severe PAPR, and sensitivity to CFO in general. The Quadrature OFDMA (Q-OFDMA) systems [1] overcome the aforementioned problems with significantly reduced complexity and power consumption in users’ terminals. Based on the concept of layered fast Fourier transform (FFT) structure [1], in Q-OFDMA systems, the intermediate domain instead of the frequency domain is used for data transmission, which results in a loss of the subcarrier orthogonality. At receiver, the orthogonality is recovered by FFT operations. The specific architecture of Q-OFDMA systems could achieve better BER performance than OFDMA when SNR is above a threshold depending on the channel condition and the modulation schemes [1]. It has been shown that advanced equalizers can significantly decrease the SNR threshold due to the large frequency diversity in Q-OFDMA systems.

In terms of minimizing the BER, the optimum maximum likelihood (ML) [2] detector is able to utilize both the diversity and coding gain furnished by frequency-selective fading channels. However, in most practical systems, linear equalizer (LE) [2]–[4] and decision feedback equalizer (DFE) [2]–[4] have been designed for complexity reasons. Turbo equalization [5]–[8] has been extensively studied when SNR and channel impulse response (CIR) are precisely known to the receiver. In cases where such information is not available or time varying thus need to be tracked, channel information should be estimated. Methods [9]–[11] attempt to perform estimation and equalization jointly, which increase the system performance at the cost of intractable complexity.

In this paper, we propose an iterative receiver for joint estimation, equalization, and decoding for the Q-OFDMA systems based on the turbo processing principle. The estimator makes use of training symbols and the soft decoded data information to track the channel frequency response. The equalizer can use the re-estimated channel to detect the transmitted data iteratively until the satisfactory outcome is obtained. We can judiciously choose estimation, equalization and decoding algorithms according to the performance/complexity tradeoff.

The rest of this paper is organized as follows. In section II, Q-OFDMA system is presented. The iterative (turbo) joint channel estimation and signal detection technique is proposed in section III and IV. Simulation results are shown in section V. Section VI concludes this paper. The following notations will be used throughout the paper. Matrices and vectors are denoted by symbols in bold face, \( x \), \( X \), \( \mathbf{x} \), \( \mathbf{X} \) and \( \mathbf{X}^{-1} \) respectively. \( (\cdot)^T \) and \( (\cdot)^H \) represent inverse, transpose and Hermitian conjugate. \( x \), \( \hat{x} \) and \( \tilde{x} \) denote symbol \( x \) in time-domain, intermediate-domain and frequency-domain, respectively.

II. REVIEW OF THE Q-OFDMA SYSTEM

The baseband of Q-OFDMA system is shown in Fig. 1. At the transmitter, each user’s data is first encoded, interleaved and mapped to a certain constellation. Unlike the subchannel in conventional OFDMA systems, which is defined in the one-dimension frequency domain, subchannels in Q-OFDMA systems are defined over an array of two dimensions in the intermediate domain [1]. This array is \( P \times Q \), where both \( P \) and \( Q \) are powers of 2, and \( N = PQ \) is the equivalent to the total

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number of subcarriers in ordinary OFDMA systems. Assume that at a specific frame, user $k$ occupies $M$ subchannels.

After all users’ data is assigned, the symbols of user $k$’s at subchannel $q$ can be extracted from the symbol matrix as

$$\mathbf{x}_q = (\tilde{x}_{0,q}, \tilde{x}_{1,q}, \tilde{x}_{2,q}, \cdots, \tilde{x}_{p-1,q})^T,$$

then the transmitter transforms the signals from intermediate domain to time domain. Based on the layered FFT/IFFT concept [1], the transform process includes the weighting operator, the per-row IFFT, and the column-wise read-out (i.e. interleaving). After adding cyclic prefix (CP) whose length is longer than the delay spread of a channel, the time domain samples are transmitted over a multipath fading channel.

At $k$th user’s receiver, after removing the CP, the time-domain samples are then serial-to-parallel converted to a $P \times Q$ array. Compute $Q$-point FFT for each row, then collect the data at subcarrier $q$ from each FFT output and arrange them into a vector. After weighting, the received symbol in intermediate-domain can be formulated as [1]

$$\tilde{y}_q = \sqrt{Q} \mathbf{H}_q \mathbf{x}_q + \tilde{\mathbf{n}}_q,$$

where the $P \times P$ circulant matrix $\mathbf{H}_q$ represents the dispersive channel, with $[\mathbf{H}_q]_{i,j} = h((i-j) \mod P)Q + q$, where $h(\cdot)$ denotes the channel response in the intermediate domain. $\tilde{\mathbf{n}}_q$ contains AWGN samples, each having zero mean and variance $\sigma^2_n$, which are same as that of noise samples introduced in the received time domain signals.

Before performing channel estimation and signal detection, a Fourier transform is implemented to the received intermediate-domain symbol $\tilde{y}_q$, which is transformed to frequency domain as

$$\mathbf{F}_P \tilde{y} = \sqrt{Q} \mathbf{F}_P \mathbf{H} \mathbf{x} + \mathbf{F}_P \tilde{\mathbf{n}},$$

where $\mathbf{F}_P$ is the normalized $P$-point DFT matrix, and the circulant matrix $\mathbf{H}$ can be diagonalized to obtain

$$\mathbf{D} = \mathbf{F}_P \mathbf{H} \mathbf{F}_P^H / \sqrt{P},$$

This scheme recovers the orthogonality between subcarriers in the frequency domain to allow for a simple one-tap equalization, similar to that for conventional OFDMA systems.

An interesting observation is that (3) actually resembles to the results obtained in precoded OFDMA systems [12], with a precoding matrix $\mathbf{F}_P$. Thus frequency diversity can be achieved without introducing any complexity relating to precoders in the transmitter, and PAPR is reduced as well.

A number of equalization methods can be used to estimate transmitted symbols $\mathbf{x}$ with $\tilde{\mathbf{y}}$, such as zero-forcing (ZF), minimum mean squared error (MMSE) and maximum a posteriori (MAP), etc. Based on the turbo concept, we construct the iterative receiver for Q-OFDMA systems by utilizing above techniques with joint channel estimation, as shown in Fig. 2.

### III. Turbo Equalization with Soft Interference Cancellation

For the $p$th element of $\tilde{\mathbf{y}}$, we rewrite (3) as $\tilde{\mathbf{y}}(p) = (\mathbf{D} \mathbf{F}_P)_p \tilde{\mathbf{x}}(p) + \sum_{k \neq p} (\mathbf{D} \mathbf{F}_P)_p, \tilde{\mathbf{x}}(k) + \mathbf{F}_P \tilde{\mathbf{n}}(p)$.

From (5) we can see the precoding matrix $\mathbf{F}_P$ breaks the orthogonal character of $\mathbf{D}$ and introduces ISI, which can be eliminated by the following turbo equalization.

The equalizer gives the MMSE estimates $\hat{\mathbf{x}}$ of $\tilde{\mathbf{x}}$ based on the received signal $\tilde{\mathbf{y}}$ and the $a$ priori information of $\tilde{\mathbf{x}}$, i.e. $E(\tilde{\mathbf{x}})$ and $\text{Cov}(\tilde{\mathbf{x}}, \tilde{\mathbf{x}})$. After passing through a demapping module, the extrinsic information for each coded bit is delivered as [6]:

$$L^E_E(d_n) = \ln \frac{P(\hat{x}(p) | d_n = 1)}{P(\hat{x}(p) | d_n = 0)}$$

$$= \ln \sum_{\forall d_n = 1} P(\tilde{x}(p) | \mathbf{d}) \prod_{n': n' \neq n} P(d_{n'})$$

$$= \ln \frac{P(d_n = 1 | \tilde{x}(p))}{P(d_n = 0 | \tilde{x}(p))} - \ln \frac{P(d_n = 1)}{P(d_n = 0)},$$

which is a function of $\hat{x}(p)$ and the $a$ priori information about the coded bits other than the $n$-th bit, i.e., $L^D_D(d_n), n' \neq n$, from the previous iteration. For the initial equalization stage, no $a$ priori information is available and hence we have $L^E_E(d_n) = 0, \forall n$. The extrinsic information $L^E_E(d_n)$, which is independent of $L^D_D(d_n)$, is deinterleaved and fed into the decoder as the $a$ priori information for the decoder. Based on the $a$ priori LLR $L^E_E(c_n)$, the decoder provides the $a$ posteriori
LLR of each coded bit as follows:
\[
L_D^t(c_n) = \ln \left( \frac{P\{L_E(c_n) \mid c_n = 1\}}{P\{L_E(c_n) \mid c_n = 0\}} \right)
= \ln \left( \frac{P(c_n = 1 \mid L_E(c_n))}{P(c_n = 0 \mid L_E(c_n))} \right) - \ln \frac{P(c_n = 1)}{P(c_n = 0)},
\]
(9)

At the last iteration, a hard decision is made as
\[
\hat{b}_n = \arg \max_{b \in \{0,1\}} P(b_m = b \mid L_E(c_n))).
\]
(11)

Here, the interleaver/deinterleaver module shuffles coded bits to decorrelate errors introduced by the decoder/equalizer, and assure, locally in several iterations, $d_n$ are independent and $L_D(d_n)$ are true a priori information on the $d_n$, which make the iterative error correction possible.

### A. MMSE Criteria

To perform MMSE estimation, we require the statistics $\tilde{x}(p) \triangleq E[\tilde{x}(p)]$ and $\tilde{v}(p) \triangleq Cov[\tilde{x}(p), \tilde{x}(p)]$ of the symbols $\tilde{x}(p)$, which can be computed by the a priori LLR of the coded bits, $L_D(d_n)$. For simplicity, we assume BPSK modulation is used in the following analysis. The soft estimates and their variance are defined as [6]
\[
\tilde{x}(p) = \tanh \left( \frac{L_D(d_n)}{2} \right),
\]
(12)
\[
\tilde{v}(p) = 1 - |\tilde{x}(p)|^2,
\]
(13)

Define
\[
\tilde{x}^p = (\tilde{x}(1), \cdots, \tilde{x}(p-1), 0, \tilde{x}(p+1), \cdots, \tilde{x}(P))^T,
\]
\[
\tilde{v}^p = \text{Diag}(\tilde{v}(1), \cdots, \tilde{v}(p-1), 1, \tilde{v}(p+1), \cdots, \tilde{v}(P)),
\]
(14)

a soft interference cancellation is performed on $\tilde{y}$ to obtain
\[
\tilde{z} = y - DF_p \tilde{x} + DF_p \tilde{v},
\]
(16)
\[
\tilde{z} = DF_p \tilde{x} + DF_p \tilde{v} + DF_p \tilde{w},
\]
(17)

where the filter $w_p$ is chosen to minimize the MSE between the coded bit $\tilde{x}$ and the filter output $\tilde{z}$, i.e.
\[
w_p = \arg \min E\{\|\tilde{z} - \tilde{x}\|^2\}
= Cov[\tilde{y}, \tilde{y}]^{-1} Cov[\tilde{y}, \tilde{x}]
= \left( \sigma_n^2 I + DF_p \tilde{v}^p (DF_p)^H \right)^{-1} DF_p \tilde{v},
= \left( \sigma_n^2 I + DF_p \tilde{v}^p (DF_p)^H \right)^{-1} DF_p \tilde{v}.
\]
(19)

We apply (16) to (20) and formulate the MMSE estimate as
\[
\tilde{x}(p) = \tilde{x}(p) + w_p^H (y - DF_p \tilde{x})
= w_p^H \left( \tilde{y} - DF_p \tilde{x} + \tilde{z}(p)DF_p \tilde{v} \right),
\]
(21)
whose statistics mean $\mu_\pm(p)$, $\tilde{x} \in \mathbb{B}$ (for BPSK, $\mathbb{B} = \{+1,-1\}$), and variance $\sigma_\pm^2(p)$ are computed as
\[
\mu_\pm(p) = w_p^H \left( E[\tilde{y}(\tilde{x}(p)) + \tilde{v}(p)DF_p \tilde{v}] \right)
= \tilde{x}^p w_p^H DF_p \tilde{v},
\]
(22)
\[
\sigma_\pm^2(p) = E \left[ \left( \tilde{x}(p) - \mu_\pm(p) \right)^2 \right]
= w_p^H \left( DF_p \tilde{v} + \tilde{x}(p)DF_p \tilde{v} \right).
\]
(23)

Thus the output extrinsic LLR $L_E^t(d_n)$ (6) of the equalizer, is given by
\[
L_E^t(d_n) = \ln \left( \frac{P(\tilde{x}(p) \mid d_n = 1)}{P(\tilde{x}(p) \mid d_n = 0)} \right)
= \ln \left( \frac{P(\tilde{x}(p) \mid \tilde{x}(p) = +1)}{P(\tilde{x}(p) \mid \tilde{x}(p) = -1)} \right)
= 2\tilde{x}(p)\mu_{\pm(1)} + \tilde{v}(p) \sigma_{\pm(1)}^2(p)
= 2w_p^H \left( \tilde{y} - DF_p \tilde{x} + \tilde{z}(p)DF_p \tilde{v} \right) \left( 1 - (DF_p \tilde{v})^H w_p \right).
\]
(24)

For the initial iteration, we have $L_D(d_n) = 0, \forall n, \tilde{x}(p) = 0$ and $\tilde{v}(p) = 0 \forall p$, then the MMSE linear equalizer solution is simplified to
\[
w_p = \left( \sigma_n^2 I + DD^H \right)^{-1} DF_p \tilde{v},
\]
(25)

and the corresponding MMSE output and LLR are given by
\[
\tilde{x}(p) = (w_p^H)^H \tilde{y},
\]
(26)
\[
L_E^t(d_n) = \frac{2(w_p^H)^H \tilde{y} \tilde{v}(p)}{1 - (DF_p \tilde{v})^H w_p}.
\]
(27)

For alleviating the high complexity of computing $w_p$ for each iteration, in the first several iterations, we utilize the coefficient matrix $w_p'$ for the first iteration to compute $\tilde{x}(p)$ and $L_E^t(d_n)$ according to (26) and (27).

In the following iterations, approximately perfect a priori LLR $|L_D(d_n)| \to \infty, \forall n$ is available, which leads to $\tilde{x}^p = (\tilde{x}(1), \cdots, \tilde{x}(p-1), 0, \tilde{x}(p+1), \cdots, \tilde{x}(P))^T$, and $\tilde{v}(p) = 0, \forall p$. $w_p$ is then simplified to
\[
w_p'' = \left( \sigma_n^2 I + DF_p \tilde{v} (DF_p)^H \right)^{-1} DF_p \tilde{v},
= \frac{DF_p \tilde{v}}{\sigma_n^2 + (DF_p \tilde{v})^H DF_p \tilde{v} \tilde{v}}.
\]
(28)

### B. Matched Filter Criteria

Analyze (25), we find in the first iteration, channel $D$ which is estimated based on the training sequence, may not be reliable. In order to reduce the complexity, the operator of
matrix inverse can be bypassed by replacing MMSE equalizer with an approximate matched filter as [13]

\[ w_p = \frac{DF_p e_p}{\sigma_n^2 + Tr[DD^T]}, \] (29)

C. Complexity Analysis

Complexity is defined as the number of complex multiplications required in processing each frame. The complexity of the turbo equalizer mainly comes from the MMSE equalizer, MAP decoder and the order of iterations. For each iteration, the MMSE equalizer performs several FFT operations, whose complexity is \( O(P/2\log_2 P) \) for Radix-2 algorithms, and several matrix operations whose complexity is \( O(P^2) \). For the MAP decoder, the complexity of soft output Viterbi algorithm (SOVA) with five iterations is twice as that of Viterbi algorithm, and the ratio becomes three with ten iterations [14].

In [1] we found that larger \( P \) leads to more reduction in complexity of Q-OFDMA without turbo receiver and lower PAPR at the transmitter, and better CFO robustness. Thus in Q-OFDMA systems with turbo receiver, \( P \) should be chosen carefully within system constraints according to the complexity/performance tradeoff.

IV. Turbo Channel Estimation

As a result of (3), channel estimation can be easily implemented by transmitting carefully chosen training symbols \( \bar{x}_T \), such that each element in \( F_p \bar{x}_T \), has unity magnitude. However, the estimation based on training symbols may be not reliable, especially when the channel is time varying and channel tracking needed. In this section, we propose an iterative channel estimation technique in conjunction with data detection. The idea is firstly use training symbols to perform an initial estimation, then the soft data information delivered by decoder will be utilized in estimation. At last iteration, when the decoding information from decoder becomes reliable, advanced estimators, e.g. maximum likelihood or MMSE estimator, are employed to provide further improvement.

From (4) we can see \( DF_p = F_p H \), which is a frequency response of channel. Therefore, we can use \( \tilde{H} = DF_p \) as the channel estimates for Q-OFDMA systems. The channel estimation method is summarized as the following several steps:

1) Initial channel estimation.

\[ (\tilde{H}_{p,p})_1 = \frac{\bar{y}(p)}{\bar{x}_T(p)} = \tilde{H}_{p,p} + \Delta_T(p), \] (30)

where \( \bar{x}_T(p) \) is the training symbols, \( \Delta_T(p) \) is AWGN with zero mean and variance \( (\sigma_n^2 + \sigma_{ISI}^2) \). Once the initial channel estimates are obtained, the detected soft data symbols \( \bar{x} \) are achieved by (12) for BPSK modulation.

2) Iterative channel estimation. In this stage, data-aided LS channel estimation is utilized.

\[ (\tilde{H}_{p,p})_2 = \frac{\bar{y}(p)}{\bar{x}(p)} = \tilde{H}_{p,p} + \Delta(p), \] (31)

Similar to the initial estimation stage, it can be shown that \( \Delta(p) \) has zero mean and variance \( (\sigma_n^2 + \sigma_{ISI}^2) \).

3) Final channel estimation. In the last iteration, the decoding information from decoder becomes very reliable, MMSE estimator [2] is able to provide an further improvement.

V. Simulations

In this section we present the BER performance of Q-OFDMA systems with different receivers, including linear ZF, MMSE and iterative (turbo) receiver. The parameter \( N \) is fixed at 1024, 16 users equally sharing 64 subcarriers in Q-OFDMA systems. System imperfections such as CFO and PAPR distortions are not introduced in the simulation. In each simulation result, BER is averaged over a number of channel realizations. Each user’s data is encoded with 1/2-rate convolutional code, and a rectangle interleaver is applied to the coded bits before modulation. SOVA is used for decoding. The Initial channel coefficients are estimated by matched filter scheme over two consecutive training symbols. Two types of channel models are simulated to compare systems performance. One is the HIPERLAN channel model E from European Telecommunications Standards Institute (ETSI), which is a dense indoor multipath channel model with non-line-of-sight conditions. The other is the SUI3 channel model from IEEE802.16, which is a sparse channel model with only a few taps and small normalized delay spread. In either case, the length of the guarding interval is set to be 64, and channel impulse response longer than 64 is truncated to have 64 taps to avoid ISI.

We depict the simulation results in Fig. 3 for systems with QPSK modulation under HIPERLAN channel model. From the figure we can see that the proposed iterative (turbo) receiver scheme performs better than Q-OFDMA systems with linear ZF and MMSE equalizations. At BER=10\(^{-4}\) level, the Q-OFDMA systems with 2 iterations can achieve it at 17dB SNR, which is about 2dB lower than MMSE equalized Q-OFDMA without iteration process. And Q-OFDMA systems with more iterations get better performance. Fig. 4 shows BER performance for systems with 64-QAM modulation, under SUI3 channel model. Subcarriers have very high correlation due to very limited number of multipath signals. In this case, the influence of frequency diversity is weakened, while the noise propagation is highlighted in Q-OFDMA systems. However, we can see a similar trend, in BER performance of Q-OFDMA systems with different order of iterations, to that of Fig. 3.

VI. Conclusions

In this paper, based on the turbo concept, we proposed an iterative (turbo) equalization in conjunction with channel estimation for Q-OFDMA systems to mitigate the noise enhancement effect and improve the BER performance. In channel estimation, the training symbols and soft decoded data are utilized to track the channel frequency response and suppress the ISI caused by channel estimation errors in Q-OFDMA systems. We can judiciously choose estimation, equalization and decoding algorithms according to the performance/complexity tradeoff. Analysis and simulation results have shown that, the proposed iterative receiver approach
Fig. 3. BER performance comparison between Q-OFDMA systems with different receivers in WLAN channel model, with QPSK modulation.

Fig. 4. BER performance comparison between Q-OFDMA systems with different receivers in Wimax channel model, with 64-QAM modulation.

can improve the Q-OFDMA systems’ BER performance with acceptable complexity.

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