Understanding the Impact of Neighborhood Information on End-to-End Fairness in Multi-hop Wireless Networks

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Abstract—Fair resource allocation for end-to-end flows is an important yet challenging problem in multi-hop wireless networks. Recent research on fair resource allocation mainly focuses on two directions: 1) theoretical methods for joint optimization of scheduling and rate allocation, which are inherently hard to implement; 2) heuristic rate allocation solutions, which are practical for implementation, but simplify the scope of resource sharing regions using different neighborhood models. One key question that remains unanswered is how well these heuristic neighborhood-aware rate allocation solutions approximate the fair point defined in the theoretical optimal resource allocation framework.

To evaluate the impact of different neighborhood models on end-to-end fairness, we first establish a baseline fairness model (i.e., proportional fairness) using the price-based resource allocation framework. In this framework, price represents the cost of the resource usage incurred by unit flow. The rate of a flow is directly linked to its price, which is the aggregated price of the links it traverses. The link price is the sum of the prices of all the resource sharing regions which it belongs to. Obviously, when the resource sharing regions are approximated by different neighborhood models in the heuristic solutions, link prices will manifest as different values. To quantify the deviations of these heuristic neighborhood-aware solutions from the baseline proportional fairness model, we introduce a normalized fairness index model. Six different neighborhood models are evaluated. Our study makes two significant observations: 1) symmetric knowledge on the construction of a neighborhood is important in achieving fairness. Such a knowledge has not been considered in any of the existing neighborhood-aware rate allocation algorithms; 2) while 1-hop neighborhood information brings noticeable gain in fairness, the knowledge of 2-hop neighborhood information does not necessarily bring additional benefit.

Index Terms—End-to-end fairness, price-based resource allocation, multi-hop wireless network

I. INTRODUCTION

A multi-hop wireless network consists of a collection of wireless nodes without a fixed infrastructure. Nodes within the transmission range of each other communicate directly, while nodes that are far away communicate via relays of intermediate nodes. In such a network, each end-to-end flow traverses multiple hops from a source to a destination. It is important to allocate the limited bandwidth resources in multi-hop wireless networks to contending flows, in a way that is both efficient with respect to resource utilization, and fair across contending multi-hop flows.

Resource allocation for end-to-end flows is an extremely challenging problem in multi-hop wireless networks. The key challenge comes from the complicated wireless resource contention model, namely location-dependent contention and spatial channel reuse. Depending on the scheduling algorithm, transmission of unit flow along a link may block the transmissions of different sets of flows, thus virtually uses different amounts of resources. In recent years, significant progress has been made on this topic. Existing works on this problem largely fall into two categories: 1) Theoretical methods for joint optimization of scheduling and rate allocation [4] [9] [13] [16], where the scope of resource sharing regions is rigorously defined by the schedulability of link-level flows. Though these methods provide a theoretically sound optimal and fair resource allocation solution, they can hardly be implemented in a cost-efficient way due to the intrinsic complexity in the optimal multi-hop scheduling; 2) Heuristic rate allocation solutions [14] [10], which simplify the scope of resource sharing regions using different neighborhood models and partially rely on the underlying packet scheduling (e.g., IEEE 802.11) to resolve the resource contention among link-level flows. These works are practical for implementation, but their fairness properties are usually only evaluated on special topologies, as their concept of fairness is not well-defined on general topologies. To date, the following essential question on end-to-end fairness still remains unanswered: how well these practical heuristic neighborhood-aware rate allocation solutions approximate the optimal point defined in the theoretical resource allocation framework.

The objective of this paper is to evaluate the impact of different neighborhood models on end-to-end fairness. Towards this goal, we first establish a baseline fairness model using the optimal resource allocation framework [16]. In this framework, the optimization objective is defined as maximizing the aggregated utility of all flows and the resource constraints are characterized by the necessary condition of schedulability defined using maximal cliques in the contention graph of the wireless network. It is shown [7] that this optimal resource allocation framework can achieve different fairness models, when different utility functions are specified. In this work, we consider the proportional fairness model as our baseline fairness model, which can be achieved under logarithm utility function.

To understand the root cause of unfairness and evaluate different heuristic rate allocation solutions that use various neighborhood models to approximate the resource sharing regions, we consider the price-based model which provides a distributed solution to resource allocation. In the price-based model, price represents the penalty of the resource usage incurred by unit flow (i.e., congestion penalty). The price of a
flow is the aggregated price of the links it traverses. And the link price is the sum of the prices of all the maximal cliques (scope of resource sharing regions), to which it belongs. The rate of a flow is then determined by its price so that its net profit, which is the difference between its utility and the cost its pays, is maximized. Obviously, when the resource sharing regions are approximated by different neighborhood models in the heuristic solutions, link prices will be manifested as different values. We introduce a normalized fairness index to quantify the deviations of these heuristic neighborhood-aware solutions from baseline fairness models,

Six different neighborhood models are evaluated in this paper: 1) clique approximation, which provides clique price estimation for the optimal clique-based solution in IEEE 802.11-style networks via the achievable capacity measurement; 2) asymmetric neighborhood (1-hop and 2-hop, respectively), which approximates the resource sharing region using link-centered neighborhood with different scopes, as in the work of [10]; 3) symmetric neighborhood (1-hop and 2-hop, respectively), which improves the asymmetric neighborhood model by ensuring the maximal neighborhood knowledge symmetrically shared by the links within the neighborhood; and 4) single link, where only the congestion penalty observed on its own link is considered, as in traditional TCP.

This paper makes the following contributions to the field. First, it presents a price-based fairness model and a normalized fairness index model, where heuristic rate allocation solutions with different neighborhood information can be compared within a common framework. With the advance of the wireless communication technology, medium access and routing protocols, the solution space of the fairness problem may continue to evolve, but its nature of fairness resource allocation remains unchanged. This fairness theoretical framework can effectively decouple the “core” of the problem and its other components, so that the basic problem formulation and its solving methodology survive. Second, extensive simulation study is conducted over a variety of carefully designed and random topologies. Our study makes two significant observations: 1) symmetric knowledge on the construction of a neighborhood is important in achieving fairness. Such a knowledge has not been considered in any of the existing neighborhood-aware rate allocation algorithms; 2) while using 1-hop neighborhood information brings noticeable gain in fairness compared with link-only solutions, the knowledge of 2-hop neighborhood information does not bring additional benefit from 1-hop neighborhood information.

The rest of this paper is organized as follows. We first introduce the price-based fairness model in multi-hop wireless networks in Sec. II. Then the neighborhood models to be evaluated and the fairness index model are presented in Sec. III. Finally we present our evaluation results in Sec. IV and conclude the paper in Sec. V.

II. Baseline Fairness Model

A. Network Model

We consider a multi-hop wireless network that consists of a set of nodes \( V \), where only nodes within the transmission range of each other can communicate directly and form a wireless link. We model such a network as a bidirectional graph \( G = (V, E) \), where \( E \subseteq 2^L \) denotes the set of wireless links. In such a network, there exist a set of end-to-end flows, denoted as \( F \). Each flow \( f \in F \) has a rate of \( x_f \). We use \( x = \{x_f, f \in F\} \) to denote the flow rate vector. Flow \( f \) goes through multiple hops in the network, passing a set of wireless links \( E(f) \). A single-hop data transmission in the flow \( f \) along a particular wireless link is referred to as a subflow of \( f \). Obviously, there may exist multiple subflows along the same wireless link. We use the notation \( L(L \subseteq E) \) to represent a set of wireless links in \( G \), such that each of the wireless links in \( L \) carries at least one subflow. A link \( l \in L \) is called an active link. The rate vector of active links is denoted as \( y = (y_l, l \in L) \), where \( y_l = \sum_{f \in E(f)} x_f \).

Flows in the multi-hop wireless network contend for channel resources in a location-dependent manner. Here we consider the protocol model [5], where two subflows contend with each other if either the source or destination of one subflow is within the transmission range of the source or destination of the other. Among a set of mutually contending subflows, only one of them may transmit at a given time. Thus the aggregated rate of all subflows in such a set can not exceed the channel capacity \( C \).

B. Optimal Resource Allocation

We first briefly review the optimal resource allocation framework and its relationship to end-to-end fairness. In Kelly’s classical optimal resource allocation framework [7], each flow \( f \) is associated with a utility, which is a function of its rate \( x_f \). The objective of resource allocation is to maximize the aggregated utility of all flows, subject to the constraint of network resource availability.

The key challenge in applying this resource allocation framework to multi-hop wireless network comes from the modeling of wireless resource sharing unit (region). In wired networks, a single link represents a resource sharing unit, where the capacity of a link can be used to characterize the constraints on flows contending for its bandwidth. However, in the case of multi-hop wireless networks, channel resource is shared in a location dependent way. Thus the capacity of a wireless link is interrelated with other wireless links in its vicinity. As a matter of fact, characterizing the channel resource sharing region is related to the issue of schedulability, i.e., whether rate vector \( y \) is schedulable given the channel capacity and the network topology. It is known [6] that establishing the sufficient and necessary condition of schedulability in a wireless network involves finding the independence number of a graph, which is an NP-hard problem.

Several approximation models are proposed in the existing literature [6], [16], [8], [1]. In this paper, we adopt the maximal
clique approximation model [16] for two of its properties: 1) better approximation factor (compared with the interference set models [8], [11]) and 2) similarity to the neighborhood approximation models which are widely used in the existing rate allocation heuristics [10], [14]. In this clique-based wireless resource allocation framework, the resource sharing regions are characterized by maximal cliques\(^1\) in the wireless link contention graph of the network. Here wireless link contention graph is defined as \( G_c = (V_c, E_c) \), where the vertex set corresponds to the wireless links \( V_c = L \), and there exists an edge between two vertices if the subflows along these two wireless links contend with each other. In a wireless link contention graph, the vertices in a maximal clique represent a maximal resource sharing region, in which at most one subflow may transmit at any given time. Formally, let \( Q \) be the set of all maximal cliques in \( G_c \). For a maximal clique \( q \) in the wireless link contention graph \( G_c \), \( V(q) \subseteq L \) is the set of its vertices. The resource constraint under this clique model is formulated based on the necessary condition of schedulability, as follows:

\[
\forall q \in Q, \sum_{f \in V(q)} y_f \leq C \tag{1}
\]

Let the utility function for an end-to-end flow \( f \in F \) be \( U_f(x_f) \), which represents the degree of satisfaction of its associated end user at rate \( x_f \). This function is increasing, strictly concave and continuously differentiable. The optimization of rate allocation can be formulated into the following nonlinear optimization problem:

\[
\text{P : maximize } \sum_{f \in F} U_f(x_f) \tag{2}
\]

subject to \( R \cdot x \leq C \tag{3} \)

\( x \geq 0 \) \tag{4}

The objective function (2) of the optimization problem is to maximize the aggregated utility of all flows. In inequality (3), the clique-flow matrix \( R = \{R_{qf}\} \) represents the “resource usage pattern” of each flow, where \( R_{qf} = |V(q) \cap E(f)| \) represents the number of subflows that flow \( f \) has in the clique \( q \). Let \( C = (C_q, q \in Q) \) be the vector of achievable channel capacities in each of the cliques. Under ideal scheduling, \( C_q = C \). This optimization constraint characterizes the schedulability condition of wireless channel resource. P achieves Pareto optimality with respect to the resource utilization and end-to-end fairness when appropriate utility functions are specified. In this paper, we use proportional fairness as our baseline fairness model, which can be achieved when the utility function takes the logarithm function form.

\( C. \) Price-based Fairness Model

Though the optimal resource allocation problem \( P \) defines the proportional fairness model, it gives little information on whether fairness can be achieved or approximated under different resource sharing models, and why, if it can not. To analyze the root cause of (un)fairness and evaluate different heuristic end-to-end rate allocation mechanisms that are based on different approximations of resource sharing regions, we examine the dual problem \( D \) of \( P \), given as follows:

\[
D : \min_{\mu \geq 0} D(\mu) \tag{5}
\]

where

\[
D(\mu) = \sum_{f \in F} \max_{q : E(f) \cap V(q) \neq \emptyset}(U_f(x_f) - x_f) \sum_{q : E(f) \cap V(q) \neq \emptyset} \mu_q R_{qf} + \sum_{q \in Q} \mu_q C_q \tag{6}
\]

\( \mu = (\mu_q, q \in Q) \) is a vector of Lagrange multipliers, and may be interpreted as the cost or penalty, of a unit flow accessing the resource sharing region characterized by the maximal clique \( q \). In other words, \( \mu_q \) is the price of clique \( q \). Dual problem \( D \) can be solved via a gradient projection algorithm [16]. This leads to an iterative algorithm that can be executed in a distributed way to achieve the optimal solution of \( P \). In particular, the price \( \mu_q \) at clique \( q \) is adjusted as follows:

\[
\mu_q(t + 1) = [\mu_q(t) - \gamma(C_q - \sum_{f : E(f) \cap V(q) \neq \emptyset} x_f \cdot R_{qf})]^+ \tag{7}
\]

This price update algorithm reflects the resource demand and supply relationship at \( q \). Specially, if the demand \( \sum_f x_f \cdot R_{qf} \) is higher than the supply \( C_q \), the price will increase; otherwise, the price will decrease. When the resource demand matches the capacity, the system enters the equilibrium state. The flow rate for \( f \) can be adjusted at the source of the flow so that its net benefit (difference between utility and cost) is maximized:

\[
\text{maximize } U_f(x_f) - \lambda_f x_f \tag{8}
\]

\( \text{Fig. 1. Wireless Flow Price Model} \)

Here \( \lambda_f \) is the price of a flow \( f \) and can be interpreted in the following two alternative ways, as show in Fig. 1.

\[
\lambda_f = \sum_{q : E(f) \cap V(q) \neq \emptyset} R_{qf} \mu_q \tag{9}
\]

\[
= \sum_{t \in E(f)} \mu_t = \sum_{t \in E(f)} \sum_{q \in V(q)} \mu_q \tag{10}
\]

In Eq. (9), flow \( f \) needs to pay for all the resource sharing regions (i.e., maximal cliques in our baseline model) it uses. For each maximal clique, the cost is the product of the number

\(^1\)In a graph, a complete subgraph is referred to as a clique. A maximal clique is defined as a clique that is not contained in any other cliques.
of wireless links that \( f \) traverses in this region and its price. In the second representation (10), flow price is the aggregated price of all wireless links it passes. For each wireless link, its price is the aggregated price of all the regions that it belongs to. This pricing model reflects the fundamental concept of fairness in a wireless network. Essentially, if two subflows are within the same resource sharing region, they should share the same price as the cost of using this region. If a link belongs to multiple regions, all the region prices should be aggregated into the link price. As the transmission along this link would influence all the links in these regions, it should receive the penalty from all. Obviously, when the resource sharing regions are approximated by different neighborhood models in heuristic solutions [14], [10], link prices will be manifested as different values, thus affect the fairness properties of end-to-end flow rate allocation.

### III. Evaluation Metrics

#### A. Neighborhood Approximation Models

In our baseline fair resource allocation model (which we denote as OPT), a maximal clique is regarded as an independent resource sharing region with capacity \( C \). While theoretically sound, this model has two limitations that need to be addressed before it can be applied as a practical solution of wireless rate allocation. First, Eq. (1) only gives an upper bound on the rate allocations to the wireless links. In practice, however, such a bound may not be tight, especially with carrier-sensing-multiple-access-based wireless networks (such as IEEE 802.11). In this case, the achievable channel capacity \( C_q \) needs to be estimated at each region \( q \). Second, calculating the price of a maximal clique involves information exchange within the 3-hop neighborhood [16], which is inevitably expensive. To limit the communication overhead, different neighborhood models can be used to approximate the maximal clique construction. The goal of this paper is to examine the impact of various approximation models on fairness. In particular, we consider the following approximation models.

1) **Clique approximation (Cliq)**: This approximation model adopted the same definition of resource sharing region (i.e., maximal clique) as the baseline model (OPT). The difference of this model from the baseline model is that it does not assume an ideal scheduling algorithm which works with the rate allocation to jointly optimize the resource allocation. Instead, it performs online estimation of the region capacity \( C_q \) under the IEEE 802.11 protocol. In particular, it uses the approach presented in [11] to measure the achievable bandwidth \( C_l \) of each wireless link \( l \) based on its historical data transmission results and aggregates the available bandwidth within the region as the region capacity, i.e., \( C_q = \sum_{l \in V(q)} C_l \).

2) **Asymmetric neighborhood – 1-hop (NB1a) and 2-hop (NB2a)**: In this model, a resource sharing region is centered at a wireless link and includes its \( n \)-hop neighboring links \((n = 1 \text{ or } 2)\). Fig. 2 shows three 1-hop asymmetric neighborhood regions, each centered at one link. This model aligns well with the existing heuristic neighborhood-aware rate allocation solutions [14], [10].

3) **Symmetric neighborhood – 1-hop (NB1s) and 2-hop (NB2s)**: This model improves the asymmetric neighborhood model by ensuring that the maximal neighborhood knowledge is symmetrically shared by the links within the neighborhood. Specifically, each link will generate its own neighborhood as a neighborhood candidate. If the neighborhood is contained by another neighborhood, it will adopt the larger neighborhood. Fig. 2 also illustrates the 1-hop symmetric neighborhood model. As shown in the figure, different from the asymmetric neighborhood model \((NB1a)\) which has three neighborhoods, the symmetric neighborhood model only has one large neighborhood, which overshadows the other two smaller ones.

4) **Link only (MAC)**: In this approach, a link only considers itself as a resource sharing region. It relies on the underlying (possible suboptimal) MAC scheduling protocol (i.e., IEEE 802.11 in our study) to resolve the resource contention among wireless links. This model well characterizes the TCP rate control mechanism.

In all the above neighborhood approximation models \((NB1a, NB2a, NB1s, NB2s, MAC)\), the neighborhood capacity \( C_n \) is estimated using the same method as Cliq. The rate allocation methods of these approximation models follow the similar price-based approach as in the baseline model. Specifically, let \( \mu_n \) be the neighborhood price, it is updated based on the following equation

\[
\mu_n(t+1) = [\mu_n(t) - \gamma(C_n - \sum_{f \in E(f) \cap V(n) \neq \emptyset} x_f \cdot R_{n,f})]^+
\]

(11)

where \( R_{n,f} \) is the number of subflows of \( f \) in the neighborhood \( n \). The link price \( \mu_l = \sum_{f \in V(n)} \mu_n \) is the aggregated price of all the neighborhoods it belongs to.

#### B. Normalized Fairness Index Model

To compare the fairness achieved by the above methods and evaluate the impact of neighborhood information, we introduce the following fairness index model. Let the allocated rate vector in the baseline fairness model be \( \boldsymbol{x}^* = \{x_f^* | f \in F \} \), and the rate vector of the neighborhood model to be evaluated as \( \boldsymbol{x} = \{x_f | f \in F \} \). The normalized flow rate vector is defined as \( \tilde{x} = \{\tilde{x}_f, f \in F\} \), where \( \tilde{x}_f = \frac{x_f}{\sum_{f \in F} x_f} \). The rate-based fairness index for \( \boldsymbol{x} \) is then defined as follows:

\[
F(x) = \frac{\left(\sum_{f \in F} \tilde{x}_f\right)^2}{|F| \sum_{f \in F} \tilde{x}_f^2}
\]

(12)
where $|F|$ is the number of flows in $F$. This fairness index is bounded between 0 and 1. The higher the fairness index is, the better a rate allocation achieves fairness.

C. Discussion

Two fairness models are often used in network resource allocation, namely max-min fairness [2] and proportional fairness [7]. Existing works on fair wireless rate allocation [15][10] usually use the max-min fairness model to evaluate their solutions. In this paper, we adopt the proportional fairness model where region prices and link prices are used as the basis of fairness definition. This approach reflects the recent work on fairness [3], which states that comparing flow rates should not be used for fairness indexing in production networks. Instead, the fairness should be determined by how flows share out the cost in the network. Our fairness model is also easier to be generalized to handle the different neighborhood models.

It is also important to note that our approximation models provide an abstraction for the heuristic neighborhood-aware rate allocation solutions. They may not fully characterize the details of individual solutions. For example, the work of [14] actually uses a node-centered neighborhood model, where the neighborhood region includes the node itself and the nodes which can interfere with this node’s signals. This creates asymmetry in the NB1a and NB2a models, as the sender and receiver nodes may have different neighborhood regions. The implementation details are also abstracted out in our model. For example, to calculate the region prices (congestion signals), different solutions have applied different methods, including using distributed queue lengths, channel conditions, etc., and delivered the prices using the active queue management mechanism. The relationship along price-based resource allocation, active queue management, and congestion control has been extensively discussed in [12]. Our approximation models abstract out these details so that they will not introduce unnecessary noise to our study and thus allow us to focus on evaluating the impact of neighborhood information.

IV. Evaluation Results

A. Simulation Setup

We implement the approximation models in ns-2. Unless explicitly mentioned, all the experiments use the following settings. RTS/CTS is enabled in the IEEE 802.11. The bandwidth of the channel is 1Mbps, and its propagation model is the two-ray ground reflection model. The transmission range and interference range are both 250m.

B. Special Topologies

We first experiment on a set of special topologies as in Fig. 3.

Fig. 4 shows the price and the rate convergence of all the approximation models for scenario 1. As shown in Fig. 1, $l_2$ and $l_3$ are within the same clique sets, so ideally their prices should converge to the same value as in the optimal approach. In this scenario, the transmissions of $f_1$ and $f_3$, or $f_2$ and $f_4$ interfere with each other, while $f_3$ and $f_4$ tend to have much larger rates. This dues to the well known exposed terminal problem, which leads to the asymmetric estimation results of the channel bandwidth. As a result, the allocated rates of these flows converge to different values.

Fig. 5 shows the normalized flow rates (over baseline rates), link prices and the normalized link prices (over baseline prices) for scenario 1. The link prices of the MAC approach directly reflect the scheduling of the MAC protocol. It is severely affected by the exposed terminal problem and shows the most asymmetric rates as in the results.

The clique-based approach is also affected by the exposed terminal problem in scenario 1. Flows $f_1$ and $f_4$ have the largest rate and smallest rate separately in Fig. 5. However, with symmetric distribution of flows as in scenario 2, the clique-based approach can exactly follow the optimal approach. Overall, the clique-based is a stable method that always performs better than the MAC approach, and most times better than NB1s and NB2s. As it is a theoretical framework, its implementation can be complicated compared with other approaches.

In an asymmetric topology, usually the Cliq approach performs the best, as it follows the same concept as the optimal solution. The fairness index of NB2s closely follows the Cliq approach, with NB1s after, and the MAC is the most unfair one.

In the NB1a approach as in scenario 1, the neighborhood of $l_2$ is $(l_1,l_2,l_3)$. But $l_1$ has the neighborhood of $(l_1,l_2)$ without knowing $l_3$, so its neighborhood is contained by the neighborhood of link $l_2$. The link price with a covered neighborhood goes to extremes of either very small eg. $l_4$ or large price eg. $l_1$, as shown in Fig. 5. $l_1$ always tries to balance with $l_2$, that is, if flow rate of $l_1$ is much higher than the link capacity, $l_2$ will silently take the penalty; if flow rate of $f_1$ is smaller than the link capacity, $f_2$ can utilize it without notifying $f_1$. So in a 2-hop connection, the links viewing the larger neighborhood will over-use the channel if available capacity is sensed, or get over-punished when the demand in the neighborhood is high. NB2a also has the problem. Affected
by the covering problems of the neighborhood information, NB1a and NB2a cannot provide reasonable results. Thus in the following scenarios, the results of NB1a and NB2a are no longer presented.

The NB1s performs the same as the Cliq approach if all the flows form a connected graph. Considering a graph constructed by all the links that have flows in the topology, we call it a flow graph. Assuming undirected links, in the graph, if any node can find a route from one node to any other node, it becomes a connected flow graph. As links can find their 2-hop neighbors via the 1-hop neighbors, the NB1s approach performs the same as the Cliq approach as in Fig. 5.

NB1s considers information sharing between directly connected neighbors. However, it does not take into account the contention between the unattached links which are still within the interference range of each other. So the 1-hop neighborhood does not carry its full responsibility of the contention. Scenario 2 is an isolated flow graph. In such a graph, all flows do not share any links or nodes, but they may still contend for channels when in the interference range of each other. When all the flows are unattached from other flows, NB1s performs the same as the MAC approach as shown in Fig. 6.

NB2s has a much larger neighborhood compared with other approaches, so it always tends to make the prices of groups of links equal. Especially in a small dense region where almost all the flows contend with each other, NB2s can sometimes outperform the Cliq approach. As in Fig. 5 and Fig. 6, NB2s acts in an egalitarian manner when other approaches have varying flow rates and link prices, because all the flows in scenario 1 and 3 are considered in the same neighborhood as in NB2s.

In scenario 3 when too many exposed terminal pairs exist, none of the approaches really help, and the flow rates diverge from the optimal values. The simulation results are presented in Fig. 7.

Table I displays the fairness indices of all the approaches.

### C. Grid and Random Topologies

Grid and Random topologies are studied in this subsection. First, a 5 × 5 grid topology is simulated. First 20 per-hop horizontal flows are deployed on the topology and then 5 horizontally paralleled multi-hop flows are tested.

Simulations show that in the per-hop flow scenario, the NB2s performs much better with fairness index of 0.838, compared with the Cliq, NB1s and MAC with fairness indices of 0.650, 0.647, 0.600. In such a scenario when flows are independent in the sense of originating from different source nodes, NB2s even the link prices in the largest region and tries to make the short flows coordinate with each other.

In the multi-hop scenario, the fairness indices of NB2s, Cliq, NB1s and MAC become 0.770, 0.762, 0.784, 0.722. NB1s turns to be the relatively best approach, followed by Cliq, NB2s and MAC. In the NB1s approach, the clique-concept within a flow is correctly used, so the flow prices are correctly constrained along each path. In addition, flows do not consider the information from other flows, which in turn is beneficial in a symmetric topology. In this scenario, though NB2s tries to equalize prices of different links, a flow price may not follow as it is determined by all the link prices along the path.

We further study the fairness level of the heuristic approaches in random topologies. 25 nodes are deployed over a 1000 × 1000 area. 2 sets of node topologies are randomly generated. 7 different sets of 15 or 25 per-hop flows are established in the topologies as in Fig. 8. Topo 1 – 3 use the first topology, and topo 4 – 7 use the second topology. The rate fairness indices are shown in Table II. The dense topologies have smaller fairness indices compared with the sparse ones.
15 flows in topo 1 mainly form three separated groups. NBCs model performs the best in this scenario. In the NBCs model, flows attached with other flows can correctly find cliques via its neighbors. In the top group, small regions composed of connected links symmetrically interfere with one another. The other two groups at the bottom distribute their own resources without contending with each other. The NB2s model performs the worst in this scenario, as it evenly distributes the resources in each isolated group.

Comparing topo 2 with topo 1, the NB2s model has the same neighborhood as in topo 1, so its fairness index does not get better. The NBCs model no longer performs well in topo 2, as the flows become more unattached and asymmetric in the top region.

Topo 3 has 10 more flows than topo 1 and 2. With increased number of flows in the top isolated region, NB2s model outperforms all the other approaches, especially when the top isolated region becomes symmetric centered by the node in the middle. The NBCs model performs the second best in this scenario.

Topo 4-7 are implemented on the other node topology, and the simulation results are consistent with the first three topologies. Topo 7 is quite symmetric with two unattached flows on the two sides and connected flows in the middle. The fairness indices of both NBCs and NB2s are larger than the MAC model. Note the Cliq model almost always has the largest fairness index.

V. Conclusion

This paper aims at understanding the gap between heuristic fair rate allocation solutions and the optimal solution. It characterizes the heuristic solutions using neighborhood models to differentiate the shared resource regions. Then a price-based fairness model is established where the impact of neighborhood information on end-to-end flow fairness can be evaluated on a common framework. The simulation-based study has revealed several important properties, including the importance of symmetric knowledge on the construction of a neighborhood and the limitation of 2-hop neighborhood information. We believe that these properties discovered in this paper have significant implications to future fair rate allocations in multi-hop wireless networks.

References

Fig. 8. Random Topologies

Fig. 4. Instantaneous Price and Rate of Scenario 1


