The role of long memory in hedging effectiveness

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Abstract

A joint fractionally integrated, error-correction and multivariate GARCH (FIEC-BEKK) approach is applied to investigate hedging effectiveness using daily data 1995–2005. The findings reveal the proxied error-correction term has a long memory component that theoretically should affect hedging effectiveness. When the FIEC model empirical conditions are satisfied, the FIEC-BEKK hedging strategy outperforms the OLS benchmark out of sample in terms of both variance reduction and hedger utility. A bootstrap exercise indicates that the variance reduction is statistically significant.

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1. Introduction

Several distinct approaches have been developed to estimate the optimal hedge ratio (OHR), also known as the minimum-variance hedge ratio. The first or benchmark approach regresses the spot return on the futures return using ordinary least squares (OLS) and the estimated OHR is given by the slope coefficient. A second approach is based on the error-correction (EC) model while a third uses the generalised autoregressive conditional heteroscedasticity (GARCH) model to estimate time-varying OHRs. There has been extensive debate on which model generates the best hedging performance (see inter alios, Baillie and Myers, 1991; Ghosh, 1993; Park and Switzer, 1995; Kavussanos and Nomikos, 2000; Harris and Shen, 2003; Yang and Allen, 2004). Notably, recent studies such as those of Lien et al. (2002) and Moosa (2003) find that the basic OLS approach clearly dominates the alternatives.

The above approaches to estimating the OHR assume that the basis (also called the futures premium)—defined as the difference between contemporaneous futures and spot prices—is stationary or \( I(0) \). However, recent studies have found that the stock index basis may contain a long memory component or that it is integrated of order \( d \), where \( 0 < d < 1 \). In this context, Lien and Tse (1999) show theoretically that this will affect the OHR and empirically investigate this for the Nikkei Stock index futures. To our knowledge, the role of basis long memory has not been investigated over a range of assets and thus it is difficult to assess its relevance for hedging effectiveness.

The motivation of the paper is to explore the relationship between basis long memory and hedging effectiveness measures, employing spot daily data for eight assets (five commodities, a stock market index and two exchange rates) and their corresponding futures contracts 1995–2005. The paper contributes to the existing literature in several ways. First, we establish the novel finding that the basis is a fractionally integrated process and that this is a stylised fact across both commodities and other asset classes. This finding is robust since the results from both parametric and
semi-parametric tests yield the same conclusion. One implication is that the Granger (1986) fractionally integrated error-correction (FIEC) model may well be more appropriate for estimating the OHs.

Second, we combine a FIEC model with the BEKK model proposed by Engle and Kroner (1995) to allow for both long memory and time-varying conditional heteroscedasticity in estimating the OH. An out-of-sample hedging comparison exercise indicates that a modified FIEC hedging strategy can outperform the OLS benchmark in terms of risk reduction. This runs contrary to the extant literature in which the OLS benchmark is rarely beaten. However, our result is conditional on both the spot and futures returns being dependent on the fractional cointegration variable which applies for just two of our sample assets, Gold and the S/Pound exchange rate.

Third, we implement out-of-sample tests for both the statistical and the economic significance of differences between competing hedging approaches while the extant literature largely fails to do this. We use the hedger’s utility function to evaluate the economic benefits of hedging and this shows that the FIEC-BEKK approach is superior to the OLS approach for hedging the Gold and S/Pound contracts. Finally, a wild-bootstrap technique confirms the statistical superiority of the variance reduction achieved by the FIEC-BEKK over the OLS methodology for these two contracts. This is an important result given the widespread use of both Gold and the S/Pound as hedging instruments.

The rest of the paper is organised as follows. Section 2 presents the empirical methodology. Section 3 describes the data and analyses of our empirical results. Section 4 evaluates relative hedging effectiveness while a final section concludes.

2. Methodology

2.1. Fractional integration

The autoregressive fractionally integrated moving average (ARFIMA) model by Granger and Joyeux (1980) and Hosking (1981) allows the modelling of persistence or long memory where $0 < d < 1$ and $d$ is a differencing or memory parameter. A time series $y_t$ follows an ARFIMA $(p, d, q)$ process if

$$\phi(L)(1 - L)^d y_t = \mu + \Theta(L)\varepsilon_t, \quad \varepsilon_t \sim iid(0, \sigma^2), \quad (1)$$

where $\phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$ and $\Theta(L) = 1 - \theta_1 L - \cdots - \theta_q L^q$. For $0 < d < \frac{1}{2}$, the process remains stationary but contains long memory so that shocks disappear hyperbolically rather than geometrically. Contrastingly, for $\frac{1}{2} < d < 1$, the relevant series is non-stationary, the unconditional variance grows at a more gradual rate than when $d = 1$ but is mean reverting.

The memory parameter $d$ can be estimated by several approaches. The most widely used technique is the log-periodogram (hereafter the GPH statistic) estimator because of its semi-parametric nature (Geweke and Porter-Hudak, 1983; Robinson, 1995a). This requires only weak assumptions on the short-memory process $\varepsilon_t$ in Eq. (1). There are a number of other semi-parametric estimators including the Gaussian semi-parametric (GSP) estimator (see Robinson, 1995b; Velasco, 1999b) extensively studied in Robinson and Henry (1999). Recently, Arteche (2006) has also proposed an additional log-periodogram regression based estimator that achieves a reduction of the asymptotic bias and a faster convergence because a larger bandwidth is permitted. However, we employ the GPH technique because of its wide application in the literature. Additionally, we check the robustness of our results by applying a parametric estimator. Geweke and Porter-Hudak demonstrate that for frequencies near zero, $d$ can be consistently estimated from the least squares regression:

$$\log(I(\lambda_j)) = \beta_0 - d \log(4 \sin^2(\lambda_j/2)) + \varepsilon_j, \quad j = l + 1, l + 2, \ldots, m, \quad (2)$$

where $I(\lambda_j)$ is the sample spectral density of $y_t$ evaluated at the frequencies $\lambda_j = 2\pi j/T$, $T$ is the number of observations and $m$ is small compared to $T$. One of the advantages of the GPH technique is that hypothesis about the least-squares estimate of $d$ can be tested using standard $t$-statistics (Pynnönen and Knif, 1998; Hassler et al., 2006). For the stationary range, $-\frac{1}{2} < d < \frac{1}{2}$, Robinson (1995a) shows that the GPH estimate is consistent and asymptotically normally distributed. Moreover, Velasco (1999a) demonstrates that the estimate of $d$ is consistent for $\frac{1}{2} < d < 2$ and asymptotically normally distributed for $\frac{1}{2} < d < \frac{7}{4}$ when the data are differenced. It is noted that there is the possibility of finite sample bias in the GPH estimator due to strongly autoregressive short memory (see, Choi and Zivot, 2007). For comparative purposes, the parametric ARFIMA $(p, d, q)$ models computed by exact maximum likelihood are also
considered. Concerning this method, the first differenced series is applied to satisfy the stationary/invertibility condition \(-0.5 < d < 0.5\) and the resulting estimate of \(d\) is then adjusted by 1.

2.2. Estimation of the OHR

Let \(\Delta s_t\) and \(\Delta f_t\) represent the log returns on spot and futures contracts between period \(t\) and \(t-1\). Then the OHR is the ratio of the covariance between spot and futures price changes to the variance of futures price changes and is given by

\[
b^* = \frac{Cov(\Delta s_t, \Delta f_t)}{Var(\Delta f_t)}. \tag{3}
\]

The conventional approach to estimating (3) employs the OLS regression technique. Specifically, the relevant regression can be written as

\[
\Delta s_t = \alpha + \beta \Delta f_t + \epsilon_t, \tag{4}
\]

where the estimate of the OHR is given by the estimate of the slope coefficient \(\beta\). However, recent studies have found long memory behaviour in the basis for some asset classes. In other words, there may be a ‘fractional cointegration’ relationship between the spot and futures prices. In this case, replacing \(\epsilon_t\) with \(z_t\) in Eq. (1), we obtain

\[
\Phi(L)(1 - L)^d z_t = \mu + \Theta(L)\epsilon_t, \quad \epsilon_t \sim iid(0, \sigma^2). \tag{5}
\]

Following Lien and Tse (1999), the FIEC model can be written as

\[
\Delta s_t = \alpha_s + \sum_{i=1}^m \beta_{si} \Delta s_{t-i} + \sum_{j=1}^n \gamma_{sj} \Delta f_{t-j} + \zeta_s [(1 - L)^d - (1 - L)] z_t + \epsilon_{st}, \tag{5a}
\]

\[
\Delta f_t = \alpha_f + \sum_{i=1}^m \beta_{fi} \Delta s_{t-i} + \sum_{j=1}^n \gamma_{jf} \Delta f_{t-j} + \zeta_f [(1 - L)^d - (1 - L)] z_t + \epsilon_{ft}. \tag{5b}
\]

where \(z_t\) is proxied by the basis, i.e., \(z_t = f_t - s_t\) (see Lien and Tse, 2002). Clearly, the OHR \((b^*)\) can be estimated from the residuals of (5) and (6)

\[
b^* = \frac{Cov(\epsilon_{st}, \epsilon_{ft})}{Var(\epsilon_{ft})}. \tag{7}
\]

To estimate a time-varying hedge ratio, the BEKK framework of Engle and Kroner (1995) is employed. The extant literature commonly employs the diagonal GARCH and constant correlation (CC) GARCH. However, these give no guarantee of a positive semi-definite covariance matrix and the CC assumption fails to hold for most financial time series (Moschini and Myers, 2002; Yang and Allen, 2004). The BEKK model overcomes both issues. This specification can be expressed as follows:

\[
H_t = W'W + A' H_{t-1} A + B' \epsilon_{t-1} \epsilon_{t-1}' B, \tag{8a}
\]

\[
\epsilon_t | \Omega_{t-1} \sim N(0, H_t), \tag{8b}
\]

where \(W, A, B\) are 2 \times 2 matrices of parameters, \(\epsilon_t = [\epsilon_1, \epsilon_2]\), \(\Omega_{t-1}\) is the conditioning information set at time \(t-1\) and \(H_t = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}\) is the conditional covariance matrix. Moreover, we combine the FIEC model with the BEKK specification. The FIEC-BEKK model is therefore used to estimate the dynamic time-varying hedge ratios

\[
b^* = \frac{\sigma_{sf,t}}{\sigma_{ff,t}}. \tag{9}
\]

This is the one-day hedge ratio. To calculate the hedge ratio over multiple days, we use iterations of Eq. (8a).
3. Data and empirical analysis

Data on eight different futures contracts were collected to undertake a comparative analysis. The contracts are for soybeans, cocoa, heating oil, gold, live cattle, S&P 500, US Dollar/Pound and US Dollar/Yen. Both daily closing spot and futures contract settlement prices spanning the period January 1995–April 2005 are obtained from DataStream yielding 2582 observations per contract. The futures prices are those from the nearest contract and contracts are rolled over to the next contract on the first business day of the contract month. Data for the period January 1, 1995 to April 6, 2004 (2332 observations) are used for in-sample analysis. Out-of-sample analysis is carried out using the remaining data in the sample for the period April 7, 2004 to April 5, 2005 (250 observations).

As a preliminary test we first employ the augmented Dickey–Fuller (ADF) test to assess the order of integration of spot and futures prices and the basis. The results indicate the presence of one unit root in both spot and futures price series, but a stationary or \( I(0) \) process in the basis for all assets.

Next we employ the semi-parametric GPH procedure to estimate the fractional differencing parameter for the basis. This method requires choosing a truncation parameter \( m \) to determine the number of Fourier frequencies to be considered. Geweke and Porter-Hudak (1983) suggest the use of \( m = n^{0.5} \), where \( n \) is the number of observations. This is the typical truncation parameter value used in the literature and we follow this convention. The GPH procedure is implemented using differenced data, thus the resulting estimates of \( d \) are increased by 1. If \( d < 0.5 \) then \( d \) was reestimated using data in levels. Also note that \( l \) is set equal to zero in (2) indicating no trimming of the harmonic frequencies (Kellard and Sarantis, 2008). The computations are performed using Ox version 3.4 (see Doornik, 1999). The results are reported in Table 1.

Contrary to the ADF results, \( t \)-tests for all assets reject the no long memory and unit root hypotheses of \( d = 0 \) and 1, respectively. They indicate the basis is neither \( I(1) \) nor \( I(0) \) but instead is better characterised by long memory behaviour with \( 0 < d < 1 \). We also undertake a parametric test for long memory using ARFIMA \((1, d, 1)\) models to check the GPH results for the basis. The results in Table 2 once again confirm that, for all assets, the basis is a fractionally integrated process.

The implication of this long memory finding in the basis may challenge the extant hedging literature since most researchers rely on the assumption that the basis is stationary. Lien (2005) points out that if the model is misspecified, it is difficult to measure hedge effectiveness through residual portfolio variances. It is therefore important to examine the impact of allowing for long memory in the basis when OHRs are estimated.

Table 1
GPH tests for the basis \((f_t - s_t)\)

<table>
<thead>
<tr>
<th></th>
<th>Soybeans</th>
<th>Cocoa</th>
<th>Heating oil</th>
<th>Gold</th>
<th>Live cattle</th>
<th>S&amp;P 500</th>
<th>S/Pound</th>
<th>$/Yen</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d_{GPH})</td>
<td>0.481</td>
<td>0.700</td>
<td>0.226</td>
<td>0.178</td>
<td>0.784</td>
<td>0.659</td>
<td>0.437</td>
<td>0.584</td>
</tr>
<tr>
<td>((m = n^{0.5}))</td>
<td>(0.106)</td>
<td>(0.106)</td>
<td>(0.106)</td>
<td>(0.106)</td>
<td>(0.106)</td>
<td>(0.106)</td>
<td>(0.106)</td>
<td>(0.106)</td>
</tr>
<tr>
<td>(\tau_d = 1)</td>
<td>-4.92</td>
<td>-2.84</td>
<td>-7.34</td>
<td>-7.79</td>
<td>-2.05</td>
<td>-3.23</td>
<td>5.33</td>
<td>3.95</td>
</tr>
<tr>
<td>(\tau_d = 0)</td>
<td>4.55</td>
<td>6.64</td>
<td>2.14</td>
<td>1.69</td>
<td>7.43</td>
<td>6.25</td>
<td>4.14</td>
<td>5.53</td>
</tr>
</tbody>
</table>

Note: Numbers in parentheses below the estimates for \(d_{GPH}\) are standard errors \((\sigma_d)\).
\(\tau_d = 1\) row reports the test statistic \((d_{GPH} - 1)/\sigma_d\).
\(\tau_d = 0\) row reports the test statistic \((d_{GPH} - 0)/\sigma_d\).

Table 2
ARFIMA \((1, d, 1)\) tests for the basis \((f_t - s_t)\)

<table>
<thead>
<tr>
<th></th>
<th>Soybeans</th>
<th>Cocoa</th>
<th>Heating oil</th>
<th>Gold</th>
<th>Live cattle</th>
<th>S&amp;P 500</th>
<th>S/Pound</th>
<th>$/Yen</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d_{ARFIMA})</td>
<td>0.506</td>
<td>0.696</td>
<td>0.413</td>
<td>0.101</td>
<td>0.794</td>
<td>0.562</td>
<td>0.443</td>
<td>0.645</td>
</tr>
<tr>
<td>((0.110))</td>
<td>(0.034)</td>
<td>(0.053)</td>
<td>(0.024)</td>
<td>(0.045)</td>
<td>(0.025)</td>
<td>(0.045)</td>
<td>(0.049)</td>
<td></td>
</tr>
<tr>
<td>(\tau_d = 1)</td>
<td>-4.51</td>
<td>-8.86</td>
<td>-10.98</td>
<td>-32.4</td>
<td>-4.53</td>
<td>-13.19</td>
<td>12.42</td>
<td>7.31</td>
</tr>
<tr>
<td>(\tau_d = 0)</td>
<td>4.62</td>
<td>20.31</td>
<td>7.73</td>
<td>3.62</td>
<td>17.46</td>
<td>16.95</td>
<td>9.88</td>
<td>13.29</td>
</tr>
</tbody>
</table>

Note: Numbers in parentheses below the estimates for \(d_{ARFIMA}\) are standard errors \((\sigma_d)\).
\(\tau_d = 1\) row reports the test statistic \((d_{ARFIMA} - 1)/\sigma_d\).
\(\tau_d = 0\) row reports the test statistic \((d_{ARFIMA} - 0)/\sigma_d\).
Table 3

<table>
<thead>
<tr>
<th>Asset</th>
<th>OLS</th>
<th>Cocoa</th>
<th>Heating oil</th>
<th>Gold</th>
<th>Live cattle</th>
<th>S&amp;P 500</th>
<th>$/Pound</th>
<th>$/Yen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soybeans</td>
<td>0.891</td>
<td>0.865</td>
<td>0.909</td>
<td>0.490</td>
<td>0.213</td>
<td>0.923</td>
<td>0.710</td>
<td>0.842</td>
</tr>
<tr>
<td>FIEC (n0.5)</td>
<td>0.890</td>
<td>0.865</td>
<td>0.901</td>
<td>0.552</td>
<td>0.200</td>
<td>0.927</td>
<td>0.732</td>
<td>0.853</td>
</tr>
</tbody>
</table>

Next we apply the FIEC model following Lien and Tse (1999). The estimated $d$ obtained from the GPH technique is used in the FIEC model in calculating the fractional cointegration variable $z^*_t = [(1 - L)^d - (1 - L)]z_t$ (Eqs. (5) and (6)). For instance, as the estimated $d$ for Soybeans is 0.481, the fractional cointegration variable can be defined as $z^*_t = [(1 - L)^{0.481} - (1 - L)]z_t$. We have employed the AIC criterion to choose the optimal number of lags $k$ for our models. To conserve space, the parsimonious mean equations are not reported here. However, it is worth noting that the fractional cointegration variable is significant in the estimation of (5) and (6) only for Gold and the $$/Pound. For all other assets, either spot price or futures price is independent of the fractional cointegration variable. Lien and Tse (1999) suggest that such independence provides a rationale for why OHRs generated by FIEC models may not differ that much from other models in practice. Specifically, if $\zeta_s$ and/or $\zeta_f$ are zero in (5) and (6), respectively, it can be clearly seen that the FIEC OHR of (7) tends to the OLS OHR of (3). As a corollary then, only Gold and the $$/Pound are expected to have markedly different optimal hedge ratios generated by the different methodologies. Table 3 displays the OLS and FIEC constant OHR estimates.

The results provide new empirical support for the Lien and Tse proposition. The differential in magnitude between the FIEC and OLS OHRs is largest for the $$/Pound and particularly for Gold while it is small or nonexistent for the other assets. It seems that the significance of the fractional cointegration variable is a key determinant in OHR size.

4. Out-of-sample hedging effectiveness

Long memory in the basis can clearly affect OHR estimation under particular empirical conditions. The question then is whether it also impacts on out-of-sample hedging effectiveness. In particular, can the FIEC-BEKK hedging approach improve on the OLS method in terms of either statistical or economic significance? Given the results of the previous section, we focus our comparison on Gold and the $$/Pound.

Hedging performance is typically assessed on the out-of-sample percentage variance reduction of the hedge portfolio relative to the unhedged position. The variance of the estimated optimal hedged portfolio can be characterised as

$$Var(\Delta s_t - \hat{b}^* \Delta f_t),$$

where $\hat{b}^*$ is the computed hedge ratio for each hedge method. As Lien and Tse (1999) argue that the performance of the models may vary according to the hedge horizon, we follow their paper and consider out-of-sample hedge horizons of 5, 10, 20 and 40 days. The results are summarised in Table 4.

The FIEC-BEKK approach exhibits better out-of-sample performance than OLS across all hedging horizons. For example, for the Gold futures contract, the FIEC-BEKK has a 55.44% out-of-sample variance reduction for a 20-day hedge horizon as compared with a 52.82% variance reduction for OLS. The FIEC-BEKK model reduces the 10-day variance for the $$/Pound futures contract by 75.33% as against 74.75% for OLS. However, are these differences statistically significant?

The statistical significance of any hedging improvement is rarely considered in the extant literature. To rectify this, a wild bootstrap is applied to assess the statistical significance of the out-of-sample hedging performance difference between the FIEC-BEKK and OLS models. The wild bootstrap allows for inference which is robust to non-normality and heteroscedasticity of unknown form. It is widely used in finance applications (see Clatworthy et al., 2007). The wild bootstrap samples from a two-point distribution and multiplies this random variable with a rescaled version of the residual. This procedure is repeated 10,000 times and a new estimate of the return variance is found for each bootstrapped series.

Table 5 reports the means and standard deviation for the out-of-sample series. For Gold and the $$/Pound, the mean variances are close to the actual sample in Table 4 and all the $t$-statistics are significant, except for the 5-day hedge.
Table 4
Out-of-sample hedging effectivenessa

<table>
<thead>
<tr>
<th></th>
<th>Gold</th>
<th>S/Pound</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Varianceb</td>
<td>Variancec</td>
</tr>
<tr>
<td>5-day</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>0.3255</td>
<td>52.8287</td>
</tr>
<tr>
<td>FIEC-BEKK</td>
<td>0.3226</td>
<td>53.2469</td>
</tr>
<tr>
<td>10-day</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>0.3256</td>
<td>52.8137</td>
</tr>
<tr>
<td>FIEC-BEKK</td>
<td>0.3208</td>
<td>53.5125</td>
</tr>
<tr>
<td>20-day</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>0.3256</td>
<td>52.8178</td>
</tr>
<tr>
<td>FIEC-BEKK</td>
<td>0.3075</td>
<td>54.4443</td>
</tr>
<tr>
<td>40-day</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>0.3259</td>
<td>52.7741</td>
</tr>
<tr>
<td>FIEC-BEKK</td>
<td>0.3132</td>
<td>54.6195</td>
</tr>
</tbody>
</table>

bVariance of the hedged portfolio in Eq. (10).
cPercentage variance reduction is calculated using the formula: $1 - \frac{\text{Var}(\lambda_{t+1} - b_t \lambda_t)}{\text{Var}(\lambda_t)}$.
dDaily utility is calculated based on Eq. (11) and a coefficient of risk aversion of 4 and a zero expected return from the hedged portfolio.
eUtility improvement is the increase/decrease in daily utility by using the FIEC-BEKK hedging strategy relative to the static OLS strategy.

Table 5
Mean and standard deviation of the bootstrapped unconditional variance distributiona

<table>
<thead>
<tr>
<th></th>
<th>Gold</th>
<th>S/Pound</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Meanb (MV)</td>
<td>Standard deviation (SD)</td>
</tr>
<tr>
<td>5-day</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>0.3229</td>
<td>0.0018</td>
</tr>
<tr>
<td>FIEC-BEKK</td>
<td>0.3201</td>
<td>0.0018</td>
</tr>
<tr>
<td>10-day</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>0.3230</td>
<td>0.0018</td>
</tr>
<tr>
<td>FIEC-BEKK</td>
<td>0.3183</td>
<td>0.0018</td>
</tr>
<tr>
<td>20-day</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>0.3230</td>
<td>0.0018</td>
</tr>
<tr>
<td>FIEC-BEKK</td>
<td>0.3107</td>
<td>0.0017</td>
</tr>
</tbody>
</table>

bIn the wild bootstrap, the Rademacher 2-point distribution is used with 10,000 replications.
c$t$-Statistics are calculated using the formula: $\frac{\text{MV}(\text{FIEC-BEKK}) - \text{MV}(\text{OLS})}{\text{SD}(\text{FIEC-BEKK})}$.

case with Gold. This demonstrates that the FIEC-BEKK hedging strategy is statistically superior to the OLS hedging method.

While the FIEC-BEKK procedure is statistically superior, it does not follow that it is also superior in economic terms. Implementation of such dynamic hedging strategies is patently more costly since it requires frequent updating and rebalancing of the hedged portfolio. Therefore, we use the hedger’s utility function to evaluate the economic benefits of the competing hedging methods. Consider an investor with a mean-variance expected utility function:

$$E_t U(x_{t+1}) = E(x_{t+1}) - \lambda \sigma_t^2(x_{t+1}),$$

(11)
where \( \lambda \) is the degree of risk aversion and \( x_{t+1} \) the hedged portfolio return. Following other empirical studies (Park and Switzer, 1995; Lee et al., 2006), we assume \( \lambda \) takes the value of 4 and the hedged portfolio expected return is zero. Table 4 reports the utility improvements from the FIEC-BEKK and the static OLS hedging methods. For example, with the 20-day Gold hedge, the variance of the hedged portfolio returns for OLS and FIEC-BEKK is 0.3256 and 0.3075, respectively. If an investor uses the OLS method for hedging, he/she obtains an out-of-sample utility of \( U(x_{t+1}) = -4(0.3256) = -1.3024 \). Similarly, with the FIEC-BEKK method, he/she obtains \( U(x_{t+1}) = -4(0.3075) = -1.23 \).

The net benefit from using FIEC-BEKK over OLS hedging for the 20-day Gold contract is 0.0724 \( \pm y \), where \( y \) represents the transaction cost from portfolio rebalancing. Therefore, the FIEC-BEKK strategy will be preferred if \( y \leq 7.24\% \). Note that the price of one gold contract with the price of $414.63 per ounce is $41463 ($414.63 * 100oz/contract). As a typical round-trip commission is $10--$20, the transaction cost will be around 0.02--0.05%. Clearly, with a typical round-trip transaction cost of around 0.02--0.05%, the FIEC-BEKK hedge results in a utility improvement in this case for an investor with a mean-variance utility function and \( \lambda = 4 \), even after accounting for transaction costs. The utility improvement from the FIEC-BEKK over the OLS hedging method for the S/Pound futures contract is smaller than Gold even if the round-trip transaction cost of around 0.008--0.017% (the price of a S/Pound contract at $1.8379 per pound is $114,870, i.e., $1.8379*62,500 pounds/contract) is smaller in this case.

5. Conclusions

The long memory properties of the spot-futures basis and its impact on optimal hedge ratio issues have not been systematically investigated by the extant literature. Employing a daily data set 1995--2005 that includes commodities, foreign exchange and a stock index, the results suggest the basis is a fractionally integrated process. A joint fractionally integrated, error-correction and multivariate GARCH (FIEC-BEKK) approach is implemented to capture this feature of the data plus time-varying conditional heteroscedasticity.

Our results indicate that the FIEC-BEKK hedging strategy can outperform the OLS benchmark in terms of risk reduction and hedger utility across a range of out-of-sample horizons of up to 40 days over a period of one year. This is an unusual finding insofar as the OLS benchmark is rarely beaten in the hedging literature. However, this result is conditional on both the spot and futures returns being dependent on the error-correction term which applies for just two of our eight sample assets, Gold and the S/Pound. A bootstrap exercise indicates that the FIEC-BEKK hedging strategy is indeed statistically superior to OLS for these two assets. Long memory matters in hedging effectiveness, conditionally!

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