MIMO Maximum Likelihood Soft Demodulation Based on Dimension Reduction

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Introduction

- Spatial multiplexing multi-input multi-output (MIMO) systems
  - Can increase the data rate linearly with the number of antennas.
  - Need to employ a receiver that effectively handles the interference among the multiple spatial streams.

- Receivers for spatial multiplexing MIMO
  - Hard detectors
    - Equalizers: linear equalizer (LE) and decision feedback equalizer (DFE)
    - Maximum likelihood (ML) hard detectors
  - Soft demodulators
    - Soft demodulators using an equalizer output
    - MIMO ML soft demodulators: soft version of MIMO ML hard detectors

- We present a novel approach for MIMO ML soft demodulation.
  - In practical wireless systems, soft demodulators are almost exclusively used.
  - MIMO ML soft demodulation produces best performance in general.
System Model (1)

- Spatial multiplexing MIMO transmitter and receiver
  - $N_T$ transmit antennas
  - $N_R$ receive antennas
  - $N_S$ spatial streams with $N_S \leq \min\{N_R, N_T\}$
System Model (2)

- Receive signal model
  \[ y = H x + z \]
  - \( y \in \mathbb{C}^{N_R \times 1} \): receive signal vector
  - \( H \in \mathbb{C}^{N_R \times N_S} \): effective channel matrix including MIMO precoding
    - Known at the receiver.
  - \( x \in \mathbb{C}^{N_S \times 1} \): transmit symbol vector
  - \( z \in \mathbb{C}^{N_R \times 1} \): circularly symmetric complex Gaussian random noise vector

  \[ f(z) = \frac{1}{\pi^{N_R}} \exp(-||z||^2) \]
Problem Formulation- MIMO ML Hard Detection

- **MIMO ML hard detection**
  - Finding the transmit symbol vector that is most likely transmitted:
    \[
    \hat{x} = \arg \max_{x \in X} f(y|x)
    \]

- **X**: set of all possible \(M^{NS}\) transmit symbol vector candidates with the modulation order of \(M\)

- \(f(y|x) = \frac{1}{\pi NR} \exp \left(-||y - Hx||^2\right)\)

- Equivalent to finding the transmit symbol vector that minimizes the Euclidean distance (ED) \(||y - Hx||^2\) between \(y\) and \(Hx\):
  \[
  \hat{x} = \arg \min_{x \in X} ||y - Hx||^2
  \]
Problem Formulation- MIMO ML Soft Demodulation

- MIMO ML soft demodulation
  - Calculating log likelihood ratio (LLR) of each coded bit $b_{s,n}$, the $n$-th bit of the $s$-th stream:

  $$L(b_{s,n}) = \log \left( \frac{P\{y|b_{s,n} = 1\}}{P\{y|b_{s,n} = 0\}} \right)$$

  - Almost the same as calculating approximate LLR derived using Max-Log-MAP approximation:

    $$\tilde{L}(b_{s,n}) \triangleq \min_{x \in X_{s,n}^{(0)}} \|y - Hx\|^2 - \min_{x \in X_{s,n}^{(1)}} \|y - Hx\|^2$$

- $X_{s,n}^{(b)}$: set of transmit symbol vector candidates with $b_{s,n}=b$
Problem Formulation - Hard vs Soft

- MIMO ML hard detection

\[ \hat{x} = \arg \min_{x \in X} \| y - Hx \|^2 \]

- There exist low-complexity algorithms that try to find the optimum \( x \) without calculating all the EDs for all transmit symbol vectors.
  - Sphere decoding, M-algorithm, K-best, etc.

- MIMO ML soft demodulation

\[ \tilde{L}(b_{s,n}) \triangleq \min_{x \in X_{s,n}^{(0)}} \| y - Hx \|^2 - \min_{x \in X_{s,n}^{(1)}} \| y - Hx \|^2 \]

- Looks similar to the hard detection problem.
- Much more difficult in reality.
  - The search space for the transmit symbol vector with the minimum ED is limited to \( X_{s,n}^{(b)} \).
  - Partitioning transmit symbol vector candidates into \( X_{s,n}^{(0)} \) and \( X_{s,n}^{(1)} \) is all different depending on \( s \) and \( n \).
Dimension Reduction Approach (1)

- We propose a dimension reduction approach for soft demodulation.
  - Central idea: reduce the dimension for soft demodulation.

- Assume that we are interested in calculating the LLRs only for the last $N_S^{so}$ streams and we don’t care about the first $N_S^{ha}$ streams, where $N_S^{ha} + N_S^{so} = N_S$.

- Partition the transmit symbol vector and the channel matrix:

$$
y = \begin{bmatrix} H^{ha} & H^{so} \end{bmatrix} \begin{bmatrix} x^{ha} \\ x^{so} \end{bmatrix} + z$$

$$= H^{ha} x^{ha} + H^{so} x^{so} + z.$$

- $x^{so} \in C^{N_S^{so} \times 1}$ and $x^{ha} \in C^{N_S^{ha} \times 1}$.
- $H^{so} \in C^{N_R \times N_S^{so}}$ and $H^{ha} \in C^{N_R \times N_S^{ha}}$. 
Dimension Reduction Approach (2)

- LLR for the \(n\)-th bit of the \(s\)-th stream for \(N_S - N^{so}_S + 1 \leq s \leq N_S\):

\[
\tilde{L}(b_{s,n}) = \min_{x \in X^{(0)}_{s,n}} \|y - Hx\|_2^2 - \min_{x \in X^{(1)}_{s,n}} \|y - Hx\|_2^2
\]

\[
= \min_{x^{so} \in X^{so,(0)}_{s,n}} \left( \min_{x^{ha} \in X^{ha}} \|y^{ha}(x^{so}) - H^{ha}x^{ha}\|_2^2 \right)
\]

\[
- \min_{x^{so} \in X^{so,(1)}_{s,n}} \left( \min_{x^{ha} \in X^{ha}} \|y^{ha}(x^{so}) - H^{ha}x^{ha}\|_2^2 \right).
\]

- \(y^{ha}(x^{so}) \triangleq y - H^{so}x^{so}\)

- \(X^{(b)}_{s,n} = \left\{ \begin{bmatrix} x^{ha} \\ x^{so} \end{bmatrix} \mid x^{ha} \in X^{ha}, x^{so} \in X^{so,(b)}_{s,n} \right\} \)
Dimension Reduction Approach (3)

- Dimension reduction soft demodulation for the last $N_{S}^{so}$ streams

1. For each $x^{so}$, use an efficient MIMO ML hard detector to find the transmit symbol subvector $\hat{x}^{ha}(x^{so})$ with the minimum ED without actually calculating all EDs:

$$\hat{x}^{ha}(x^{so}) = \arg\min_{x^{ha} \in X^{ha}} \| y^{ha}(x^{so}) - H^{ha} x^{ha} \|^2.$$ 

2. Calculate the corresponding minimum ED:

$$D(x^{so}) = \| y^{ha}(x^{so}) - H^{ha} \hat{x}^{ha}(x^{so}) \|^2.$$ 

3. After calculating the minimum EDs for all $x^{so}$, calculate the LLR:

$$\tilde{L}(b_{s,n}) = \min_{x^{so} \in X^{so,(0)}_{s,n}} D(x^{so}) - \min_{x^{so} \in X^{so,(1)}_{s,n}} D(x^{so}).$$

- LLR for other than the last $N_{S}^{so}$ streams can be calculated by rearranging the transmit symbol vector and solving the same problem.
Complexity Analysis

- **Complexity measure**
  - Average number of visited nodes in a tree search

- **Complexity comparison**

<table>
<thead>
<tr>
<th>$N_S$</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimension reduction</td>
<td>406 ($N_S^{so}=1$)</td>
<td>6,344 ($N_S^{so}=1$)</td>
</tr>
<tr>
<td></td>
<td>2,098 ($N_S^{so}=2$)</td>
<td>29,618 ($N_S^{so}=2$)</td>
</tr>
<tr>
<td></td>
<td>8,465 ($N_S^{so}=3$)</td>
<td>1.34×10⁶ ($N_S^{so}=4$)</td>
</tr>
<tr>
<td>Exhaustive</td>
<td>69,905</td>
<td>4.58×10⁹</td>
</tr>
</tbody>
</table>

- $M = 16$
- Hard detector used for dimension reduction approach: sphere decoder

- To reduce the complexity further, a suboptimal hard detector can be used instead of an optimal hard detector.
Simulation Results (1)

- Simulation Parameters
  - General setting: WiMAX (IEEE 802.16) radio conformance test
  - Convolutional turbo code with code rate of 1/2
  - Vehicular-A 60 km/h
  - High antenna correlation model with 4 transmit and 4 receive antennas
  - 4 spatial streams
• $N_{\mathcal{S}}^{\text{so}} = 1$ for dimension reduction soft demodulation (DRSD) (Resulting in the lowest complexity.)

• 16 QAM for all streams
Conclusions

- We proposed a dimension reduction soft demodulator (DRSD).
  - A DRSD reduces the dimension of the search space for soft demodulation.
  - A DRSD relies on a hard detector for the rest of the dimension.

- A DRSD with an optimal hard detector significantly reduces complexity compared to the exhaustive ML method.

- A DRSD with a suboptimal hard detector achieves further complexity reduction with little performance degradation.
Appendix- Additional Simulation Results (1)

- $N_S = 2$, $N_{So} = 1$, and 64 QAM for all streams
Appendix- Additional Simulation Results (2)

- $N_S = 4$, $N_S^{so} = 1$, and 4 QAM for all streams